Essays in Macroeconomics: Environments with Informational and Financial Frictions.

by

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To Nahil for her enormous love and patience.
Curriculum Vitae

The author was born in Caracas, Venezuela on March 5, 1975. He attended Universidad Central de Venezuela from 1992 to 1997, and graduated with a Bachelor degree in Economics. After graduating with a Master of Science in Operational Research at Universidad Central de Venezuela in 2001, he came to the University of Rochester in the Fall of 2003 and began graduate studies in Economics. He received fellowships from the Department of Economics at the University of Rochester and from the Banco Central de Venezuela. In 2005, he received a Master of Arts degree from the University of Rochester and went on to pursue his research in Macroeconomic Dynamics under the direction of Professor Árpád Ábrahám.
Acknowledgments

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Abstract

Frictions may crucially influence the behavior of economies and policy design. As a consequence, using macroeconomic models that explicitly incorporate different types of frictions has become popular. In this dissertation, we move in that direction by incorporating either informational or financial frictions into different environments and analyzing their main implications. In the context of informational frictions, we focus on repeated moral hazard problems emerging from the non observability of agents’ actions. With respect to financial frictions, we focus on agents’ limited access to capital markets.

In the first chapter, we study how the presence of moral hazard in labor contracts shapes the wage distribution in a search environment with on-the-job offers. Since moral hazard induces residual wage dispersion as a mechanism to encourage workers’ effort (incentives effect), one would intuitively argue that it unambiguously leads to more wage inequality. However, this reasoning overlooks the fact that the presence of moral hazard affects the worker’s continuation values resulting from job offers, as well as the effort schedule. These changes are such that the wage profile associated with job offers shifts down and becomes flatter across productivity levels and educational levels which leads to less wage dispersion (wage offers effect). We calibrate the model to the U.S economy. The main result is that the wage offers effect more than offsets the incentives effect. In fact, eliminating the information friction increases wage dispersion by approximately 7%.

In the second chapter, we consider the problem of unemployment insurance in
a repeated moral hazard framework. Unlike the existing literature, in our environment, unemployed workers can secretly participate in a hidden labor market. This extension substantially modifies the properties of the optimal payment schedule. In particular, for workers with relatively low continuation values, the payment profile becomes flatter in order to keep them out of the hidden labor market. However, for sufficiently large unemployment spells, payments suddenly drop to zero and participation in the hidden labor market jumps. Altogether, the optimal unemployment insurance system in an economy with a hidden labor market is simple, it has a relatively flat phase and prescribes no payments for long term unemployed workers. These features resemble those present in the unemployment insurance programs of many countries.

In the third chapter, we explore to what extent firms’ indirect access to international funds shapes the business cycle of a small open economy. In particular, we model a firm dynamics environment in which firms can only obtain funds through a domestic intermediary and sovereign default is costly. In our environment, each firm’s interest rate depends on the whole structure of the corporate sector, and not exclusively on idiosyncratic factors as standard models of firm dynamics suggest. This interdependency introduces spill over effects among firms that work as an amplification/propagation mechanism, reinforcing the negative correlation between the sovereign interest rate and output, and increasing the interest rate volatility and persistence. We also find that the impact of these spill over effects on aggregate output is partially offset by precautionary capital formation of mature firms.
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Foreword

This dissertation compiles three papers developed during my doctoral studies at the University of Rochester. Each chapter corresponds to one paper. The second chapter is a joint work with Juan Sanchez.
Chapter 1


1.1 Introduction

There is substantial evidence that pay-by-performance is a common practice. For instance, MacLeod and Malcomson (1997) report that 23% of young workers in the U.S., and 34% of workers in the U.K., received some kind of performance-related pay in 1990.\(^1\) It is well known that this practice may emerge as an optimal response to a moral hazard problem in labor contracts. As the literature establishes, moral hazard reduces risk sharing; as a consequence, in the context of labor contracts, it should be considered a source of wage dispersion. Nevertheless, to the best of my knowledge, there is no previous work investigating the impact of this informational friction on the wage profile in an environment that incorporates realistic dynamics of the labor market in particular, the high degree of job mobility.

This paper fills that gap by studying how the presence of moral hazard shapes the cross-sectional wage distribution in a search environment with endogenous worker effort and firm competition over workers leading to job-to-job transitions.

\(^1\)Moreover, they stress that the fraction of workers rewarded on good performance is higher once one takes into account promotions based on historical performance.
A moral hazard problem in labor contracts arises naturally. If the worker is risk adverse, a state-independent wage is the cheapest way to deliver a certain level of utility. However, if the worker’s wage is independent of the output realization, the worker has no incentive to exert any level of effort. That is, if the firm is not able to monitor the worker, there exists a conflict between the provision of incentives and the provision of insurance. The optimal balance in this conflict implies that the provision of insurance in firm-worker relationships is smaller under the informational friction.

Firm competition also emerges naturally in an environment with on-the-job search. I assume that whether an employed worker has an outside job offer is private information; once the worker reveals it, the incumbent and the new potential employer compete over the worker. Firm competition is modeled similar to Postel-Vinay and Robin (2002) in which the current employer and the relevant competitor engage in a second price auction for the worker. Clearly, the worker reveals that she has gotten an offer only if she benefits from triggering a firm competition. Notice that firm competition may lead to wages variation due to job-offer arrival even in the absence of job switching.

This environment combines two important and interconnected sources of residual wage dispersion: search frictions and an informational friction in the form of moral hazard. In this context, wages are not set according to worker productivity but instead as a solution to an optimal contractual problem that balances the provision of incentives and insurance, and that takes into account that employed workers may obtain job offers triggering firm bargaining.

How does wage dispersion arise among ex ante identical workers in this framework? On the one hand, the search frictions induce residual wage dispersion emerging from different histories of on-the-job offers and different transitions from/to unemployment. For instance, the presence of search frictions introduces a wage differential between those lucky workers who get on-the-job offers and those who do not. Moreover, since all job offers are not alike, search frictions also introduce wage differentials among those workers getting on-the-job offers. On the other hand, moral hazard induces wage dispersion among identical workers because it reduces the firm’s potential for risk sharing, making the history of
productivity shocks relevant for the wage sequence of the worker. More precisely, under lack of observability of worker effort, the optimal contract prescribes wage dispersion across realizations of output, as a mechanism to encourage the desired level of effort. I refer to this mechanism as the _incentive effect_. In contrast, if the firm observes worker effort, it would provide full insurance to her with respect to uncertainty in a productivity factor that stochastically depends on worker effort.

Based on the incentive effect, one would intuitively argue that moral hazard unambiguously leads to more wage dispersion. However, this reasoning overlooks the fact that the presence of moral hazard affects both (1) the lifetime utility the worker gets when triggering firm competition and (2) the effort schedule. These changes modify the wages profile, in particular the wage schedule associated with job offers. I refer to this mechanism as the _wage offer effect_. The main contribution of this paper is to identify this mechanism and quantify its importance in a framework calibrated to the U.S. economy.

More precisely, under moral hazard workers get smaller lifetime utilities from firm competition than in the observable effort case.\(^2\) This shifts down the wage profile associated with job offers. Moreover, in the calibration, it is the case that the reduction in these lifetime utilities is similar across outside competitor types. Henceforth, all other things being equal, the concavity of utility with respect to consumption implies that the reduction in the wage profile is greater for workers getting offers from more productive firms; that is, the wage schedule associated with job offers becomes flatter.

Moral hazard also reduces the effort level required by the firm for any promised utility. This is because under moral hazard, it is more expensive to require a certain level of effort given that incentives have to be provided through wage dispersion. This force also shifts down the wage profile associated to job offers. Moreover, the reduction in effort levels caused by the presence of moral hazard is greater for more educated workers and for workers getting offers from more productive firms.\(^3\) This non-parallel shifting reinforces the flattening in the wage profile.

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\(^2\)Notice that moral hazard reduces the value of the firm for any firm-worker match and hence, reduces the highest lifetime utility a given firm is willing to offer to a given worker. In the environment, when firm competition is triggered, the worker gets the highest lifetime utility the less productive firm is willing to offer.

\(^3\)Notice that firm competition implies that well educated workers and workers having offers
schedule associated with job offers.

The wage offer effect leads to less wage dispersion for the following reasons. First, it reduces the wage differential between workers who have gotten an on-the-job offer and those who have not. Second, it compresses the wage structure among workers getting on-the-job offers. Finally, it reduces the wage differential across educational groups.

In sum, the presence of moral hazard affects the wage structure of the economy through two main channels. The first one is the standard mechanism associated with the provision of incentives, which suggests that moral hazard leads to higher wage dispersion. The second one emerges from the interaction between the informational friction and the search frictions. In particular, the moral hazard problem affects the wage schedule associated with job offers in such a way that mitigates the history of job offer arrivals as a mechanism to generate wage dispersion. In as much as these two forces affect the wage dispersion in opposite directions, how moral hazard shapes the wage structure is a nontrivial question that requires a quantitative framework to be addressed.

The main result of this paper is that, in the benchmark calibration of the environment, the existence of moral hazard actually reduces wage inequality. This result challenges the intuitive argument—based exclusively on the incentive effect—that moral hazard leads to more wage dispersion. The explanation of this results relies on the quantitative importance of the wage offer effect that more than offsets the effect of the incentive mechanism. More precisely, the net effect of eliminating moral hazard from the benchmark calibration is approximately 7 percent; in other words, wage dispersion is 7% greater when effort is observable! Since the incentive effect alone implies more wage dispersion, I can interpret this level as a lower bound for the magnitude of the wage offer effect.4

This result suggests that improvement in the technology to monitor work-

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4I found that in the benchmark calibration the incentive effect is quantitatively small. This is due to the high job finding rate associated with the calibrated economy together with the fact that a relevant offer makes the history of productivity shock irrelevant for wage determination.
ers may lead to higher wage dispersion; and henceforth, can be considered as a potential driving force of the increase in wage dispersion experienced in recent decades.\textsuperscript{5}

The quantitative analysis also suggests that endogenous changes in effort from removing the informational friction seem to be relatively more important than changes in promised utilities, but both play a role as driving forces behind the wage offer effect.

\section*{Related Literature}

This paper parallels Manoli and Sannikov (2005), whose environment also features job mobility and moral hazard. However the two papers differ in multiple aspects. In their environment, the incumbent’s bidding strategy is itself a device to provide incentives. While it is ex-ante efficient, ex-post it leads to inefficient outcomes. As a matter of fact, the incumbent may let the worker switch to a less productive firm even if, ex-post, it is profitable for the incumbent to bid and keep the worker. More importantly, the incumbent’s bidding strategy involves situations in which the worker is worse-off after getting an outside offer. Such a strategy is not implementable in an environment in which whether the worker gets an offer is private information. In contrast, in the framework presented here, a worker never switches to a less productive firm and is never hurt from getting an offer. The two papers also differ in the main research question. In particular, Manoli and Sannikov (2005) focus on job dynamics instead of wage dispersion. Thus, the two papers complement each other. Finally, they provide an analytical characterization of the bidding strategy and the job dynamic in a continuous time environment, whereas I use a discrete time environment to study some properties the job dynamics and some properties of the optimal contract. In addition, I provide a quantitative framework that is used to address a question that is quantitative in nature. To the best of my knowledge, this is the first quantitative

\textsuperscript{5}The increase in wage dispersion has been well documented in the literature, see for instance Golding and Katz (2007) for the evolution of educational wage differential and Bosworth et al. (2001) for the evolution of wage differential between workers in the top and workers in the bottom of the wage distribution. Finally Katz and Autor (1999) stress that residual wage dispersion have also significantly contributed to the increase in overall wage dispersion.
framework studying the wage structure in an environment that combines moral hazard and job mobility.

This paper also adds to the quantitative macroeconomic work exploring the sources of wage inequality. A recent example is Hugget et al. (2006), in which a model of risky human capital formation is used to study the sources of lifetime inequality. They found that initial conditions—initial human capital and ability to learn—are more important in determining lifetime differences in wealth than differences in the history of shocks during the lifetime. I depart from their work in several aspects. For instance, instead of using a spot market in which the worker sells her labor, I explicitly model the worker-firm relationship as a long-term contractual arrangement, facing informational and search frictions as well as a commitment problem emerging from the fact that the worker can either quit or accept an outside offer.

This paper also relates with Postel-Vinay and Robin (2002). One of their contributions is to provide a variance decomposition that allows them to measure the importance of search friction in the cross-sectional wage dispersion. They quantify its contribution to around 50% in a French data-set. As I have argued, the importance of search frictions—leading to heterogeneity in the history of job offers—as a mechanism to introduce wage dispersion is crucially affected by the importance of the moral hazard problem through the wage offer effect. I adopt firm competition over workers in a similar fashion; however, my framework brings informational frictions in the form of moral hazard into the firm-worker relationship. Moreover, I introduce endogenous worker effort in the labor relationship, which is not present in traditional search environments, but plays an important role in the determination of wages.

Finally, this paper relates to the growing literature of applied dynamics contracts. An early application of the dynamic contract approach to labor market issues is found in Thomas and Worral (1988). They show that in the optimal self-enforcing contract, wages are less variable than in a competitive spot market, which is more consistent with the stylized facts on wage formation. However, they do not evaluate how moral hazard interacts with firm competition over workers to shape the wage structure.
1.2 The Model

1.2.1 The environment

The economy is populated by a large number of risk averse workers, $I_w$, and a large number of risk neutral firms, $I_f$. When a firm is born, it is matched with a worker who can be either unemployed or employed. If the firm does not hire the worker, then it dies. Firms differ in time invariant productivity level, $z$, and each firm employs only one worker; as a consequence, the number of firms, $I_f$, is equivalent to the employed population, and $I_w - I_f \geq 0$ is the number of unemployed workers.

While unemployed, the worker obtains an unemployment benefit, $b$, and receives a job offer with probability $\pi^u$. The job offer comes from a $z$-type newly born firm with probability $\Pi(z)$.

While employed, the worker gets a job offer from a newly born firm with probability $\pi^s$. The firm type is drawn from the distribution $\Pi(z)$.

Whether the employed worker has an outside offer is non-observable for the incumbent.

When a worker—either employed or unemployed—and a newly born firm are matched, they engage in a long term contract. The firm commits to the contract, however the worker can either quit or move to another job. Firm-worker contracts end exogenously with probability $\lambda$. In this case the worker becomes unemployed and the firm dies. The firm-worker relationship can also end endogenously due to quitting or to a worker’s job-to-job transitions. In any case, an endogenous separation also implies that the firm disappears.

Workers die with probability $(1 - \psi)$ and they differ from each other in their human capital level, $h$. When a worker dies, she is replaced by an unemployed worker with the same human capital such that the population remains constant at $I_w$. I assume that the level of human capital is invariant across the lifespan of the worker.

A $z$-type firm manages the following technology

\begin{equation}
Y(A, z, h),
\end{equation}

---

Notice that both employed and unemployed workers draw job offers from the same distribution for firm types. Since existing firms do not make offers, job offers come exclusively from newly born firms. Hence, $\Pi(z)$ represents the probability that a $z$-type firm is born.
where $A$ represents a productivity factor depending on the worker’s *non-observable* effort, $e$, as follow:

$$A = \begin{cases} 
    A^- & \text{with probability } 1 - \pi(e), \\
    A^+ & \text{with probability } \pi(e).
\end{cases} \tag{1.2}$$

I assume that $A^+ > A^-$ and that $\pi$ is a strictly increasing and strictly concave function, while effort is assumed to belong to the compact set $\mathbb{E} \equiv [0, \pi]$. Lack of observability of worker effort and outside offer arrivals are the only informational frictions in the economy.

Different attitudes toward risk between the workers and the firm together with the non-observability of effort, introduces a standard moral hazard problem: the firm would like to fully insure the workers against uncertainty in $A$, but this would lead to too little worker effort. The firm’s contract is designed in order to optimally balance the tradeoff between insurance and incentives.

Instantaneous workers utility is represented by

$$u(c) - \gamma(e), \tag{1.3}$$

where $u$ is a strictly increasing, strictly concave, continuous, and bounded above function representing utility from consumption. The function $\gamma$ represents the disutility due to effort and it is strictly increasing, convex, and continuous. Then, $u(c) - \gamma(e)$ is also bounded above.

