Critical Thinking: 
The Effects of Difficult Problems on the Workshop Experience

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1 Introduction

“Critical Thinking” is a term with various and broad definitions (Paul et al. 1997). However, when used to describe a person’s thought process while problem-solving, the term often implies that the person is utilizing the cognitive strategy known as metacognition (van Gelder, 1997; Willingham, 2007). A person is practicing metacognition if they are thinking about thinking (Willingham, 2007). Most experts agree that critical thinking is an essential skill that is very difficult to teach (van Gelder, 1997; Willingham, 2007).

For our research project we wanted to investigate the effects that the inclusion of a critical thinking problem would have on the workshop experience, so we asked the question, “To what extent does replacing one of the regular workshop questions with a single critical thinking question affect the workshop experience?” We hypothesized that the students would enjoy the change and would see the critical thinking questions as having a positive impact on the workshop experience. More specifically, we expected the student response to be that they found the inclusion to be worth the trade-off, that the critical thinking problems improved the workshop experience, that the inclusion helped them to understand the material better, that it made them think differently, and that they found the questions to be of an appropriate level of difficulty.
2 Method

To start our investigation, we began searching for and creating critical thinking problems. We considered three criteria when choosing problems:

- The problem must encourage metacognition.
- The problem must be of an appropriate level of difficulty.
- The problem must be computer science related.

We used the first criterion to ensure that the problem involved critical thinking. We chose the second criterion to ensure that the students could approach the problem while still being challenged, and we chose the third criterion because we were leading a computer science workshop. We ended up including four different critical thinking problems in four separate workshops.

Each of the four problems that we ended up using fell into one of three categories: problems that we wrote entirely on our own (see Figure 1), problems that we modified to be of an appropriate level of difficulty, and problems that we took verbatim from Project Euler (see Figure 2). (Project Euler is a website that compiles interesting and challenging programming questions.) The problem appearing in Figure 2 is an excellent example of a problem that satisfies our three criteria: firstly, the problem is computer science related because it requires programming to be solved; secondly, the problem is of an appropriate level of difficulty because its solution (see Figure 3) requires knowledge of only introductory computer science material; thirdly, the intuitive and often initial route taken when solving the problem results in a lengthy runtime (the program takes a significant amount of time to execute), but if students stop and recognize that their initial line of thinking is not working then they can create a minor modification to the code and decrease the runtime by a factor of 1000.

After we had completed the critical thinking problem in our final workshop, we asked the students to fill out a survey about what effect they felt the inclusion of the critical thinking problems had on the workshop experience. A picture of the survey appears below in Figure 4.
Collatz Conjecture

The Collatz Conjecture was first proposed by Lothar Collatz in 1937. It states

**Proposition 1** (Collatz Conjecture). Take any natural number \( n \). If \( n \) is even, divide it by 2, and if \( n \) is odd, multiply it by 3 and add 1. If this procedure is repeated, it will always result in 1.

As an example, take 13. First we multiply 9 by 3 and add 1 to get 40. 40 is even, so we divide it by 2 to get 20. 20 is even, so we divide it by 2 to get 10, and so on. Continuing on this way we see that the sequence of numbers is

\[
13 \rightarrow 40 \rightarrow 20 \rightarrow 10 \rightarrow 5 \rightarrow 16 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1
\]

(a) Show that the Collatz Conjecture holds for all \( 1 \leq n \leq 1000 \).

(b) Modify your method to print out how many steps each number requires to reach 1. In the example above, 9 took 19 steps.

(c) Which number requires the most steps?

Figure 1: A critical thinking question about the Collatz Conjecture that we wrote entirely on our own.

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**Integer right triangles**

**Problem 39**

14 March 2003

If \( p \) is the perimeter of a right angle triangle with integral length sides, \([a,b,c]\), there are exactly three solutions for \( p = 120 \).

\( (20,48,52), (24,45,51), (30,40,50) \)

For which value of \( p \leq 1000 \), is the number of solutions maximised?

Figure 2: A critical thinking question taken from Project Euler (Question 39, 2003).
public class Q39
{
    public static void main(String[] args)
    {
        int a,b,c;
        int[] p = new int[1001];
        for(a = 0;a<1000;a++)
        {
            for(b = 0;b<1000;b++)
            {
                for(c=0;c<1000;c++)
                {
                    if((Math.pow(a,2)+Math.pow(b,2) == Math.pow(c,2))&&((a+b+c)<=1000))
                    {
                        //System.out.println(a+ " + b + " +c);
                        p[a+b+c]= p[a+b+c]+ 1;
                    }
                }
            }
            int index = 0;
            int max = p[1];
            for(int i=1; i<1001; i++)
            {
                if(p[i]>max)
                {
                    index = i;
                    max = p[i];
                }
            }
        System.out.println(index);
    }
}
Critical Thinking Research Survey

Thank you for taking the time to fill out this survey for our research project. Please circle your opinion on the following statements and give comments for the last two questions. Feel free to leave any additional comments on the back of this sheet.

The critical thinking questions made the workshop better.
Strongly Disagree  Disagree  Neither Agree nor Disagree  Agree  Strongly Agree  N/A

The critical thinking questions helped me understand the course material better.
Strongly Disagree  Disagree  Neither Agree nor Disagree  Agree  Strongly Agree  N/A

The critical thinking questions were too difficult
Strongly Disagree  Disagree  Neither Agree nor Disagree  Agree  Strongly Agree  N/A

The critical thinking questions were frustrating
Strongly Disagree  Disagree  Neither Agree nor Disagree  Agree  Strongly Agree  N/A

The critical thinking questions were within my capability to solve
Strongly Disagree  Disagree  Neither Agree nor Disagree  Agree  Strongly Agree  N/A

The critical thinking questions forced me to think in a different way
Strongly Disagree  Disagree  Neither Agree nor Disagree  Agree  Strongly Agree  N/A

Approaching the questions with another person made them easier
Strongly Disagree  Disagree  Neither Agree nor Disagree  Agree  Strongly Agree  N/A

Replacing one or two of the workshop questions with a critical thinking question was a good tradeoff
Strongly Disagree  Disagree  Neither Agree nor Disagree  Agree  Strongly Agree  N/A

In what ways was this effective?
In what ways could this have been improved?

Figure 4: Survey given to students after final workshop.

3 Results

We were pleased to see that the responses to the survey mostly matched our initial expectations/hypotheses (see Figure 5). In response to the open ended questions at the end of the survey, students said that the critical thinking problems “encouraged [them] to think differently,” “encouraged group problem solving,” “helped keep the workshop interesting,” and “improved [their] problem-solving skills.” One of the main critiques that the students offered was that some of the
problems were not relevant to the current workshop/course material.

![Figure 5: A graph of the average responses to the survey.](image)

4 Conclusion

Based on the responses to the survey, we believe our hypotheses to be true. Also, we would recommend including critical thinking problems in future workshops. However, if we were to continue including critical thinking problems, we would make a greater effort to ensure that the problems were relevant to the current course material.

5 References