Time is discrete and it is assumed that the workers and the firms discount the future at a common rate $\beta = \frac{1}{1+r}$. As is common in the repeated moral hazard literature, I restrict workers from asset accumulation. I believe that this assumption—equivalent to full observability in assets— does not change my main results. First, Abraham and Pavoni (2006) show that with hidden assets there exists scope for a partial risk sharing as in the model with full asset observability. More importantly, my main result relies on the quantitative importance of the wage offer effect, which may be reinforced by lack of observability of assets.\(^7\)

\(^7\)Hidden assets makes the contractual problem of the firm more difficult. Therefore, the value of the firm—and hence the wage offered to capture workers— should react more when
It remains to make explicit the features of the bargaining process. I assume that when a firm hires an unemployed worker it has all the bargaining power; hence, it gets all the surplus of the match. In contrast, when two firms compete over a worker, both of them are in symmetric positions. They both have the same bargaining power and they both know the competitor’s type as well as the human capital of the worker. The competition between firms implies that the worker moves for sure if the outside firm is willing to offer a more attractive contract than the incumbent. In the cases in which both firms are willing to offer the same contract, the worker moves with probability $1/2$. It is important to remark that a job offers may induce wage changes even if the worker does not switch job.\(^8\) This bargaining process—explained in more detail below—closely follows Postel-Vinay and Robin (2002).

1.2.2 The Optimal Contract: Recursive Representation

When a firm is matched with a worker, it offers a long-term contract that is optimally designed under two kinds of constraints. The first is an informational friction rooted in the non-observability of workers’ effort. The second is a lack of commitment problem emerging from workers’ ability to quit or to switch jobs.

Given the informational friction, the prescribed effort must be consistent with the worker’s best response. As a consequence, since the cost of effort is assumed to be strictly increasing, full insurance is incompatible with worker positive effort. In other words, the informational friction implies some spread in wages across states of nature introducing some degree of wage dispersion.

Regarding the commitment problem, the contract is designed such that it is never in the interest of the worker to quit to unemployment and job-to-job transitions may occur only if the outside competitor is as least as productive as the current employer. Notice that the worker may get an offer from a more productive firm and yet not call for firm competition. This is the case when the outsider is not willing to offer a better contract than the one the incumbent is eliminating effort observability if assets were also non-observable.

\(^8\)There exist some offers that does not lead either to job switching or to wage change. This is the case when the outside competitor is not willing to offer a higher lifetime utility than the one the current contract promised in absence of an outside offer.
offering. When the worker reveals she got an offer, she moves with probability one if the outside offer comes from a more productive firm, and with probability 1/2 if the outside offer comes from a firm as productive as her current employer. If the revealed outside offer comes from a less productive firm, the worker does not move but gets a wage gain. This is the case because the outside firm is willing to offer a more attractive contract than the one the current employer is offering, otherwise the worker would not trigger firm competition.

An important feature of the contractual problem is its history dependence; that is, at any particular state, in any moment of time, the actions prescribed and the the payoffs to each party depend on the whole history of previous states, previous actions and previous payoffs. Given this history dependence, the problem—in principle—becomes intractable from a quantitative point of view. Fortunately, as stressed in Abreu et al. (1990), this sort of repeated moral hazard problem can be recursively represented using the promised utility approach, in which history is encoded in a single variable, the continuation value.

Let $U$ be the utility the contract has to deliver to the worker. In addition, let $U^i$ be the promised utility under the realization of $A^i$, with $i \in \{+, -\}$. Then, a recursive contract can be defined as follow:

**Definition 1.1. A Recursive Contract, $C$, is a collection of functions that for each triple $(z, h, U)$ specifies a prescribed worker effort, $e^{z,h}(U)$, a current period wage $w^{z,h}(U)$, and continuation values for the worker with no outside offer, for any realization of $A$, $\{U^{+,z,h}(U), U^{-,z,h}(U)\}$. In addition, a recursive contract implies continuation values for the worker when she gets an offer from any $\tilde{z}$-type firm, under the different realizations of $A$. I represent such values by $U^{+,z,\tilde{z},h}(U)$ and $U^{-,z,\tilde{z},h}(U)$ when $A$ equals $A^+$ or $A^-$, respectively.

It is important to mention that in the determination of the continuation values associated with job offer arrivals, $U^{+,z,\tilde{z},h}$, three elements are involved. First, the best offer the $\tilde{z}$ type outside competitor is willing to offer. Second, the continuation value the contract would deliver when the worker reports that no offer has arrived, $U' \in \{U^{+,z,h}(U), U^{-,z,h}(U)\}$. If the former is not higher than the latter, the worker does not reveal she has an offer, competition between firms is not triggered, and hence, the job offer has no effect. I refer to this case as an irrelevant offer.
The third and last component determining these continuation values is the best offer the current employer is willing to offer to the worker. When the firm is choosing \( \{U^{+,z,h}(U), U^{-,z,h}(U)\} \), it takes into account how these decisions affect the continuation values associated with job offer arrivals. The worker also takes into account this relation when choosing her best response to the offered contract.

As mentioned, the prescribed effort, \( e^{z,h}(U) \), has to be consistent with the best response of the worker to the contract. I follow the first order condition approach to incorporate this incentive compatibility requirement into the contractual problem.\(^9\)

For any arbitrary worker-firm match \((h, z)\) and any arbitrary contract \(C\), the worker’s best response satisfies the following problem:\(^{10}\)

\[
\max_{e \in [0, e]} \ u(w) - \gamma(e) + \beta \psi(1 - \lambda)\{(1 - \pi^s)[U^+ \pi(e) + (1 - \pi(e))U^-] + \pi^s EO\} + \lambda \beta \psi U_{un}^h \\
EO = \sum_{\tilde{z}} \Pi(\tilde{z})[\pi(e)U_{o+}^{+,\tilde{z}}(U) + (1 - \pi(e))U_{o-}^{-,\tilde{z}}(U)].
\]

(1.4)

In (1.4), \( EO \) is the expected continuation value for a surviving worker how gets a job offer, while \( U_{un}^h \) is the value of being unemployed for a worker with human capital \( h \) (to be specified below).

The first order condition for an interior solution of problem (1.4) is\(^{11}\)

\[
(1 - \lambda)\psi \beta \pi'(e)\{(1 - \pi^s)(U^+ - U^-) + \pi^s E_{\tilde{z}}[U_{o+}^{+,\tilde{z}}(U) - U_{o-}^{-,\tilde{z}}(U)]\} = \gamma'(e). \quad (ICC)
\]

I use (ICC) to represent the set of incentive compatible effort levels. This expression is a standard condition indicating that, in the optimum, the marginal benefit and the marginal cost of effort have to be the same. Equation ICC suggests that job mobility makes harder the provision of incentive in the sense that a higher spread, \( U^+ - U^- \), is needed to induce a certain level of effort as \( \pi^s \)

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\(^9\)Previous literature stresses that this approach may need to be ex-post verified if the worker’s problem is not convex, see Abraham and Pavoni (2006). In this case, the worker’s problem is convex provided that \( U^+ \geq U^- \).

\(^{10}\)To facilitate the notation, I drop the arguments \((U, z, h)\). For instance, \( u \) represents \( u^{z,h}(U) \) while \( U_{o+}^{+,\tilde{z}} \) represents \( U_{o+}^{+,z,\tilde{z},h}(U) \).

\(^{11}\)I assume that \( \pi \) is sufficiently; hence, the worker’s best response satisfies interiority.
increases. Intuitively, this is the case because under worker mobility, firm’s control over worker’s expected continuation values under different realizations of $A$ is dismissed. This is clear from equation ICC and from the fact that, for any $\tilde{z}$, $U^+ - U^- \geq U_{\tilde{z}}^+(U) - U_{\tilde{z}}^-(U)$, and typically strictly greater. In addition, the contractual problem must satisfy the promised keeping constraint defined as follows:

$$u(w) - \gamma(e) + \beta\psi(1 - \lambda)((1 - \pi^s)[U^+\pi(e) + (1 - \pi(e))U^-] + \pi^sEO) + \lambda\beta\psi U_{\text{un}}^h = U.$$  

(PKC)

The existence of exogenous separations and firm competition introduces important features in this promised keeping constraint that are not present in other standard repeated moral hazard environments. First, the (PKC) includes a component in which the firm has no control whatsoever, $U_{\text{un}}^h$, and one component in which the firm has very little control, $EO$. In addition, this second component varies across firm types. This is the case because the worker’s current employer type imposes an upper bound on the maximum upgrade in lifetime utility the worker can obtain from firm competition. Specifically, the more productive the worker’s current employer, the higher the expected continuation value associated with job offers, $EO$. Hence, a more productive firm faces a more relaxed promised keeping constraint in the sense that, all else equal, it can satisfy the promised keeping constraint with a lower wage.

The incentive compatibility constraint as well as the promised keeping constraint are affected by the value of being unemployed, $U_{\text{un}}^h$. Then, a full description of the contractual problem requires an explicit specification of this term. Given my assumption that unemployed workers have no bargaining power, it is easy to

---

12For any pair $U^+, U^-$ and any $\tilde{z}$, there are three possible cases. First, if $U^+, U^-$ are such that the offer from the $\tilde{z}$-type firm is irrelevant for any realization of $A$, then $U_{\tilde{z}}^+(U) - U_{\tilde{z}}^-(U) = U^+ - U^-$ and hence, the spread $U^+ - U^-$ required to encourage a certain level of effort is independent of $\pi^s$. Second, if $U^+, U^-$ are such that the offer from the $\tilde{z}$-type firm is relevant for any realization of $A$, then $U^+ - U^- > U_{\tilde{z}}^+(U) - U_{\tilde{z}}^-(U) = 0$. Finally, it can be that the offer is irrelevant only under the good realization of $A$: in this case, $U^+ - U^- > U_{\tilde{z}}^+(U) - U_{\tilde{z}}^-(U) > 0$. In the last two cases, the spread $U^+ - U^-$ to require a certain level of effort is increasing in $\pi^s$. 

---
show that $U^{h}_{un} = \frac{u(b)}{1-\beta \psi}$.\(^{13}\)

Finally, the promised utilities $(U^{-}, U^{+})$ must prevent workers quitting and must be feasible to be delivered.\(^{14}\) I call the set satisfying such conditions the admissible utilities set, $AU$. On one hand, $AU$ is bounded below by $\frac{u(b)}{1-\beta \psi}$ to guarantee that the worker never quits.\(^{15}\) On the other hand, here—as in other standard applications—the term $u(c) - \gamma(e)$ is bounded above by zero. As a consequence, $AU$ has to be bounded above by zero as well. However, the existence of exogenous separation $\lambda$, imposes a tighter upper bound for deliverable promised utility, $U^{upper} = \frac{\beta \psi \lambda}{1 - \beta \psi(1 - \lambda)} U^{h}_{un} < 0$.\(^{16}\) As a result, $AU = \left[ \frac{u(b)}{1-\beta \psi}, \frac{\beta \psi \lambda}{1 - \beta \psi(1 - \lambda)}, \frac{u(b)}{1-\beta \psi} \right]$.

Now we have all the components to state the contractual problem. To that end, define $V^{z,h}(U)$ as the value for a $z$-type firm when hiring a worker with human capital $h$ to whom the contract delivers a utility of $U$. In addition, let $B^{z,\tilde{z},h}(U')$ be the continuation value for the current $z$-type firm when bargaining with a $\tilde{z}$-type firm over a worker with human capital $h$, to whom was promised utility $U'$ if no offer arrives.

The constrained efficient contract $C^{*}$, solves the following functional equation problem:

\(^{13}\)It follows trivially from $U^{h}_{un} = u(b) + \beta \psi \left[ \pi^{u} V^{\tilde{z}} + (1 - \pi^{u}) U^{h}_{un} \right]$. Where $VO^{\tilde{z}}$ is the value for an unemployed worker when she gets an offer from a $\tilde{z}$-type firm, which equals $U^{h}_{un}$ given the cited assumption.

\(^{14}\)Given the relatively small value of being unemployed relative to the value of the match in the calibration, endogenous quitting is inefficient. In other words, in the calibration any type of firm makes positive profit with any type of workers when delivering the value of unemployment.

\(^{15}\)The possibility of quitting prevent form immizerization even for those workers that never get job offer.

\(^{16}\)This expression can be obtained as follows

$$u(w) - \gamma(e) + \beta \psi(1 - \lambda) \left[ \pi^{u} E O + (1 - \pi^{u}) \ast (\pi(e) U^{+} + (1 - \pi(e)) U^{-}) \right] + \beta \psi \lambda U^{h}_{un}$$

\begin{align*}
\text{Bounded above by 0} & \quad \text{Bounded above by } U^{upper} \\
\Rightarrow \beta \psi(1 - \lambda) U^{upper} + \beta \psi \lambda U^{h}_{un} = U^{upper} \Rightarrow \\
U^{upper} & = \frac{\beta \psi \lambda}{1 - \beta \psi(1 - \lambda)} U^{h}_{un}.
\end{align*}
\[
V^{z,h}(U) = \max_{e,w,U^+,U^-} y(z,h,A_t)(1 - \pi(e)) + y(z,h,A_h)\pi(e) - w + \]
\[
\text{Current period expected payoff }
\beta(1 - \lambda)\psi(1 - \pi^s) \left[ V^{z,h}(U^-)(1 - \pi(e)) + V^{z,h}(U^+)(1 - \pi(e)) \right] \]
\[
\text{Expected continuation value if the workers gets no offer }
\beta(1 - \lambda)\psi\pi^s \sum_{\tilde{z}} \Pi(\tilde{z})[B^{z,\tilde{z},h}(U^-)(1 - \pi(e)) + B^{z,\tilde{z},h}(U^+)(1 - \pi(e))] . \quad (1.5)
\]
\[
\text{s.t.}
(ICC), (PKC), (U^+, U^-) \in AU^2, w > 0, e \geq 0 .
\]

For the problem to be fully specified, it is needed to define \((U^+_o, z, \tilde{z}, h, U^-_o, z, \tilde{z}, h, B^o_{z,\tilde{z},h})\) representing the continuation values for the worker and the firm whenever the worker gets an outside offer.

Suppose a \(z\)-type incumbent competes against a \(\tilde{z}\)-type firm over a worker with human capital \(h\). Define \(U^{z,h}_*\) as the solution to \(V^{z,h}(U^{z,h}_*) = 0\). Since \(V^{z,h}(U)\) is strictly decreasing in \(U\), the value \(U^{z,h}_*\) represents the maximum utility the \(z\)-type firm is willing to offer to a worker with human capital \(h\). Let \(U' \in \{U^+, U^-\}\) be the utility the contract would deliver in the absence of a job offer. Then one of the following mutually exclusive situations arise:

1. \(U' \leq U^{z,\tilde{z},h}_*\) \(\Rightarrow\) the worker does not reveal the offer, gets \(U'\) and remains at current employer who gets \(V^{z,h}(U')\). I refer to this case as an irrelevant offer.

2. \(U' < U^{z,\tilde{z},h}_*\) and \(U^{z,\tilde{z},h}_* > U^{\tilde{z},\tilde{z},h}_*\) \(\Rightarrow\) the worker reveals the offer, gets \(U^{z,\tilde{z},h}_*\) and remains at her current employer who gets \(V^{z,h}(U^{z,\tilde{z},h}_*)\). In this case, even though there is no job-to-job transition, the outside offer affects the path of the continuation values and, hence, the path of wages.

3. \(U' < U^{z,\tilde{z},h}_*\) and \(U^{z,\tilde{z},h}_* = U^{\tilde{z},\tilde{z},h}_*\) \(\Rightarrow\) the worker reveals the offer, gets \(U^{z,\tilde{z},h}_*\) and remains at her current employer with probability 0.5 in which case the incumbent gets \(V^{z,h}(U^{z,\tilde{z},h}_*)\). With probability 0.5 the worker moves and the incumbent disappears.
4. $U' < U^{*,\check{z},h}$ and $U^{*,z,h} < U^{*,\check{z},h} \Rightarrow$ the worker reveals the offer, gets $U^{*,\check{z},h}$ and moves to the new job. The incumbent disappears.

In other words, if a relevant offer arrives, the two firms start a second-price auction for the worker. The more productive firm gets the worker and offers to her the highest lifetime utility the losing firm is willing to offer. This competition assumed here is similar to the one presented in Postel-Vinay and Robin (2002).

An important difference arises from the existence of potential irrelevant offers coming from more productive firms. More precisely, in this environment the $z$-type current employment may have negative value at some moment of the worker-firm relationship—i.e $U' > U^{*,z:h}$—therefore, if the $\check{z}$-type outside competitor is only a little more productive than the current employer, it may be the case that $U' > U^{*,z:h} > U^{*,\check{z},h}$ and the worker does not report getting an offer and remains at the current employer. In Postel-Vinay and Robin (2002) the worker always moves when getting an offer from a more productive firm. As I argue next, irrelevant offers from more or equally productive firms are not frequent events under the calibration.

These outcomes of the competition process implies the following specification for the continuation values:

\[
U^{+,*\check{z},h}(U) = \begin{cases} 
    U^{+,*\check{z},h}(U), & \text{if } U^{+,*\check{z},h}(U) \geq U^{*,\check{z},h}, \\
    \min\{U^{*,z,h}, U^{*,\check{z},h}\} & \text{if } U^{+,*\check{z},h}(U) < U^{*,\check{z},h}.
\end{cases}
\]

\[
U^{-,*\check{z},h}(U) = \begin{cases} 
    U^{-,*\check{z},h}(U), & \text{if } U^{-,*\check{z},h}(U) \geq U^{*,\check{z},h}, \\
    \min\{U^{*,z,h}, U^{*,\check{z},h}\} & \text{if } U^{-,*\check{z},h}(U) < U^{*,\check{z},h}.
\end{cases}
\]

\[
B^{z,\check{z},h}(U') = \begin{cases} 
    V^{z,h}(U') & \text{if } U' \geq U^{*,\check{z},h}, \\
    V^{z,h}(U^{*,\check{z},h}) & \text{if } U' < U^{*,\check{z},h} \text{ and } U^{*,z,h} > U^{*,\check{z},h}, \\
    0.5V^{z,h}(U^{*,\check{z},h}) & \text{if } U' < U^{*,\check{z},h} \text{ and } U^{*,z,h} = U^{*,\check{z},h}, \\
    0 & \text{if } U' < U^{*,\check{z},h} \text{ and } U^{*,z,h} < U^{*,\check{z},h}.
\end{cases}
\]
Before moving to the quantitative analysis I establish a result related to the incentive effect. In particular I show that effort observability implies that $U^+ = U^-$ while effort non-observability yields $U^+ > U^-$ if effort is non-zero. This implies that moral hazard introduces wage dispersion because it makes the history of the effort-dependent shocks relevant for the worker’s wage path.

**Lemma 1.1.** Consider problem (1.5), $e > 0 \Rightarrow U^+ > U^-$. Under effort observability and concavity of $V^{z,h}(U)$ with respect to $U$, in the optimal contract the firm fully insures the worker against the effort-dependent productivity shock; that is, $U' \equiv U^+ = U^-$. 

*Proof.* See Appendix.

1.3 **Quantitative Analysis**

The contractual problem presented above cannot be fully characterized analytically; hence, a further analysis requires a numerical solution. First, I provide explicit specification for the functional forms and a strategy to calibrate the set of structural parameters of the model. With this quantitative economy, I can numerically solve the contractual problem (1.5), and compute the desired statistics from the stationary distribution induced by the optimal contract and by the exogenous randomness associated with the calibration. I use this quantitative economy to answer the main research question about the role of moral hazard in shaping the wage structure.

1.3.1 **Functional Forms and Agent Types**

The utility from consumption is given by a constant relative risk aversion (CRRA) function, that is

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma},$$

where $\sigma > 1$ is the coefficient of relative risk aversion. The cost of effort is defined by a quadratic function, $\gamma(e) = e^2$. 
The probability of getting the high realization of the effort dependent productivity factor $A$, $\pi(e)$, is defined by the following exponential function:

$$\pi(e) = 1 - \exp(-\rho e).$$

I represent the set of firm types by the discrete set $\mathbb{Z} = \{z_1, z_2, z_3\}$ with $z_1 < z_2 < z_3$. Hence, the probability distribution defining the firm birth process, $\Pi(z)$, can be represented by two values, $(\Pi_1, \Pi_2)$, where $\Pi_i$ is the probability that an $i$-type firm is born. In addition, I represent the set of worker types by $\mathbb{H} = \{h_1, h_2\}$. I interpret $h_1, h_2$ as the human capital for a worker with no college and with college respectively. I define $f$ as the proportion of workers with a college degree in the economy.

The technology is given by the function $Y(z, h, A) = zAh$. Non-concavity of $Y(\cdot)$ with respect to $h$ is irrelevant in the sense that the parametrization $\{zh^\alpha A, \{h_1, h_2\}\}$ is equivalent to the parametrization $\{zhA, \{h_1^\alpha, h_2^\alpha\}\}$.

The vector of parameters to be specified is given by

$$\theta = (\beta, \sigma, \lambda, \pi^u, \psi, \pi_s, \rho, A^-, A^+, h_1, h_2, f, z_1, z_2, z_3, \Pi_1, \Pi_2).$$

### 1.3.2 Calibration Strategy

I calibrate the model to the US economy. As is standard, some parameters are taken from previous literature while others are set such that the model closely matches some moments of the actual economy. I set the time period to a quarter and correspondingly, $\beta = \frac{1}{1+0.04} = 0.99$, which implies an annual interest rate of 4%.

The coefficient of relative risk aversion, $\sigma$, is set to 2 which is a standard value in the quantitative macroeconomic literature. In addition I set $f$ to .25 which is approximately the proportion of workers with college in recent years.\(^{17}\)

The survival probability $\psi$ is set to .994 which is consistent with an expected working life of 40 years. As in Shimer (2005), I set $b$ to 0.4. The probability of getting a job offer while unemployed, $\pi^u$, is set to .784 which is consistent with the

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\(^{17}\)US Census Bureau report on educational attainment in the United States, 2003
average flow rate from unemployment to employment for the period 1994-2004.\textsuperscript{18} The exogenous separation rate, $\lambda$, is set to 0.044 to match an unemployment rate of 6%.

I normalize $h_1 = z_2 = 1$ and define

$$z_1 = z_2 - \Delta z, \quad z_3 = z_2 + 2\Delta z,$$

$$A^- = 1 - \Delta A,$$

$$A^+ = 1 + \Delta A.$$

With this normalization-definition, setting the parameters ($h_1, h_2, A_h, A_1, z_1, z_2, z_3$) reduces to setting ($h_2, \Delta z, \Delta A$).

Residual wage dispersion in this model depends on history in two dimension. The first is due to search frictions: two identical workers in the same firm may have different wages due to a different history of job offers. The key parameters here are ($\pi^s, Z, \Pi_1, \Pi_2$). The second source of residual wage dispersion is rooted in the moral hazard problem: two identical workers in the same firm may have different wages due to a different history of the realization of the productivity factor $A$. The key parameters in this friction are ($\Delta A, \rho$). As a consequence, wage structure may be not enough to separate both effects when pinning down the parameters ($\pi^s, Z, \Pi_1, \Pi_2, \Delta A, \rho$). Fortunately, the job arrival rate together with the birth rate—and not the moral hazard parameters—crucially determines the job mobility and the distribution of workers across firm types. Therefore,—under the assumption that there exists a positive relation between firm productivity and firm size—$\pi^s$, together with ($\Pi_1, \Pi_2$), can be calibrated as follows. Let ($\mu_1, \mu_2$) be the measure of workers in small size (or low productive) firm, and medium size (or medium productive) firms respectively. In addition, let $jj$ be the fraction of employed workers changing from job. Then, I can pin down ($\pi^s, \Pi_1, \Pi_2$) from the

\textsuperscript{18}The monthly series was taken from http://robert.shimer.googlepages.com/flows.
following $3 \times 3$ system:\footnote{This is actually an approximation. This system allows for some transitions that may not occur in the artificial economy. For instance, in the system it is imposed that 1/2 of competition among identical firms leads to job transitions. However, some of these offers are irrelevant in which case the worker does not move with probability 1. More precisely, in the artificial economy, transitions also depends on $U' \in U^+, U^-$. Therefore, computing the exact job-to-job transition rate requires solving the model. As I ex-post verify, the job-to-job transition rate associated with the artificial economy is very close to the one implied by this system.}

$$
\mu_1 = \mu_1(1-\lambda)\psi(1-\pi^s) + \pi^s \mu_1(1-\lambda)\psi \Pi_1 + \frac{U_{un}}{1-U_{un}} \pi^u \Pi_1,
$$
\hspace{1cm}
(1.6)

$$
\mu_2 = \mu_2(1-\lambda)\psi(1-\pi^s) + \pi^s \mu_2(1-\lambda)\psi(\Pi_1 + \Pi_2) + \\
\text{Workers with no offer} \quad \text{Workers with offer that remain in med prod firm} \\
\frac{U_{un}}{1-U_{un}} \pi^u \Pi_2 + \pi^s \mu_1(1-\lambda)\psi \Pi_2,
$$
\hspace{1cm}
(1.7)

$$
jj = \frac{\mu_1(1-\lambda)\psi \pi^s(1-\Pi_1)}{movers \text{ from low to higher prod firm}} + \frac{\mu_2(1-\lambda)\psi \pi^s(1-\Pi_1 - \Pi_2)}{movers \text{ from medium prod to high prod firm}} + \\
0.5[\mu_1(1-\lambda)\psi \pi^s \Pi_1 + \mu_2(1-\lambda)\psi \pi^s \Pi_2 + (1-\mu_1 - \mu_2)(1-\lambda)\psi \pi^s(1-\Pi_1 - \Pi_2)]
$$
\hspace{1cm}
(1.8)

As reported in Tybout (2000), the distribution of employment shares across plant size is as follows: 8.5% of the workers are employed in plants with less than 19 workers, while 22% of the workers are employed in plants with between 20 and 99 workers. Thus, I set $(\mu_1, \mu_2) = (0.085, 0.22)$. Regarding the job to job transition rate, Fallick and Fleischman (2004) report that on average 2.6% of employed workers change jobs each month. Thus, I set $jj = 0.076$. Using this information and the system (1.6)-(1.8) I obtain $(\pi^s, \Pi_1, \Pi_2) = (0.33, 0.403, 0.358)$.\footnote{For the benchmark calibration I obtain job-to-job transitions that imply a monthly rate of 2.3%}

I set $\Delta_A$ to 0.35 and perform a robustness check on this parameter. Specifically, I experiment with two alternative values: $\Delta_A = \{.25, .45\}$. In both cases, I
recalibrate the remaining parameters as indicated in the calibration strategy. The results are presented in the appendix. In both cases, the main result of the paper holds: removing the informational friction leads to more wage dispersion.

The remaining parameters—$(\rho, \Delta z, h_2)$—are calibrated such that model matches the following three targets: (1) the college premium, $m_1 \equiv \frac{w_c}{w_{hs}}$; where $w_c$ is the average wage for workers with college and $w_{hs}$ is the one for worker with no college, (2) the the ratio $m_2 \equiv \frac{w_{80}}{w_{20}}$; where $w_{80}$ is the average wage for the top 20% of the wage distribution while $w_{20}$ is the average wage for the bottom 20% of the wage distribution and, (3) the average pay-by-performance compensation as a proportion of total earnings, $m_3$. In order to compute this last moment in the model, I interpret $w^{z,h}(U^-)$ as the straight wage and $w^{z,h}(U^+)$ as the straight wage plus bonuses associated with individual performance.

I set the college premium, $m_1$, to 1.78, as reported in Golding and Katz (2007) for the year 2000. Based on Bosworth et al. (2001) I set $m_2 = 3.1$. Finally, I set $m_3 = .05$. Lemieux et al. (2006) reported a median share of 3.5%. Other sources report that for recent years, the variable pay over total payroll is around 10% (Industry Week Magazine, reporting a Hewwit’s survey). However, some fraction of this variable payments is in the form of common bonuses that have nothing to do with individual performance, thus I believe $m_3 = 5\%$ is a good compromise. Under these targets, the values for the parameters are $(\rho, \Delta z, h_2) = (3.01, 0.25, 2.20)$. Table 1.1 shows that the parameterized model closely matches the corresponding moments in the data.

<table>
<thead>
<tr>
<th>Moment</th>
<th>Model</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_1$</td>
<td>1.787</td>
<td>1.784</td>
</tr>
<tr>
<td>$m_2$</td>
<td>3.151</td>
<td>3.1</td>
</tr>
<tr>
<td>$m_3$</td>
<td>0.049</td>
<td>0.05</td>
</tr>
</tbody>
</table>

For this benchmark calibration, the artificial economy reproduces a coefficient of variation for wages of .487. Dunne et al. (2004) finds that for 1999 the coefficient

\[21\text{For men, this moment moved from around 2.6 in the 60’s to around 3.2 in the 80’s and 90’s. Likewise, for full time women, the ratio increases from around 2.4 in the 60’s to around 3 at the end of the 90’s.} \]
of variation of wages in the U.S manufacturing sector was 0.681. Given that this model lacks many other sources of wage heterogeneity, I do not expect the model to match the overall wage dispersion.

The results of the calibration strategy are summarized in table 1.2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
<th>Basis and Targets</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b$</td>
<td>0.4</td>
<td>Consumption when unemployed</td>
<td>Shimer (2005)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.99</td>
<td>Discount factor</td>
<td>Annual r=4%</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>2</td>
<td>Relative risk aversion coefficient</td>
<td>Standard</td>
</tr>
<tr>
<td>$\pi^u$</td>
<td>0.784</td>
<td>Finding Job rate</td>
<td>Average U-E flow</td>
</tr>
<tr>
<td>$\psi$</td>
<td>0.994</td>
<td>Survival Probability</td>
<td>Working life 40 years</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.044</td>
<td>Exogenous Separation</td>
<td>To match $U_{un} = 6%$.</td>
</tr>
<tr>
<td>$\pi^*$</td>
<td>.33</td>
<td>Prob. of job offer while working</td>
<td>Solving (1.6)-(1.8)</td>
</tr>
<tr>
<td>$\Pi_1$</td>
<td>0.403</td>
<td>Birth rate for low-prod. firms</td>
<td>To match $(jj, \mu_1, \mu_m)=$</td>
</tr>
<tr>
<td>$\Pi_2$</td>
<td>0.358</td>
<td>Birth rate for med.-prod. firms</td>
<td>(0.076, 0.085, 0.22)</td>
</tr>
<tr>
<td>$\rho$</td>
<td>3.01</td>
<td>Parameter in $\pi(e) \equiv \text{prob of } A_h$</td>
<td>To match $(m_1, m_2, m_3)$</td>
</tr>
<tr>
<td>$\Delta_\lambda$</td>
<td>0.26</td>
<td>Spread in firm type</td>
<td>To match $(m_1, m_2, m_3)$</td>
</tr>
<tr>
<td>$h_2$</td>
<td>2.2</td>
<td>High human capital</td>
<td>To match $(m_1, m_2, m_3)$</td>
</tr>
<tr>
<td>$\Delta_A$</td>
<td>0.35</td>
<td>Spread in $A$</td>
<td>Robustness Check</td>
</tr>
</tbody>
</table>

1.3.3 Results

In this section I show the main result of this paper. In particular, I study how a moral hazard problem in firm-worker relationships shapes the cross-sectional wage structure in an environment with job mobility. To that end, I compute (1) the college premium, (2) the ratio $w_{80}/w_{20}$, and (3) the coefficient of variation of wages for the quantitative economy presented above under two alternative assumptions regarding effort observability.

In the first environment, I assume that effort is not observable. In this case, the contractual problem is represented by problem (1.5), parameterized according to the calibration procedure previously described. I label this the moral hazard case (MH). As stressed, under this environment to prescribe non-zero effort requires $U^+ - U^-$ to be strictly positive; in other words, the worker is not insured against
uncertainty in $A$. This lack of insurance implies wage dispersion even among workers with identical job-offer histories.

In the second environment, I assume that effort is observable, I label it the no moral hazard case (NMH). Under observability of effort, the contractual problem is similar to problem (1.5) but the constraint ICC is not required. In this case, I use the exact same parametrization used in the moral hazard case, summarized in table 1.2. As stated in lemma 1.1, in this case $U^+ = U^-$; that is, the optimal contract provides full insurance to the workers against uncertainty in the effort-dependent productivity shock.

Given the provision of insurance under the no moral hazard case, one would expect less wage dispersion if the firm can observe worker effort. However, the quantitative results indicate that in the benchmark economy, the presence of moral hazard actually reduces wage dispersion or, equivalently, eliminating the informational friction leads to higher wage dispersion. The increase in wage dispersion is between 5 and 7 percent depending on the measure. The measures of wage dispersion for the two environments are presented in Table 1.3.

The explanation of this unexpected result relies on the impact of the informational friction in the wage schedule associated with on-the-job offers. I refer to this impact as the *wage offer effect*. This wage schedule changes due to two main forces: (1) changes in lifetime utilities emerging from firm bargaining and, (2) changes in the effort schedule.

First, the informational friction reduces the value of the firm for any match $(z, h)$. As a consequence, it reduces the highest lifetime utility the $z$-type firm is willing to offer to the $h$-type worker, as illustrated in figure 1.1. This induces smaller wages associated with the arrival of on-the-job offers.

Moreover, in the quantitative economy, it is the case that the reduction of

<table>
<thead>
<tr>
<th>Measure</th>
<th>MH</th>
<th>NHM</th>
<th>($\frac{\text{NMH} - \text{MH}}{\text{MH}}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>College Premium</td>
<td>1.789</td>
<td>1.894</td>
<td>5.76%</td>
</tr>
<tr>
<td>Ratio $w_{80} - w_{20}$</td>
<td>3.151</td>
<td>3.337</td>
<td>5.90%</td>
</tr>
<tr>
<td>Coefficient of Variation</td>
<td>0.486</td>
<td>0.522</td>
<td>7.41%</td>
</tr>
</tbody>
</table>

Table 1.3: Moral Hazard Vs No Moral Hazard: Dispersion
these lifetime utilities is similar across firm types.\footnote{The reduction in $U^{*,z,h}$ is relatively modest. The average reduction in these critical utilities is similar across firm productivity and slightly smaller for highly educated workers.} Hence, by the concavity of the utility function, the reduction in wages tends to be higher for workers receiving high levels of lifetime utilities; that is, for workers getting offers from more productive firms.

Second, the wage schedule associated with on-the-job offers also changes due to variations in the effort levels required after firm competition is triggered. These effort levels differ from those in the no moral hazard case, not only because the policy function for effort changes, but also because the lifetime utilities emerging from firm bargaining change. This is illustrated in Figure 1.2. For this particular case, when introducing the informational friction effort falls from point (2) to point (1). This reduction can be decomposed as follows, a move from (2) to (1b) due to the change in the lifetime utility, and a move from (1b) to (1) due to a shift in the policy function.

Notice that the effort required by the contract is decreasing in the utility the contract has to deliver, $U$.\footnote{This is true for both the moral hazard case and the observable effort case.} This is because the higher is $U$, the more costly it is to compensate the worker for the required effort due to the concavity of the utility function. As a consequence, smaller critical utilities, $U^{*,z,h}$, leads to more effort (movement from (2) to (1b)). However, for a given level of $U$, the presence of
moral hazard reduces the required effort; that is, the informational friction shifts down the effort policy. The intuition is that with observable effort, no variation in wages across realizations of $A$ is needed to require a certain level of effort; hence, the firm does not have to compensate the worker for such wage dispersion. In other words, with observable effort it is cheaper for firms to require a certain level of effort.

Whether the effort after an on-the-job offer arrives is higher or lower with moral hazard is undetermined. In the benchmark calibration, these effort levels are lower with the informational friction indicating that the shift in the policy function dominates changes in the level of lifetime utilities emerging from firm competition.

The reduction in effort levels reinforces the reduction in the wage schedule associated with on the job offers. Moreover, the reduction in effort after job offers is greater for more educated workers and for workers having job offers from more productive firms. Recall that firm competition implies that well-educated workers and workers having offers from more productive firms enjoy higher promised utilities. The higher the promised utility the contract has to deliver, the more expensive to provide incentives. Hence, it is expected that the informational friction affects more significantly the effort levels associated with this group of workers.\footnote{The average reduction in the effort level associated with a job offer is 17\%. For low human capital workers the average reduction is 8\% while for high human capital workers the average reduction is 24\%. For workers getting an offer from a low productivity firm the average reduction}
Such a change in the effort profile leads to a greater reduction in wages for more educated workers and for workers getting offers from more productive firms.

The combination of these two forces implies that the presence of moral hazard shifts down the wage schedule associated with on-the-job offers. To see the magnitude of this effect, define $\bar{W}(z_w, z_l, h)$ as the wage the $z_w$-type winning firm pays to a $h$-type worker when the winner firm compete against a $\tilde{z}_l$-type losing firm, with $z_w \geq \tilde{z}_l$. The average drop in this schedule is 2% and the maximum drop is 8% approximately. Moreover, it is the case that this wage schedule flattens, in the sense that the shifting is greater for more educated workers and for workers getting offers from more productive outside competitors. The average drop conditional on being a low human capital worker is 1.18% while for a high human capital worker the average drop is 3.92%. For those workers getting offers from low productivity firms, the average drop in this wage schedule is less than .5%, while for workers getting offers from a medium productivity firm the average drop is close to 3%. Finally, for workers having offers from the most productive firms, the average drop is greater than 5%.

Such a change in $\bar{W}(z, \tilde{z}, h)$, once non-observability of effort is introduced, leads to less wage dispersion for the following reasons. First, it reduces the wage differential between workers who have gotten an on-the-job offer and those that have not. Second, it compresses the wage structure among those lucky workers getting on-the-job offers. Finally, it reduces the wage differential across educational groups.

1.3.4 Some Experiments

In what follows, I perform some counterfactual experiments in order to further understand the channels through which the informational friction shapes the wage structure. The first, Exercise I, is aimed at assessing the quantitative importance of the incentive effect in the calibrated economy. The other two are aimed at

\footnote{is 13%, for those getting an offer from a medium productivity the average drop is 19% and finally 22% is the average drop for workers getting an offer from a high productivity firm.}

\footnote{Let $z$ be the productivity of the current employer and $\tilde{z}$ be the productivity of the outside competitor. The outcome of the bargaining process implies $z_w = \max\{z, \tilde{z}\}$ and $\tilde{z}_l = \min\{z, \tilde{z}\}$.}
evaluating the quantitative importance of the main sources behind the wage offer effect.

More precisely, in Exercise I I use the policy functions for effort and for wages obtained in the no moral hazard environment, \( e_{NMH}, w_{NMH} \). In addition, I also use the critical utilities associated with the no moral hazard environment, \( U^{*z,h}_{NMH} \). Finally, I assume that effort has to be incentive compatible and solve for \( U^+, U^- \) using the incentive compatibility constraint together with the promised keeping constraint. Notice that if one sets \( U' = U^+ = U^- \) and uses only the promised keeping constraint to solve for \( U' \), the wage distribution is the same to the one with no moral hazard; thus, any difference in wage dispersion respect to the no moral hazard environment corresponds exclusively to the non observability of effort that requires the incentive compatibility constraint to be satisfied. Table 1.4 shows the results of this experiment.

<table>
<thead>
<tr>
<th>Measure</th>
<th>MH</th>
<th>NHM</th>
<th>Exercise I</th>
<th>((EI - NMH)_{NMH})</th>
</tr>
</thead>
<tbody>
<tr>
<td>College Premium</td>
<td>1.789</td>
<td>1.894</td>
<td>1.914</td>
<td>1.06%</td>
</tr>
<tr>
<td>Ratio ( w_{80} - w_{20} )</td>
<td>3.151</td>
<td>3.337</td>
<td>3.533</td>
<td>5.87%</td>
</tr>
<tr>
<td>Coefficient of Variation</td>
<td>0.486</td>
<td>0.522</td>
<td>0.529</td>
<td>1.34%</td>
</tr>
</tbody>
</table>

As expected, wage dispersion increases relative to the no moral hazard case. However, with the exception of the ratio \( w_{80}/w_{20} \), the increase in the wage dispersion associated with the incentive effect is modest.\textsuperscript{26} This small effect may be explained by the high on-the-job offer arrival rate in the calibrated economy together with the fact that the history of productivity shocks becomes immaterial whenever a relevant on-the-job offer arrives. The higher impact on the ratio \( w_{80}/w_{20} \) can be explained by the fact that now workers at the top of wage distribution—for whom the fraction of relevant job offers is relatively smaller—get further wage gains as a reward for good performance.

Now I focus on the main forces driving the changes in the wage schedule associated with on-the-job offers, \( \bar{W} \). To that end, consider a variation of problem

\textsuperscript{26}The increase in the ratio \( w_{80}/w_{20} \) translates to an increase in the variance of wages of 5.4%. However, the average wage increases from 1.466 to 1.525. Then, the coefficient of variation of wages increases only modestly.
(1.5) in which a policy function for effort, $e_{\text{given}}^{z,h}(U)$, and a set of critical lifetime utilities, $U_{\text{given}}^{*,z,h}$, are imposed. More precisely consider the following functional equation problem

$$
\tilde{V}^{z,h}(U|e = e_{\text{given}}^{z,h}(U), U_{\text{given}}^{*,z,h}) = \max_{w, U^+, U^-} \left( \begin{array}{l}
\text{current period expected payoff} \\
\beta(1 - \lambda)\psi(1 - \pi^s) [V^{z,h}(U^-)(1 - \pi(e)) + V^{z,h}(U^+)]
\end{array} \right)
\begin{array}{l}
\text{Expected continuation value if the worker gets no offer} \\
\beta(1 - \lambda)\psi\pi^s \sum_{\tilde{z}} \Pi(\tilde{z}) [B^{z,\tilde{z},h}(U^-)(1 - \pi(e)) + B^{z,\tilde{z},h}(U^+)]
\end{array}
\begin{array}{l}
\text{Expected continuation value if the worker gets an offer}
\end{array}
$$

s.t.

$$(ICC), (PKC), (U^+, U^-) \in AU^2, w > 0, e \geq 0.$$
case; therefore, this change in effort may dampen the importance of changes in lifetime utilities. Hence, the results presented in Experiment III may represent a lower bound on the contribution of the change in $U^*$ to the increase in wage dispersion. Similarly, since effort levels are relatively high in experiment II, the results may represent an upper bound on the importance of changes in effort levels as a force behind the wage offer effect.

Table 1.5: “Decomposing” the wage offer effect

<table>
<thead>
<tr>
<th>Measure</th>
<th>MH</th>
<th>NHM</th>
<th>Exp II</th>
<th>(E_{III} - MH)</th>
<th>Exp III</th>
<th>(E_{III} - MH)</th>
</tr>
</thead>
<tbody>
<tr>
<td>College Premium</td>
<td>1.789</td>
<td>1.894</td>
<td>1.829</td>
<td>2.19%</td>
<td>1.797</td>
<td>0.45%</td>
</tr>
<tr>
<td>Ratio $w_{80} - w_{20}$</td>
<td>3.151</td>
<td>3.337</td>
<td>3.328</td>
<td>5.32%</td>
<td>3.156</td>
<td>0.16%</td>
</tr>
<tr>
<td>Coef. of Var.</td>
<td>0.486</td>
<td>0.522</td>
<td>0.509</td>
<td>4.85%</td>
<td>0.488</td>
<td>0.41%</td>
</tr>
</tbody>
</table>

1.4 Concluding Remarks

I study how a moral hazard problem shapes the wage distribution in an environment that allows for job-to-job transitions. The moral hazard problem emerges due to non-observability of worker effort and from different attitudes toward risk between employers and workers. The model features two important and interconnected sources of residual wage dispersion, namely search frictions and moral hazard in firm-worker contracts.

As is well known, the presence of moral hazard limits the firm as a vehicle for risk sharing given that, under the informational friction, a spread in future wages is required as a mechanism to encourage non-zero levels of worker effort (incentive effect). As a consequence, one would intuitively argue that non-observability of worker effort leads to higher wage dispersion.

However, I found that the informational friction also shapes the wage structure by affecting the wage profile associated with job offer arrivals. I label this mechanism the wage offer effect. Specifically, moral hazard reduces the worker’s lifetime utility emerging from firm competition, as well as the effort levels required after job offers. The changes are such that the wage profile associated with job offers not only shifts down, but also flattens in the sense that the wage reduction
is greater for more educated workers and for workers getting offers from more productive firms. The wage offer effect reduces wage dispersion for the following reasons. First, it reduces the wage differential between workers who have gotten an on-the-job offer and those who have not. Second, it compresses the wage structure among workers getting on-the-job offers. Finally, it reduces the wage differential across educational groups. As a consequence, the net effect of the informational friction in the wage dispersion becomes a quantitative question. The main contribution of this paper is to identify the existence of this mechanism and quantify its importance in a model calibrated to the U.S economy.

I found that in the calibrated economy the incentive effect is quantitatively modest. The explanation relies on the fact that the job offer arrival rate is high, together with the fact that the history of productivity shocks becomes immaterial for the wage determination whenever a relevant job offer arrives. In contrast, the wage offer effect is quantitatively important and more than offsets the incentive effect. Indeed, the main result indicates that removing the informational friction from the benchmark calibration leads to an increase in wage dispersion of approximately 7%.

I also find that the effort response when removing the informational friction seems to be quantitatively more important than the response in lifetime utilities arising from firm competition. However, both forces contribute to explain the wage offer effects.

The main result challenges the intuitive argument—based exclusively on the incentive effect—that moral hazard leads to more wage dispersion. Moreover, the result suggests that improvement in the technology to monitor workers may lead to higher wage dispersion and hence, can be considered a potential driving force behind the increase in wage dispersion experienced in recent decades.

A natural extension to this environment is to incorporate aggregate and persistent shocks and study how moral hazard, together with job mobility, shapes the business cycle and the wage response to productivity shocks. My result seems to suggest that the informational friction reduces the response of wage to productivity shocks, introducing wage rigidity. The intuition for this conjecture is that a good aggregate productivity shock can be interpreted, in this framework, as if
the worker is getting an offer from a more productive firm. As was shown, the informational friction reduces more significantly wages associated with offers from more productive firms, mitigating the impact of productivity shocks on wages.
Chapter 2

Unemployment Insurance with a Hidden Labor Market.

2.1 Introduction

Unemployed workers face a trade-off between search effort and leisure. Since effort is unobservable, the design of unemployment protection programs balances insurance and incentives. This moral hazard problem has been the object of study since the pioneering work of Shavell and Weiss (1979). In this literature it is assumed that employment status is observable and hence individuals cannot defraud the system by working and asking for unemployment payments. But what if there is a shadow economy that allows unemployed individuals to secretly work and, simultaneously, ask for unemployment payments? This paper explores the implications of a hidden labor market in the design of optimal unemployment insurance. Hopenhayn and Nicolini (2001) stress the importance of this contribution:

Our focus in discussing optimal unemployment insurance has been the group of workers whose unemployment risk is higher, namely, the less educated and less experienced workers. Often, the members of this group find job opportunities mostly in the informal sector of the economy. Therefore, it may be very hard to monitor the employment status of the worker. Our investigation assumed that while search effort is not observable, employment status is. A strong incentive problem arises when the latter does not hold, since an unemployed worker receiving benefits would not have incentives to disclose having found a job.
Table 2.1 reports three facts suggesting that incorporating a hidden labor market in the study of optimal unemployment insurance might be crucial:

- The hidden labor market or the shadow economy is important in many countries, ranging from 13-30 percent of GDP in industrialized economies to 39-76 for African countries.

- In economies with a sizeable shadow economy, unemployment is a likely event. On average, unemployment rates range from an average of 6.8 in industrialized economies to 11.6 for African countries.

- With different coverage, replacement rate, and benefits’ duration, most of the countries protect the workers against unemployment risk. Replacement rates are around 45-75 percent during 1-2 years.

In light of these facts, it is reasonable to think about economies where the hidden labor market is important, workers face high unemployment risk, and the government provides unemployment insurance. In the current paper we study the optimal unemployment insurance in economies with such features.

Incorporating a hidden labor market modifies the standard unemployment insurance problem in three dimensions. First, it imposes an endogenous lower bound for promised utility that has to be incorporated as an explicit restriction in the planner’s problem. Second, it breaks the identity between unemployment payments and consumption. Finally, when preference are non-separable in search effort and participation, the presence of a hidden labor market makes harder the encouragement of job search effort. In fact, an extra incentive compatibility constraint must be added to keep participation at the desired level. This is simply because more participation in the hidden labor market increases the cost of search effort.

Important insights can be obtained in the case of linear effort-cost function, when the unemployment insurance system can be characterized analytically. Ini-

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1 We interpret the informal sector as a hidden labor market in which the worker can always find a job but his productivity is lower than in the formal sector.

2 Also, the number of countries providing unemployment insurance programs increased from 21 in 1940 to 68 in 1997 (based on Social Security Programs Throughout the World, Social Security Administration, US, 1997).
Table 2.1: Shadow economy size, unemployment rate, and unemployment insurance

<table>
<thead>
<tr>
<th>Region</th>
<th>Shadow economy % GDP</th>
<th>Unemployment rate %</th>
<th>Coverage</th>
<th>Replacement rates %</th>
<th>Benefit duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Industrialized economies</td>
<td>Vary between 13 and 30.</td>
<td>6.8</td>
<td>Most of the countries provide and cover all the employed.</td>
<td>Vary between 40 and 75.</td>
<td>Most of the countries limit the length between 8 and 36 weeks.</td>
</tr>
<tr>
<td>Transition economies</td>
<td>Vary between 9 and 43.</td>
<td>9.2</td>
<td>Most of the countries provide and cover all the employed.</td>
<td>Vary between 50 and 75.</td>
<td>Most of the countries limit is 26 weeks. Exception Hungary, 2 years.</td>
</tr>
<tr>
<td>Latin America and the Caribbean</td>
<td>Vary between 25 and 60.</td>
<td>8.0</td>
<td>Most of the countries provide and cover all the employed.</td>
<td>Vary between 50 and 60.</td>
<td>Maximum entitlement 1 year.</td>
</tr>
<tr>
<td>Asia</td>
<td>Vary between 13 and 70.</td>
<td>4.8</td>
<td>Differs significantly. Bangladesh only commerce and industry. South Korea: all firms.</td>
<td>Vary between 50 and 55.</td>
<td>Bangladesh 30-120 days. China 1-2 years. South Korea 90-240 days.</td>
</tr>
<tr>
<td>Africa</td>
<td>Vary between 39 and 76.</td>
<td>11.6</td>
<td>Egypt excludes public sector workers. South Africa excludes highly paid workers.</td>
<td>Vary between 45 and 75.</td>
<td>Vary between 26 weeks and 3 years.</td>
</tr>
</tbody>
</table>

ially, consumption is strictly decreasing during unemployment and workers do not participate in the hidden labor market. For sufficiently large unemployment spells, the lower bound for promised utility is eventually reached. Only at that moment, the optimal contract prescribes positive participation in the hidden labor market together with zero unemployment payments. With the jump in participation, the worker smooths out the abrupt decline in payment and hence the consumption path remains smooth. In addition, once the lower bound for promised utility binds, the planner’s instruments—taxes and benefits—and prescriptions—search effort and hidden labor market participation—remain constant over time.

For the case of non-linear effort-cost function, we calibrate the model to Spain. In this quantitative exercise, the initial phase of the optimal unemployment insurance scheme resembles the one described in the case of linear effort-cost function. However, a new intermediate phase arises as a consequence of the effect of participation in the hidden labor market on the cost of search effort. In this phase, participation remains at zero but the incentive compatibility constraint associated with it binds. It implies that the unemployment payments’ profile must be flattened in order to prevent participation in the hidden labor market or, equivalently, to prevent a reduction in search effort. Thus, during this phase, incentives are introduced by decreasing promised utility. As time goes, promised utility decreases but payments stay relatively constant. Eventually, the accumulated decrease in promised utility materializes in a abrupt decline in payments and the final phase starts. There, hidden labor market participation jumps and search effort drops. The drop in search effort implies that long-term unemployed individuals have a lower probability of finding a job than workers entering to unemployment. This result differs from previous literature on optimal unemployment insurance and is consistent with empirical observations—see Bover et al. (2002).

To the best of our knowledge, this is the first paper justifying a flattening in the payment profile together with a subsequent abrupt decline. The intuition goes as follow. Initially, participation remains at zero—first two phases—to facilitate the encouragement of search effort. More specifically, zero participation facilitates the encouragement of search effort because: (i) the marginal cost of search effort remains low, and (ii) decreasing payments translates to decreasing consumption.
If payments are high enough, the worker has no incentive to participate in the hidden labor market, in spite of a steep payments’ path. Eventually, unemployment payments reach a level in which such steep profile becomes inconsistent with no participation. At that phase, the payment profile becomes flatter, participation remains at zero, and the contract encourages search effort by dynamics incentives, that is, by decreasing promised utility. For sufficiently large unemployment spells, the accumulated decrease in promised utility requires an abrupt decline in payments and, as a result, a jump in participation. Again, this variation in payments does not imply such variation in consumption because workers participate in hidden labor market to smooth consumption. Nevertheless, this drop provides incentives guaranteeing that long run unemployment remains a serious threat.

Additionally, the results indicate that a hidden labor market reinforces the importance of a type-dependent unemployment insurance system for economies in which labor mobility differs among workers. In particular, payments duration should be shorter for workers with higher separation rates. The intuition is relatively simple. When separation rates are low, jobs are more “valuable” and, therefore, the planner delays participation in the hidden labor market to keep search effort high. Naturally, this requires a longer duration of unemployment payments for those worker.

Unemployment insurance systems have been studied from two different perspectives. One approach uses quantitative general equilibrium models to study the influence of unemployment insurance systems on macroeconomics variable and welfare—see for example Hansen and Imrohoroglu (1992) and Wang and Williamson (1996). The other approach, followed in this paper, uses contract theory in a partial equilibrium environment to study the optimal design of unemployment insurance. In a moral hazard framework in which the planner can only choose unemployment payments, early contributions find that payments must decrease over time in order to encourage search effort—Shavell and Weiss (1979). A subsequent work allows the planner to choose, together with unemployment payments, an after re-employment tax, and numerically solves for the optimal contract using a recursive representation of the problem. The paper finds that extending the planner’s set of instruments does not modify the previous result of
a decreasing path for unemployment payments—Hopenhayn and Nicolini (1997).

A controversial result from this early literature is that the optimal contract leads unemployed workers toward “immiserization”. More precisely, “if the workers utility function is unbounded below, then efficiency requires that workers expected discounted utility falls, with positive probability, below any arbitrary negative level”—Pavoni (2007). To prevent this result, Pavoni (2007) imposes an arbitrary lower bound for promised utility and characterizes the resulting optimal unemployment insurance.³ Our work is related to this idea because lifetime utility cannot fall below a lower bound. However, our paper differs from Pavoni (2007) in several dimensions. It justifies the existence of a lower bound in lifetime utility by including a hidden labor market where workers can guarantee themselves a minimum consumption. This is important because the lower bound is here connected to the structure of the economy. Thus, in quantitative exercises, the lower bound in promised utility can be determined using information about hidden labor markets. Moreover, as it will be shown later, the features of the optimal contract—specially the duration of unemployment payments—depends crucially on the wage in the hidden labor market. Additionally, the presence of a hidden labor market makes possible an extra deviation: the worker can deviate from the planner’s prescription by exerting hidden labor market participation. With non-linear effort-cost function, this deviation hardens incentives provision, because it increases the marginal cost of search effort. As a consequence, it makes flatter the payment profile and differentiates it from Pavoni (2007). Finally, unemployed workers’ consumption is not necessarily equal to unemployment payments. In particular, unemployed individuals can smooth out abrupt declines in payments by working in the hidden labor market. Obviously, since consumption and payments are equal in Pavoni (2007), any abrupt decline in payments in that environment works against consumption smoothness.

The policy implications of a hidden labor market are studied in different environments and with different goals. Among others, labor regulations, wage controls, migration policies, and taxation are analyzed in frameworks with hidden

³There are several related environments where “immiserization” does not hold. For instance, see Wang (1995) and Phelan (1995).
labor markets by Banerjee (1983), Fortin et al. (1997), and Johnson et al. (1998). Surprisingly, there is no paper in this strand of literature considering the optimal design of unemployment insurance. Our work fills this gap studying the implications of a hidden labor market for the optimal unemployment insurance.

Two recent papers are related to our work because of their interest in the relation between the informal sector or “bad jobs” and optimal unemployment insurance. Hopenhayn and Nicolini (2001) study the effects of heterogeneity on the design of optimal unemployment insurance. Importantly, they comment that one way to cope the incentive problem generated by an informal sector is to design programs that require those receiving benefits to appear at the insurance office. As they mention, this monitoring schemes may come at a significant cost. We designed an incentive compatible contract that requires no monitoring. Likewise, Hopenhayn and Nicolini (2005) focus on workers’ incentives to accept bad jobs when layoffs and quits are undistinguishable. In particular, their interest is about workers accepting and then quitting those jobs just to upgrade their unemployment insurance benefits. Instead, we study the possibility that those jobs are non-observable, what implies that workers could accept them and, simultaneously, ask for unemployment payments.

2.2 Model

2.2.1 Environment

The economy is populated by a continuum of infinitely-lived risk averse workers. At each period, a worker’s labor status is denoted by $m_t$. The variable $m_t \in \{u, e\}$, where $u$ stands for unemployment and $e$ for employment. The employment history of a worker up to time $t$ is represented by $h^t = (m_0, ..., m_t)$.

In contrast to previous studies, this environment includes a secondary labor market in which workers obtain a wage, $\varpi$, for unit of participation, $s$. Henceforth, we refer to this market as the hidden labor market.

Workers employed in the formal sector—employed workers—exert a constant work effort $\bar{a}$, have productivity $\omega$, and receive wages $w$. Employment relationships end exogenously with probability $\delta$. By assumption, we rule out participa-
tion in the hidden labor market during employment as well as on-the-job search. Productivity in the hidden labor market is lower than in the formal sector, \( \varpi < \omega \).

Unemployed workers search for a job and can, simultaneously, participate in the hidden labor market. When exerting search effort \( a \), they find a job next period with probability \( p(a) \). The total effort while unemployed, \( x \), is restricted to lie in the compact set \([0, \vartheta]\). The function \( p \) is strictly increasing, strictly concave, twice differentiable and satisfies standard Inada conditions.

The instantaneous felicity function is separable in consumption and total effort

\[
u(c) - v(x). \tag{2.1}
\]

The function \( u \) reflects the utility from consumption. It is strictly increasing and bounded above by \( \bar{u} \). It is also strictly concave, twice differentiable, and satisfies Inada conditions. Similarly, the function \( v \) measures the cost of effort exerted either searching for a job, participating in the hidden labor market or working in the formal sector. We assume this function is convex, strictly increasing and twice differentiable. Notice that total effort, \( x \), equals \( \bar{a} \) if the worker is employed and \( s + a \) otherwise. Finally, workers can neither save nor borrow and they discount future with the factor \( \beta \in (0, 1) \).

We suppose there is a risk neutral planner who provides unemployment insurance and affects workers’ earnings by levying a history dependent after-reemployment tax. The planner cannot observe either hidden labor market participation, \( s \), or job search effort, \( a \). The planner can borrow and save at a constant interest rate \( r = 1/\beta - 1 \).

### 2.2.2 Optimal contract

The planner has to decide unemployment payments, after reemployment wages, search effort, and hidden labor market participation. A contract, precisely described in the next definition, collects those decisions for each possible history node.

**Definition 2.1.** A contract or unemployment insurance system, \( W \), is a collection of functions specifying unemployment payments, \( b \), wages after reem-
ployment, \( w \), search effort, \( a \), and hidden labor market participation, \( s \), for each possible history, \( h^t \), at each period; i.e., \( \mathcal{W} = \{ b_t(h^t), w_t(h^t), a_t(h^t), s_t(h^t), \forall h^t, t \} \).

Notice that a contract can be divided into two components: those representing planner’s direct instruments, \( \mathcal{I} = \{ b_t(h^t), w_t(h^t), \forall h^t, t \} \), and those representing planner’s prescriptions for non-observable workers’ actions, \( \mathcal{P} = \{ a_t(h^t), s_t(h^t), \forall h^t, t \} \).

Feasibility imposes some restrictions on the set of contracts. In particular, the planner’s prescriptions, \( \mathcal{P} \), and instruments, \( \mathcal{I} \), are required to belong to the following sets:

\[
\mathbf{A} = \{ \mathcal{P} : \text{for each } h^t \text{ and each } t, (a_t(h^t), s_t(h^t)) \in [0, \vartheta]^2, a_t(h^t) + s_t(h^t) \leq \vartheta \text{ if } m_t = u, \text{ and } a_t(h^t) = s_t(h^t) = 0 \text{ if } m_t = e \},
\]

\[
\mathbf{T} = \{ \mathcal{I} : b_t(h^t) \geq 0 \text{ for each } h^t, \text{ with equality if } m_t = e, \text{ and } w_t(h^t) \geq 0 \}.
\]

Thus, a contract \( \mathcal{W} = (\mathcal{P}, \mathcal{I}) \) is said to be a feasible contract if \( (\mathcal{P}, \mathcal{I}) \in \mathbf{A} \times \mathbf{T} \). Notice that each contract prescribes sequences for consumption, \( c \), total effort, \( x \), and one period planner’s payoffs, \( y \), as follows:

\[
c_t(h^t) = \begin{cases} 
    b_t(h^t) + s_t(h^t) \pi & \text{if } m_t = u \\
    w_t(h^t) & \text{if } m_t = e 
\end{cases}
\]

\[
x_t(h^t) = \begin{cases} 
    a_t(h^t) + s_t(h^t) & \text{if } m_t = u \\
    \bar{a} & \text{if } m_t = e 
\end{cases}
\]

\[
y_t(h^t) = \begin{cases} 
    -b_t(h^t) & \text{if } m_t = u \\
    \omega - w_t(h^t) & \text{if } m_t = e 
\end{cases}
\]  \hspace{1cm} (2.2)

Define \( \mathcal{U}_t(\mathcal{W}; h^t) \) as the workers’ promised utility associated with the contract \( \mathcal{W} \), from a particular history \( h^t \). Likewise, define \( \mathcal{V}_t(\mathcal{W}; h^t) \) as the planner’s continuation value. That is,

\[
\mathcal{U}_t(\mathcal{W}; h^t) \equiv E_t \left[ \sum_{n=t}^{\infty} \beta^{n-t} (u(c_n(h^n)) - v(x_n(h^n))) | \mathcal{W}, h^t \right],
\]  \hspace{1cm} (2.3)
In (2.3)-(2.4) expectations are taken conditional on the contract $\mathcal{W}$ and on the initial node $h^t$. This is because the actions prescribed by the contract affect the probability measure on histories.

Since search effort and hidden labor market participation are not observable, the unemployment insurance scheme has to be \textit{incentive compatible}. That is, the levels of search effort and hidden labor market participation prescribed by the optimal contract have to be consistent with workers’ best responses.

\textbf{Definition 2.2.} A contract $\mathcal{W} = (\mathcal{P}, \mathcal{I})$ is \textbf{incentive compatible} if for each history and each period, there is not an alternative feasible action $\tilde{\mathcal{P}}$ providing a higher lifetime utility for the worker than the one induced by the contract; i.e., for each $h^t$ and $t$,

\[ U_t((\mathcal{P}, \mathcal{I}); h^t) \geq U_t((\tilde{\mathcal{P}}, \mathcal{I}); h^t) \text{ for each } \tilde{\mathcal{P}} \in \mathcal{A}. \] (2.5)

In the standard dynamic moral hazard model—without hidden labor markets, Pavoni (2007) shows that the constrained efficient contract “implies a weaker form of what is known as the immiserization result: if the workers utility function is unbounded below, then efficiency requires that workers expected discounted utility falls, with positive probability, below any arbitrary negative level”. In the environment presented in this paper, we show that the planner cannot punish the worker with a promised utility below the critical value $U$, where

\[ U \equiv \frac{u(s^* \varpi) - v(s^*)}{1 - \beta}. \] (2.6)

and $s^*$ solves

\[ u'(s^* \varpi) \varpi = v'(s^*). \] (2.7)

We call $U$ the \textit{lower bound for promised utility}. Notice that $U$ is the lifetime utility the worker attains when she does not search for a job at any period, receives no
unemployment payments, and chooses participation in the hidden labor market optimally. Lemma 2.1 formally states this result.

**Lemma 2.1 (No “immiserization”).** In the optimal contract there is not “immiserization”; i.e., the promised utility provided by the constrained efficient contract $W^*$ are bounded below by $\underline{U}$.

*Proof. See Appendix.*

The intuition for this result is transparent: the hidden labor market provides a protection for the workers that prevents “immiserization”. Whenever the planner pretends to deliver a promised utility below $\underline{U}$, the worker deviates to guarantee herself at least $\underline{U}$. Notice that the assumption that payments are non-negative is crucial to prevent “immiserization” in our environment. Negative unemployment payments could be understood in this environment as taxes to the hidden labor market activity. This possibility is ruled out because we have no knowledge of this kind of taxes in actual economies.

A direct implication of the last result is that the planner’s problem (2.10)-(2.13) is not well defined whenever $U < \underline{U}$ because the set of incentive compatible contracts is empty. In contrast, we prove that for each $U \geq \underline{U}$ there is always an incentive compatible contract providing $U$ to the workers. This result is formalized in lemma 2.2.

**Lemma 2.2.** Define $C(U; h^0)$ as the set of incentive compatible contracts providing promised utility greater or equal than $U$ given initial state $h^0$; i.e.,

$$C(U; h^0) \equiv \{W : \mathcal{U}_0((P, I); h^0) \geq U\}. \quad (2.8)$$

Then, for any $U \geq \underline{U}$, the set $C(U; h^0)$ is not empty.

*Proof. See Appendix.*

Recall that $u$ is bounded above $\bar{u}$. As a consequence, the lifetime utilities the contract can deliver is bounded above by $\bar{U} \equiv \frac{\bar{u}}{1-\beta}$. Define $P_m$ as the set of promised utilities lower than $\bar{U}$ such that $C(U; h^0) \neq \emptyset$ when $h^0 = m$. From
proposition 2.1 and lemma 2.2 it transpires

\[ \mathbf{P}_u = \mathbf{P}_e = \mathbf{P} \equiv \{ U \in \mathbb{R} : \bar{U} \geq U \geq U \}. \] (2.9)

The set \( \mathbf{P} \) is the set of promised utility the contract can deliver. In order to define the contractual problem, let \( \mathbf{F} \) be the set of all feasible contracts and \( U \) be the utility the contract has to deliver to the worker. When the planner offers a contract to a worker whose initial labor status equals \( m_0 \), it solves

\[
V^{m_0}(U) = \max_{W \in \mathbf{F}} V_0(W; h^0) \tag{2.10}
\]

s.t

\[
W \in \mathbf{C}(U, m_0), \tag{2.11}
\]

\[
U_0(W; h^0) \geq U, \tag{2.12}
\]

\[
U_t(W; h^t) \in \mathbf{P}, \forall h^t, t. \tag{2.13}
\]

Equations (2.11) and (2.12) are, respectively, the incentive compatibility and the promise keeping constraints. The contract solving problem (2.10)-(2.12) is called the constrained efficient contract and is denoted by \( \mathbf{W}^* \).

### 2.2.3 Recursive representation

Due to its history dependence, the problem (2.10)-(2.13) is not tractable for quantitative purposes. Fortunately, following the lines of Spear and Srivastava (1987) and Abreu et al. (1990) the problem has a recursive representation in which the history is completely encoded in two single variables, namely, promised utility and labor status. As implied by lemma 2.1 and 2.2, the planner’s problem is well defined if and only if promised utility is not smaller than \( U_0 \). Therefore, the recursive representation must explicitly impose that promised utility belongs to \( \mathbf{P} \).

Before presenting the recursive planner’s problem we introduce the definition of a recursive contract.

**Definition 2.3.** A recursive contract or unemployment insurance system, \( \mathcal{W}^R \), is a collection of functions specifying unemployment payments, \( b \),
wages after reemployment, \( w \), search effort, \( a \), hidden labor market participation, \( s \), and promised utility in any state \( m, U^m \), for each \( U \) in \( P \); i.e., \( \mathcal{W}^R = \{b(U), w(U), a(U), s(U), U^u(U), U^e(U)\} \).

A recursive contract can be divided into two components: those representing planner’s direct instruments, \( \mathcal{I}^R = \{b(U), w(U), U^u(U), U^e(U)\} \), and those representing planner’s prescriptions for workers’ actions, \( \mathcal{P}^R = \{a(U), s(U)\} \).

In order to define the planner’s problem recursively, let \( \mathcal{E} \) be the set of one-period feasible actions and \( \mathcal{M} \) the set of one-period best response of workers, taking as given unemployment payments and promised utility. That is,

\[
\mathcal{E} \equiv \{(a, s) : (a, s) \in [0, \vartheta]^2\},
\]

\[
\mathcal{M}(b, U^e, U^u) \equiv \{\text{argmax}_{(a, s)\in \mathcal{E}} u(b + s\varpi) - v(s + a) + \beta[p(a)U^e + (1 - p(a))U^u]\}.
\]

Then, the planner’s problem is represented by the following functional equations system:

\[
V^u(U) = \max_{b, a, s, U^u, U^e} -b + \beta[p(a)V^e(U^e) + (1 - p(a))V^u(U^u)], \quad (2.14)
\]

\[
s.t.
\]

\[
u(b + s\varpi) - v(s + a) + \beta[p(a)U^e + (1 - p(a))U^u] - U = 0, \quad (2.15)
\]

\[
(a, s) \in \mathcal{M}(b, U^e, U^u), \quad (2.16)
\]

\[
(U^u, U^e) \in \mathcal{P}^2, \quad (2.17)
\]

where

\[
V^e(U) = \max_{\tilde{U}^u, \tilde{U}^e, \omega} \omega - w + \beta[\delta V^u(\tilde{U}^u) + (1 - \delta)V^e(\tilde{U}^e)]
\]

\[
s.t.
\]

\[
u(w) - v(\bar{w}) + \beta[(1 - \delta)\tilde{U}^e + \delta\tilde{U}^u] - U = 0,
\]

\[
(\tilde{U}^u, \tilde{U}^e) \in \mathcal{P}^2.
\]

Equation (2.15) is the promise keeping constraint in recursive form, while equation (2.16) is the incentive compatibility constraint. Finally, (2.17) restricts
the value of promised utility to the set $P$. We call this constraint the *feasibility of promised utility constraint* (FPUC).

The inconvenience with the previous representation is that constraint (2.16) is a complicated object which makes the problem non-tractable. A common strategy is the so-called *first order condition approach*. To describe this strategy, consider an unemployed worker who takes as given $(b, U^u, U^e)$ and chooses $(a, s) \in E$ to maximize utility.\(^5\) For such a problem the first order condition are:

\[
\begin{align*}
\beta p'(a)[U^e - U^u] - v'(s + a) &= 0, \quad (ICC1) \\
-u'(b + s\varpi)\varpi + v'(s + a) &\geq 0. \quad (ICC2)
\end{align*}
\]

Now define $M^{foc}$ as the set of all $(a, s) \in E$ such that equations (ICC1)-(ICC2) are satisfied. The first order condition approach basically replaces constraint (2.16) in the recursive problem (2.14)-(2.17) by the condition that $(a, s) \in M^{foc}$.

Since $M \subseteq M^{foc}$, the solution to the problem using the first order condition approach may differ from the solution to the original problem. A recent example is provided in Kocherlakota (2004). In our environment, given $(b, U^u, U^e)$, the worker’s problem is convex. This implies that the first order conditions are both necessary and sufficient and hence, the first order approach is valid.\(^6\)

\(^5\)Under the assumption that $p$ satisfies Inada conditions and provided that in equilibrium $U^e > U^u$, we can guarantee that the non-negativity constraint for $a$ is never binding. Likewise, we can always set $\varpi$ big enough to avoid $a + s = \varpi$. Hence, we only focus on non-negativity constraint for participation in the hidden labor market.

\(^6\)First notice that $E$ is a convex set. Second, the function $f$, defined as $f(a, s) = u(b + s\varpi) - v(a) + \beta(p(a)U^e + (1 - p(a))U^u)$, is a strictly concave function provided that $U^e \geq U^u$. 
Using the first order condition approach, the recursive planner’s problem is

\[ V^u(U) = \max_{b,a,s,U^e,U^u} -b + \beta[p(a)V^e(U^e) + (1 - p(a))V^u(U^u)], \]

(PP)

s.t.

\[ u(b + s\varpi) - v(s + a) + \beta[p(a)U^e + (1 - p(a))U^u] - U = 0, \]

(PKC)

\[ \beta p'(a)[U^e - U^u] - v'(s + a) = 0, \]

(ICC1)

\[ -u'(b + s\varpi)\varpi + v'(s + a) \geq 0, \]

(ICC2)

\[ (U^e, U^u) \in P^2, \]

(FPUC)

\[ s \geq 0, \]

where

\[ V^e(U) = \max_{\tilde{U}^u,\tilde{U}^e,w} \omega - w + \beta[\delta V^u(\tilde{U}^u) + (1 - \delta)V^e(\tilde{U}^e)] \]

s.t.

\[ u(w) - v(\bar{w}) + \beta[(1 - \delta)\tilde{U}^e + \delta\tilde{U}^u] - U = 0, \]

\[ (\tilde{U}^u, \tilde{U}^e) \in P^2. \]

Henceforth, we refer to the previous problem as (PP). We use this problem to analytically characterize the first best contract—the optimal contract without information frictions—and the constrained efficient contract with linear effort-cost function. We also use it to numerically characterize the constrained efficient contract with non-linear effort-cost function.\textsuperscript{7}

\textsuperscript{7}Since there is no moral hazard during employment, optimality requires that promised utility during employment remains constant; that is \( \tilde{U}^e = U \). To see this, notice that the first order condition with respect to \( \tilde{U}^e \) together with envelope condition yield

\[ V^{\tilde{U}^e}(\tilde{U}^e) = V^e(U), \]

which implies \( \tilde{U}^e = U \) provided concavity of \( V^e \). The functions \( V^u \) and \( V^e \) turn out to be strictly concave in all the numerical experiments we performed.
2.2.4 First best contract

As a benchmark, we characterize the optimal contract when search effort and participation in the hidden labor market are both observable. The problem is the same as in (PP) but now there is no need to introduce constraints (ICC1), (ICC2), and (FPUC).

Let $\lambda$ be the multiplier associated with constraint (PKC). Then the first order conditions satisfied by first best contract are

$$(s :) \quad -u'(b + s\varpi)\varpi + v'(s + a) \geq 0,$$

$$(b :) \quad \frac{1}{u'(c)} = \lambda,$$

$$(U^u :) \quad \lambda + V^u(U^u) = 0,$$

$$(U^e :) \quad \lambda + V^e(U^e) = 0,$$

$$(a :) \quad \beta p'(a)[V^e(U^e) - V^u(U^u)] = \lambda[v'(a + s) - \beta p'(a)(U^e - U^u)].$$

Envelope condition implies

$$V^u(U) = -\lambda \implies V^u(U^u) = -\lambda'.$$

Combining (2.19),(2.20) and (2.23),

$$\frac{1}{u'(c)} - \frac{1}{u'(c')} = 0 \implies c = c'.$$

That is, the first best allocation implies a constant stream of consumption during unemployment.$^8$

Notice that under hidden effort, the first best allocation is not incentive compatible when planner values employment state and unemployment state differently. This is clear since (2.22) is inconsistent with (ICC1) whenever the RHS is different than zero. However, if the planner has control over search effort, the RHS of (ICC1) is zero observability regarding participation in the hidden labor market becomes immaterial. This property is formally stated in lemma 2.3.

$^8$Using a similar argument, it can be shown that there is also constant consumption across labor status.
Lemma 2.3. If search effort is observable, the first best allocation is achievable even if the planner cannot observe participation in the hidden labor market.

Proof. See Appendix. \(\square\)

The reason for the last result is that once search effort is guaranteed at the social optimum level, the planner and worker’s positions regarding the optimal level of participation in the hidden labor market are aligned. However, it does not imply we can discard (ICC2) when solving the problem with asymmetric information. When search effort is not observable, the cost and the benefit of an additional unit of participation in the hidden labor market are valued differently by the planner and by the worker. In particular, the planner internalizes the marginal effect of a unit of hidden labor market participation in the cost of providing incentives to make compatible the desired search effort. In contrast, with linear effort-cost function, there is no marginal change in the cost of effort and hence this situation does not arise. This result is presented in lemma 2.4.

Lemma 2.4. If \(v\) is linear, we can solve the problem (PP) without considering (ICC2).

Proof. See Appendix. \(\square\)

2.2.5 Constrained efficient contract

Now we turn to the case in which the planner does not observe either search effort or hidden labor market participation. As mentioned, we characterize this contract through the problem labeled as (PP).

Let \(\mu_1\) be the multiplier associated with (ICC1), \(\mu_2\) the multiplier associated with (ICC2) and \(\phi\) the one associated with the non-negativity constraint for \(s\). As before, let \(\lambda\) be the multiplier associated with (PKC). Finally, let \(\xi^u\) and \(\xi^e\) be the multipliers associated with constraints (FPUC) for \(U^u\) and \(U^e\), respectively.
The first order conditions for the planner’s problem are

\[(s:) \quad \lambda[u'(b + s\varpi)\varpi - v'(s + a)] - \mu_1 v''(s + a) = \mu_2[u''(b + s\varpi)\varpi^2 - v''(s + a)] + \phi,\]  

\[(b:) \quad \lambda = \frac{1}{u'(c)} + \mu_2 \frac{u''(c)\varpi}{u'(c)} = \frac{1}{u'(c)} - \mu_2 \rho_a(c)\varpi,\]  

\[(U^u:) \quad \lambda + V^u(U^u) + \frac{\xi^u}{\beta(1 - p(a))} = \mu_1 \frac{p'(a)}{1 - p(a)},\]  

\[(U^e:) \quad \lambda + V^e(U^e) + \frac{\xi^e}{\beta p(a)} = -\mu_1 \frac{p'(a)}{p(a)},\]  

\[(a:) \quad p'(a)[V^e(U^e) - V^u(U^u)] + \mu_1[3p''(a)(U^e - U^u) - v''(s + a)] + \mu_2 v''(s + a) = 0,\]  

where \(\rho_a\) represents the coefficient of absolute risk aversion. These conditions together with complementary slackness and the corresponding constraints completely characterize the contract solving (PP).

Using envelope condition and equations (2.26)-(2.27) we can get the expression that governs the profile of consumption during unemployment spell,

\[
\frac{1}{u'(c)} - \frac{1}{u'(c')} + \varpi[\mu_2\rho_a(c') - \mu_2\rho_a(c)] + \varpi = \frac{\xi^u}{\beta(1 - p(a))} = \mu_1 \frac{p'(a)}{1 - p(a)}. \tag{2.30}
\]

There, we can see that the presence of a hidden labor market affects the profile of consumption during unemployment in three ways: (1) a direct effect represented by the term \(\varpi\) which does not show up in the standard case, (2) an indirect effect through the term \(\mu_1\) and, (3) other direct effect through the term \(\xi^u\). The term \(\varpi\) disappears in the case of linear effort-cost function because the multipliers \(\mu_2\) and \(\mu_2'\) are zero in that case. However, it is particularly important in the case of non-linear effort-cost function because it gives rise to an intermediate phase in the path of unemployment payments. This phase will be analyzed in further details in Section 4.3.

\[\text{Notice that the term } \varpi \text{ in (2.30) is a function of the absolute coefficient of risk aversion. A similar result is found in an environment with moral hazard and hidden savings. See Abraham and Pavoni (2006).}\]
In order to provide a further characterization of the problem, we must either add assumptions about functional forms, or rely on numerical solution. In the remaining of the paper we develop both strategies.

2.3 Qualitative analysis

In this section the cost of effort is defined by

\[ v(x) = \alpha x. \]  

(2.31)

With this additional assumption, we can provide a detailed characterization of the constrained efficient contract.\(^{10}\)

When the cost of effort is defined as in (2.31), the dynamic of consumption is characterized by a decreasing path, until a period when it becomes flat. This result is formalized in a set of lemmas.

Lemma 2.5. The following results hold during unemployment:

1. When the constraint for promised utility if unemployed (FPUC) does not bind—i.e., \( U^u > U \)—the consumption path is strictly decreasing.

2. The sequence of consumption is bounded below by

\[ c \equiv u^{-1}(a/\varpi). \]

This consumption level is achieved whenever unemployed workers participate in the hidden labor market.

3. For sufficiently large unemployment spell, the lower bound for utility is reached; i.e., there exist a period \( t^* \) such \( U^u_{t^*} = U \).

\(^{10}\)Some results also hold under a more general specification of \( v \). Consider \( v(a,s) = g(a) + q(s) \) with \( g'(a) > 0 \) and \( q'(s) > 0 \). Then, the sequence of consumption during unemployed is still decreasing and bounded below. Likewise, the lower bound for promised utility is eventually reached. Moreover, when lower bound for promised utility is reached, the contracts prescribes zero payments and the lower bound for consumption. However, now it is not the case that participation in the hidden labor market occurs only when the lower bound for promised utility is reached.
Under concavity of the value function, consumption and promised utility can be characterized at the lower bound for promised utility.\(^\text{11}\)

**Lemma 2.6.** If \(V^u\) is concave:

1. The lower bound for utility is a fixed point of the function \(U^u\); i.e., \(U^u(U) = U\).

2. At the lower bound for promised utility, the optimal contract prescribes the consumption’s lower bound; i.e., \(c(U) \equiv \bar{c} = c\).

Putting these results together we can affirm that initially, if \(U > U\), consumption decreases over the unemployment spell until the lower bound for promised utility is reached. From that period on, consumption remains constant at its lower bound.

With a hidden labor market consumption does not necessarily equal unemployment payments. Hence, the characterization of consumption is not enough to identify the optimal path for unemployment payments. Therefore, we characterize unemployment payments under linear effort-cost function. The main result is presented in the following proposition.

**Proposition 2.1.** If \(V^u\) is concave, \(b_{t+1} < b_t\) when the constraint (FPUC) does not bind. If constraint (FPUC) binds, \(b_{t+1} = 0\). In other words, unemployment insurance payments decrease until the lower bound for promised utility is reached. After that moment they become zero.

**Proof.** When the lower bound for \(U\) does not bind, consumption is strictly decreasing and above \(c\). Since in this phase \(s_t = 0\), \(b = c\) and, therefore, unemployment payments also decrease. When \(U = U\), \(b\) is constant at \(b\) because \(U^u(U) = U\). Suppose, by contradiction, \(b > 0\). By incentive compatibility the contract must deliver an utility consistent with worker’s best response, that is, worker’s utility is

\[
\bar{U} \equiv \max_{a,s} u(b + \varpi s) - v(a + s) + \beta[p(a)U^e + (1 - p(a))U].
\]

\(^{11}\)Stating these results conditional on the concavity of the value function and verifying that it is in fact concave in the quantitative results is the same strategy followed by Hopenhayn and Nicolini (1997).
Moreover, this utility must be as good as any deviation, in particular one in which search intensity is set to zero and s is set optimally. That is,

\[ \hat{U} \geq \max_s u(b + \varpi s) - v(s) + \beta U \equiv \hat{U}. \]

Since \( u \) is strictly increasing,

\[ \hat{U} > \max_s u(\varpi s) - v(s) + \beta U \equiv U. \]

This implies that there exist a feasible reduction in \( b \) which increase planner’s surplus. Therefore \( b > 0 \) is not optimum. \( \square \)

**Corollary 2.1.** Participation in the hidden labor market takes one of two values:

1. When lower bound for promised utility is not binding, \( s = 0 \).
2. When lower bound for promised utility is binding, \( s = s^* > 0 \).

Previous corollary highlights a key property of the optimal contract in the case of linear effort-cost function, namely: *initially, the planner does not encourage participation in the hidden labor market.* The explanation for this somehow unexpected results is that, once the planner induces participation, it completely looses ability to punish the worker through reduction in consumption. Moreover, as we will see in the numerical exercise, this property is still present, to some extent, in the case of non-linear effort-cost function.

In summary, there are two different phases during unemployment. During the first phase, payments (consumption) decrease smoothly without reaction of hidden labor market participation. It is because the payments are relatively high and hence the marginal cost of participation is higher than its marginal benefit. However, as payments decline, the marginal benefit of participation increases, and participation is eventually optimal. At that point the last phase of unemployment starts. In this phase, with positive participation, linearity and incentive compatibility uniquely determined the consumption level (\( c \)). At that level of consumption, the promised utility can be satisfied with zero payments (any positive payments deliver a higher lifetime utility). Thus, during this phase, zero payments become optimal.
A feature of the optimal unemployment insurance is certain lack of smoothness. In particular, at a certain period, payments drop to zero and hidden labor market participation jumps away from zero. This lack of smoothness should be striking just if it is reflected on consumption. This is not the case here since in this phase participation compensates payments drops, and consumption is perfectly smooth.

2.4 Quantitative analysis

In this section we carry out a quantitative analysis for the case of non-linear effort-cost function. First, we present the calibration strategy. Then, we describe the dynamics of unemployment benefits, job search effort and informal sector participation under the optimal contract. Finally, we derive welfare implications associated with the implementation of the optimal contract. The algorithm is presented in the appendix.

2.4.1 Calibration

Functional forms are standard. Preferences over consumption goods are described by a constant relative risk aversion function with parameter $\sigma$, while the cost of effort function is $v(a) = a^{\gamma}/\gamma$. Finally, the job finding probability is $p(a) = 1 - \exp(-\rho a)$.

A key preliminary question regarding the calibration is, Which economy should be considered as benchmark for our calibration? Quantitative research usually uses the U.S. economy. However, our environment naturally applies to developing countries, where unemployment rate and informal sector participation are high. Working with developing countries involves certain problems. First, it is hard to find reliable data for the informal sector. More importantly, there is not accurate information about unified and explicitly designed unemployment insurance systems. We calibrate our model using data for Spain due to the importance of its informal sector and the availability of information. In particular, we use data for the end of the 90’s, when unemployment and the size of the informal sector were relatively high. We are aware that in the last 25 years unemployment rate, informal sector participation, and labor markets regulations have changed significantly
in Spain. Therefore, the conclusions of this exercise may not apply to the current Spanish economy where the labor market performance has substantially improved. However, these results may still be valid for developing countries, whose current labor market structure is similar to that used in the calibration.

Most of the parameters are taken from previous literature. We interpret each period as a month and set the discount factor $\beta = 0.994$ (annual interest rate of 7.5%). We normalized workers’ productivity, $\omega$, and effort in the formal labor market, $\bar{a}$, to one. Data about average tenure in Spain prescribes $\delta = 0.015$. The relative risk aversion coefficient, $\sigma$, is set equal to 2, which is a common value in the real business cycle literature.

The calibration strategy uses the actual unemployment insurance scheme instead of the optimal contract. That is, for plausible combinations of $\rho$ and $\varpi$, we compute the stationary distribution of an economy in which agents take optimal decisions considering the actual unemployment insurance scheme. From this distribution, we calculate unemployment and informal sector participation. These outcomes are then compared with those in the data. The calibration consists in finding values of $\rho$ and $\varpi$ compatible with our targets. This strategy is natural since unemployment and informal sector participation are outcomes of the optimal reaction of individuals to the actual system.

The calibration matches unemployment rate and informal sector size in Spain for 1997. Unemployment rate, $\mu$, was 18.5 percent while production in the informal sector as a proportion of the GDP, $\zeta$, was 23 percent—Schneider and Enste (2000).

In Spain, unemployed workers qualify for benefits if they have contributed to social security over a minimum period in the previous years. Depending on their employment duration, unemployed workers receive payments from 4 months to a maximum of 2 years. After that period, they qualify for unemployment subsidy. A typical worker receives 70 percent replacement ratio during the first 6 months, and 60 percent for the next 18 months. Unemployment subsidy is 80 percent of

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12 According to Arranz and Garca-Serrano (2004) the average tenure in the private sector is around 6 years.
13 Here we depart from the traditional values in the literature of optimal unemployment insurance. Pavoni (2007) works with log preferences ($\sigma = 1$) while Hopenhayn and Nicolini (1997) set $\sigma = 0.5$. 
the minimum wage. This implies a replacement ratio of 30 percent after two years of unemployment.\textsuperscript{14} Taxes financing the system are set at 5 percent.

Workers decide effort and hidden labor market participation to maximize their lifetime utility, taking as given the actual unemployment insurance scheme. This scheme is represented by $(\tau, \{b_n\}_{n=1}^\infty)$, where subindex $n$ refers to unemployment duration. Since $b_n$ is constant for $n \geq n^* = 25$, the agent’s problem has the following recursive representation

$$\hat{V}_n^u = \max_{a,s} u(b_n + s\varpi) - v(a + s) + \beta[p(a)\hat{V}^e + (1 - p(a))\hat{V}^u_{n+1}], \forall n \in \{0, 1, ..., n^* - 1\}$$

$$\hat{V}_{n^*}^u = \max_{a,s} u(b_n + s\varpi) - v(a + s) + \beta[p(a)\hat{V}^e + (1 - p(a))\hat{V}_{n^*}^u], \forall n \geq n^*$$

$$\hat{V}^e = u(\omega(1 - \tau)) - v(\bar{a}) + \beta[(1 - \delta)\hat{V}^e + \delta\hat{V}_0^u],$$

where $\hat{V}^u_n$ is lifetime utility of an unemployed agent with unemployment spell $n$ and $\hat{V}^e$ corresponds to lifetime utility of an employed agent.

The backward solution to the previous problem provides policy functions $a_n$ and $s_n$ that can be used to compute the stationary distribution of agents over unemployment spell, $\bar{\Omega}$. The stationary distribution is a fixed point of the transition law

$$\Omega'_0 = \left(1 - \sum_n \Omega_n\right)\delta$$

$$\Omega'_n = (\Omega_{n-1}(1 - p(a_{n-1})) \forall n < n^*,$n

$$\Omega'_{n^*} = [\Omega_{n^*-1}(1 - p(a_{n^*-1})) + \Omega_{n^*}(1 - p(a_{n^*}))),$$

In the stationary distribution, we measure the unemployment rate and the informal sector size by

$$\mu = \sum_n \bar{\Omega}_n(1 - s_n),$$

$$\zeta = \frac{\varpi \chi}{\varpi \chi + (1 - \sum_n \Omega_n) \omega},$$

where $\chi = \sum_n \bar{\Omega}_n s_n$ is the average participation in the hidden labor market. Table 2.2 summarizes our calibration. With these parameters the artificial economy

\textsuperscript{14}This number is obtained dividing 80 percent of the minimum wage by the income per capita.
reproduces $\mu = 0.1787$ and $\zeta = 0.2326$, very close to our targets, 0.185 and 0.23, respectively.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
<th>Basis and Targets</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.994</td>
<td>Discount factor</td>
<td>Interest rate</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>2</td>
<td>Relative risk aversion coefficient</td>
<td>Standard</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>2</td>
<td>Search cost elasticity</td>
<td>Standard</td>
</tr>
<tr>
<td>$\omega$</td>
<td>1</td>
<td>Productivity in formal sector</td>
<td>Normalization</td>
</tr>
<tr>
<td>$\bar{a}$</td>
<td>1</td>
<td>Effort when employed</td>
<td>Normalization</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.015</td>
<td>Separation rate</td>
<td>Average tenure</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.045</td>
<td>Parameter in probability of finding a job</td>
<td>Unemployment rate</td>
</tr>
<tr>
<td>$\varpi$</td>
<td>0.415</td>
<td>Wage in informal sector</td>
<td>Informal sector size</td>
</tr>
</tbody>
</table>

### 2.4.2 Optimal policy

The inclusion of a hidden labor market is crucial in determining the optimal path of unemployment payments. Figure 2.1 shows the path for search effort and hidden labor market participation while Figure 2.2 depicts the optimal unemployment payments and consumption. In both figures the initial promised utility is associated with a “typical” unemployed individual with whom the planner breaks even.\(^{15}\)

Figure 2.1 indicates that the optimal participation is initially zero and search effort increases with unemployment spell. This result resembles those in Hopenhayn and Nicolini (1997). After approximately 2.5 years of unemployment, when effort peaks, participation jumps and search effort drops. After that point, participation increases and search effort decreases gradually toward fairly steady levels. Figure 2.2 shows that unemployment payments and consumption are equal and decrease smoothly while the agent is not participating in the hidden labor market.

\(^{15}\)A “typical” unemployed individual is one that entered to the job market 10 years ago. Individuals enter the market with the promised utility that solves $V^u(U) = 0$. Before setting it equal to zero, the planner’s surplus function was adjusted to account for administrative costs. According to data from the “Ministerio de Trabajo y Asuntos Sociales”, available at www.mtas.es, slightly above 10 percent of the expenditures on social assistance went to administrative costs during 1999.
When participation jumps, payments drop to zero and consumption is slightly increasing thereafter. The rise in consumption is related to the decrease in promised utility during this phase. Since payments are at zero, decreasing promised utility means that taxes after reemployment will be higher. As a consequence, unemployed individuals substitute search effort by hidden labor market participation and, therefore, enjoy higher consumption.

Some aspects are remarkable from this behavior. First, required effort is not monotonically increasing in the unemployment spell. The reason is that in order to induce an increase in participation, it is optimal to reduce the search effort such that the marginal cost of inducing participation falls. Second, in the early phase of unemployment, workers are fundamentally searchers with no participation in the hidden labor market. It is worth noting that our contract does not imply hidden labor market participation during the first 2.5 years of unemployment. Thus, it is very likely that unemployed individuals find a job before they start to participate in the hidden labor market. Finally, once participation is induced, unemployment payments drop sharply down to zero. That is, with a hidden labor market, it is optimal to give no payment for workers with a sufficiently large unemployment spell. The next subsection provides further analysis of the optimal unemployment insurance scheme.
2.4.3 More on the optimal profile of unemployment payments

In this section we take a closer look to the optimal payment scheme for our benchmark quantitative economy. In order to identify the key elements shaping the payment profile in each phase, we also show the optimal payments’ paths under two alternative environments. The first environment is the same presented in Pavoni (2007), parameterized according to Table 2.2. The planner’s problem in this case is a simplification of our problem (PP). In particular, we impose \( s = 0 \) and drop the constraint (ICC2). The second environment that we consider is one in which participation in the hidden labor market is observable by the planner. The problem is basically (PP) but without the constraint ICC2. This second alternative environment differs from Pavoni (2007) in that \( s \) is not set to zero but chosen optimally. In the three environments the lower bound for promised utility is given by the system (2.6)-(2.7).

Figure 2.3 shows the unemployment payments profile in each case. In addition, Figure 2.4 shows the left hand side of the constraint (ICC2) with a negative sign. A positive value in this figure indicates that workers want to participate in the hidden labor market more than what it is prescribed by the contract.\(^{16}\)

How does the payments’ scheme look like in each environment? First, let us focus on Pavoni (2007). Unemployment payments decrease with unemployment spell, and, more importantly, as the promised utility approaches its lower bound, payments decrease faster and faster. Eventually, the worker cannot be further punished and the payments stabilize at a positive value. For the non-observable-\( s \) environment, it is also the case that the payments profile becomes steeper with the unemployment spell. The main difference relative to Pavoni (2007) is that now payments drop to zero. This can happen because in this context workers can smooth out consumption by participating in the hidden labor market. In terms of incentive compatibility, it is remarkable that the plan prescribed by the optimal contract in this environment is not implementable if \( s \) is not observable. As Figure 2.4 shows, the constraint (ICC2) is violated by this contract. Let us

\(^{16}\)This term is meaningless for the Pavoni (2007)’s case and it is therefore excluded.
now focus on our benchmark case. In phase 1, payments decrease even faster than in the alternative environments. This happen while the constraint (ICC2) is not binding with zero hidden labor market participation. However, eventually the constraint (ICC2) becomes binding and a fast decrease in payment is not compatible with zero participation. There is where our contract starts to differ more from other environments. In this case, the payment profile becomes flatter to prevent the worker from participation in the hidden labor market. The flattening in the payment scheme is the main implication of the non-observability of hidden labor market participation. During the whole phase 2, participation is set at zero but the constraint (ICC2) is binding. Here, incentives are mainly introduced by decreasing the promised utility. Eventually, the accumulated drop in promised utility is only implementable by an abrupt decline, down to zero, in unemployment payments. The elimination of payments and the associated jump in hidden labor market participation represent the beginning of phase 3. As in the case in which participation in the hidden labor market is observable, the optimal contract provides no payment for workers with sufficiently large unemployment spells. In sum, the non-observability of the hidden labor market is responsible for the flattening in the payment schedule (phase 2), while the existence of a hidden labor market (that can be used as a source of income) is responsible for the optimality of the elimination in unemployment payments for sufficiently long unemployment spell.

Phase 2 and 3 are the novel features in the payment profile relative to the standard optimal unemployment insurance problem. The question is: How relevant are these two phases from a quantitative point of view? In order to answer this question we perform the following exercise. We start the economy with a large number of unemployed workers to whom the contract deliver the same lifetime utility. We set this initial lifetime utility such that the planner breaks even with each worker. Then, we simulate large paths for labor status, as well as for continuation values. If the worker is unemployed, we also simulate paths for participation in the hidden labor market and effort. Obviously, all the paths are generated using the optimal contact. Then, for each group of potential experience, we compute the distribution of unemployed workers across the different phases. Table 2.3 reports
Table 2.3: Conditional distribution of unemployed individuals across UI phases

<table>
<thead>
<tr>
<th>Potential experience</th>
<th>Phase 1</th>
<th>Phase 2</th>
<th>Phase 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.260</td>
<td>0.602</td>
<td>0.138</td>
</tr>
<tr>
<td>10</td>
<td>0.235</td>
<td>0.657</td>
<td>0.108</td>
</tr>
<tr>
<td>15</td>
<td>0.230</td>
<td>0.661</td>
<td>0.109</td>
</tr>
<tr>
<td>20</td>
<td>0.225</td>
<td>0.671</td>
<td>0.103</td>
</tr>
<tr>
<td>25</td>
<td>0.218</td>
<td>0.677</td>
<td>0.105</td>
</tr>
<tr>
<td>30</td>
<td>0.211</td>
<td>0.690</td>
<td>0.099</td>
</tr>
<tr>
<td>35</td>
<td>0.207</td>
<td>0.695</td>
<td>0.098</td>
</tr>
</tbody>
</table>

the results for some selected groups of potential experience.\(^{17}\)

Two aspects are remarkable from the table. First, phase 2 is quantitatively the most important. Indeed, between 60% and 70% percent of unemployed workers with five or more years in the labor market are in this phase. Second, there exist a significant measure of workers receiving no payments. As a matter of fact, around 10% of unemployed workers with five or more years in the labor market are receiving zero payments.

Finally, it is worth noticing that the preceding analysis suggests that a simple

\(^{17}\)In our model potential experience is equivalent to years in the labor market.
2-step system looks closer to the optimal payment scheme than previous literature suggested. The initial two phases have to be generous enough to keep the worker out of the secondary labor market, while the last step is characterized by no payments at all. This is important because many countries have unemployment protection programs with that structure (e.g. United States).

### 2.4.4 Budget savings

In this section we quantify the budget savings of switching from the stylized Spanish scheme presented in the previous section to the optimal program considering participation in the hidden labor market.

As described above, during the calibration we calculate the stationary distribution over unemployment spell, \( \bar{\Omega} \). With this distribution at hand, we can compute the present value of providing unemployment insurance forever using the Spanish system \( (C_S) \),

\[
C_S = \frac{1}{r} \left[ \sum_n \bar{\Omega}_n b_n - \left( 1 - \sum_n \bar{\Omega}_n \right) \omega (1 - \tau) \right].
\]

Similarly, using the solution to the recursive problem of an unemployed agent in Spain, \( (\hat{V}^u_n, \hat{V}^e) \), we transform the distribution over \( n \) to a distribution over \( U \). Then, we calculate the cost of providing the same distribution of lifetime utility with the optimal contract \( (C_N) \),

\[
C_N = - \left[ \sum_n \bar{\Omega}_n V^u(\hat{V}^u_n) + \left( 1 - \sum_n \bar{\Omega}_n \right) V^e(\hat{V}^e) \right].
\]

Important inter-temporal budget savings are associated with the implementation of the optimal contract. Given the stationary distribution of lifetime utility, the cost under the Spanish scheme is 42.18. On the other hand, providing the same lifetime utility distribution with the optimal contract costs 21.65. These savings come from two sources. First, the standard savings arising from the Hopenhayn and Nicolini (1997) type of contract; that is, from a contract considering deviation form search effort toward leisure. Hopenhayn and Nicolini (1996) calibrate this optimal contract for the Spanish economy and find savings ranging between
20%-37%. We believe the additional savings are strictly related with the implementation of a contract considering deviations from search effort toward hidden labor market participation.

### 2.5 A type-dependent unemployment insurance?

Separation rates may differ across workers. Therefore, it may be important to design a $\delta$-dependent unemployment insurance. Wang and Williamson (2002) perform an exercise in these lines. In the case of fully experience rating, in the sense that taxes in each “industry” fully fund unemployment benefits for workers in that industry, they find that the replacement ratio is lower for agents in industries where the nature of production implies frequent transitions between employment and unemployment. However, they find it is optimal to extend benefits indefinitely for each type.

In this section we answer a quantitative question: Do hidden labor markets reinforce the importance of a $\delta$-dependent unemployment insurance? To that end, we compute the optimal contract for three values of $\delta$ (0.01, 0.015 and 0.02) for an economy with an important hidden labor market ($\varpi = 0.415$)—as calibrated for Spain—and for an economy with a very small one ($\varpi = 0.05$). For the remaining parameters we use the values in the benchmark calibration in both economies.

This experiment fits in the environment of Wang and Williamson (2002) in which there are workers employed in different industries, with different separation rates, and a worker who is re-employed after an unemployment spell always returns to the same industry. Following the case of fully experience rating in Wang and Williamson (2002), we start the experiment at the level of payments for which promised utility results in a balanced budget for each $\delta$ considered.

Figure 2.5 shows that without an important informal sector the optimal payments’ dynamics is not significantly affected by the separation rate. In fact, as in Wang and Williamson (2002), it just changes the level at which the payments start without affecting payments durations. In contrast, when we consider an economy with an important hidden labor market—see Figure 2.6, the dynamics of payments varies with the separation rate. In particular, the duration of pay-
ments increases from 29 to 55 months when the separation rate increases from 0.01 to 0.02. This result clearly prescribes an easy-to-implement contract for the optimal δ-dependent unemployment insurance in an economy with a hidden labor market. That is, the duration of payments should be shorter for workers with higher separation rates. Thus, considering the existence of a hidden labor market is also relevant because it reinforces the importance of an δ-dependent unemployment insurance system.

The intuition behind this result goes as follow. The planner considers employment of workers with lower separation rates more “valuable”. Therefore, the planner is willing to delay participation in the hidden labor market by extending the initial phase of unemployment to promote high search intensity for longer periods.

2.6 Concluding Remarks

The problem of the optimal unemployment insurance has been addressed from the perspective of a principal-agent framework since the pioneering work of Shavell and Weiss (1979). Early literature mainly focuses on the moral hazard problem arising under non-observability of search effort. More recent works emphasize also the non-observability of assets. However, the implications of the existence
of a hidden labor market were unknown. This was an important omission in the unemployment insurance literature because the incorporation of a hidden labor market has non-trivial consequences. Moreover, understanding the role of a hidden labor market is highly relevant because it represents a sizeable segment of the labor market in many countries. This paper fills this gap by providing an environment in which unemployed workers can simultaneously search for a job and participate in the hidden labor market. The main conclusions are:

1. The presence of a hidden labor market imposes an endogenous lower bound for promised utility and, unless the effort-cost function is linear, an extra incentive compatible constraint must be added.

2. With linear effort-cost function, when the optimal unemployment insurance can be characterized analytically, there are two important phases. First, consumption decreases during unemployment and workers do not participate in the hidden labor market. Second, for sufficiently large unemployment spells, the lower bound for promised utility is reached. After that period, promised utility during unemployment remains constant at the lower bound. Moreover, during this phase consumption remains constant, payments become zero and participation jumps to a positive constant.

3. With non-linear effort-cost function, there exist an intermediate phase. There, participation in the hidden labor market is zero but individuals are indifferent between increasing participation or not (the incentive compatibility constraint for \( s \) is binding). This implies that the payment profile must be relatively flatter to avoid participation and to keep search effort high. This phase, together with the subsequent drop in payments, differentiate this work from previous literature. As a whole, the optimal unemployment insurance with a hidden labor market is simple—with a relatively flat phase and with no payments for long term unemployed workers—and looks similar to unemployment protection programs in many countries.

4. In the quantitative exercise, fairly large budget savings—compared with the Spanish system—are obtained when the unemployment insurance system takes into account the existence of a hidden labor market.
This paper suggests two main policy lessons. The first one has to do with the inconvenience of providing unemployment payments indefinitely. This is a very important difference with the previous literature, where efficiency requires that unemployment payments last indefinitely. The existence of a hidden labor market together with payments forever, guarantee sufficiently high consumption even after a very long period of unemployment. As a consequence, long unemployment spells are not serious threats to encourage search effort. The second lesson is associated with the provision of an type-dependent unemployment system. When the hidden labor market is not important, the optimal payments’ dynamic does not change significatively with the separation rate. However, the profile of unemployment payments—in particular payments duration—dramatically changes with the separation rate when the hidden labor market is important. Since the separation rates are usually different among worker groups, this result suggests that an type-dependent contract may be optimal. Specifically, the numerical exercise indicates that the duration of payments should be shorter for the high-separation-rate group.
Chapter 3

Foreign Debt and Economic Performance: Credit Barriers in a Firm Dynamics Model

3.1 Introduction

It is well documented that small emerging economies behave differently than developed ones. Within the context of business cycle indicators, sovereign interest rates are highly counter-cyclical, very volatile and significantly higher than the world interest rate. In addition, output is twice as volatile as in developed economies, and trade balance is strongly counter-cyclical, while it is only weakly counter-cyclical for developed economies—for recent works see Aguiar and Gopinath (2007) and Neumeyer and Perri (2005). Beyond the context of business cycles, emerging economies have a higher propensity to experience financial crises of different types e.g. currency, debt, and banking crises.

In this paper, we explore to what extent this contrasting behavior can be attributed to corporations’ limited access to international credit markets and costly sovereign default. The main result is that the interaction between firm distress and sovereign liquidation cost works as a transmission mechanism by which the effects of shocks persist, amplify, and spread out. Hence, the paper suggests a
channel through which financial integration reduces cyclical volatility.\footnote{This is in line with recent evidence. De Souza (2004) finds that for Baltic and Central Eastern European countries, financial integration leads to a reduction in cyclical volatility, both in the short and in the long run.} We also find that in this environment—characterized by sudden jumps in interest rates—mature firms over-accumulate capital in order to avoid taking expensive loans in the future. This precautionary capital formation tends to reduce the volatility and persistence of output in big firms, partially offsetting the impact of the amplification mechanisms on aggregate output.

The proposed environment focuses on the connection between firm dynamics, financial fragility and the business cycle. Corporate sector dynamics have taken center stage in explaining the occurrence of systemic financial crises in recent years. The most dramatic example is the Easter Asia crisis.\footnote{During the Asia crisis, Claessens et al. (2003) identify 664 firms as financially distressed, out of a sample of 1,472 publicly traded firms in five Easter Asian countries.} In addition, the links between the risk of financial crises, in particular debt crises, and the business cycle have been stressed in the literature of sovereign default (see Aguiar and Gopinath (2006) and Arellano (2006)). However, most of this literature uses a representative firm framework, which prevents it from fully capturing the whole dynamics of the corporate sector and its connection with financial fragility and the business cycle. In contrast, we introduce a model of firm dynamics that allows us to explicitly study how the dynamics and structure of the corporate sector affects the financial fragility of the economy and, as a result, the business cycle.

Existing models of firm dynamics assume that financial intermediaries face a risk-free state-independent interest rate as opportunity cost—as in Cooley and Quadrini (2001). The “open economy” interpretation of this type of models is that firms directly interact with international lenders. This is not a valid assumption for emerging economies where a significant fraction of corporate debt is associated with domestic contracts.\footnote{For instance—excluding China and India—between 1997 and 2003, foreign debt represented only 27% of the total debt for corporations in emerging economies (IMF Global Financial Stability Report, 2005).} This suggests that firms in developing countries face barriers to international credit markets.

To mimic this, we propose a financial structure that explicitly imposes bor-
rowing barriers to corporations. In particular, we assume that corporations can only borrow from a risky domestic intermediary who interacts with international lenders to clear the domestic credit market. We also assume that, when providing funds, the domestic intermediary charges actuarially fair interest rates.

Using a single entity—charging actuarially fair interest rates—as a channel for corporate credit in emerging economies does not seem to be a strong assumption. In many of these economies, state-banks control a sizeable share of the bank industry and it is reasonable to argue that they privilege allocation of funds over maximization of profits.\(^4\)

We model a small open economy subject to both aggregate and firm-specific shocks. It is the case that under certain realizations of the shocks, the domestic intermediary is unable to pay back her total debt with international creditors. In such cases, there exists a proportional to debt liquidation cost. Independently of who pays this cost, it introduces a wedge between what is recovered by the domestic intermediary and what is obtained by international lenders.\(^5\)

There exists strong evidence suggesting that default episodes involve important pecuniary costs for both sides of the debt contract. From the defaulter’s side, output losses in the wake of sovereign default appear to be very large, around 7\%, (see De Paoli et al. (2006)). Moreover, losses are even higher when default occurs during currency crises and/or banking crises.\(^6\) From the lender’s side, it is well documented that there are significant investor losses in sovereign debt restructurings, known as “haircuts”. Sturzenegger and Zettelmeyer (2005) find

\(^4\)The case of South Asia and the Middle East is remarkable; by 1995 they had the largest share of state ownership, at 90\% and more than 50\% respectively. By contrast, industrial countries had a prevalence of state ownership of banks close to 20\% (see Yeyati et al. (2004)). Latin America has a level of state ownership of banks similar to the developing countries’ average—more than 60\% in 1985 and close to 50\% in 1995.

\(^5\)This cost may also reflect state dependent intermediation cost (higher during crises), or stronger difficulties to liquidate firms when a large scale corporate bankruptcy—leading to sovereign default—happens. For a model dealing with costly financial intermediation in a explicit banking model see Diaz-Gimenez et al. (1992). For a model including an upper bound to the total level of capital that can be liquidated in the economy see Arellano and Kocherlakota (2008).

\(^6\)Sovereign defaults rarely occur in isolation. Often, they coincide with banking and/or currency crises. In fact, some studies suggest that almost one half of default episodes are actually triple (sovereign, currency and banking) crises, (De Paoli et al. (2006)).
that there are substantial differences between average investor losses across debt restructurings, with most of them clustered in the 25-35% range.\footnote{The authors measure “haircuts” as the difference between the net present value of original and new (restructured) instruments, using the immediate post-exchange yield of the new instrument to discount both payment streams.}

In a environment featuring corporations’ indirect access to international credit markets and costly liquidation in defaulting states, each firm’s interest rate depends on the whole structure of the corporate sector and no longer exclusively on idiosyncratic factors; that is, there are \textit{spill-over effects} among corporations. The presence of these spill-over effects substantially influences the economic performance. First, it introduces amplifying mechanisms affecting the business cycle. Second, it could fuel a contagious effect leading to a major economic crisis. To the best of our knowledge, this is the first paper studying the importance of spill-over effects arising from credit barriers and foreign debt liquidation cost in a firm dynamics model.

In our framework, the behavior of the economy is determined by the following chain of interactions. The structure of the corporate sector influences financial fragility and hence the recovery rate for the domestic lender, as well as the probability of default. This recovery rate, together with the interaction of default risk and cost of default, shapes the sovereign interest rate and the corporate interest rates schedule. Finally, the corporate interest rates schedule influences firms’ investment decisions and, as a consequence, the structure of the corporate sector.

In the numerical exercise, we compare some business cycle statistics for alternative values of the sovereign debt liquidation cost, $\mu_s$. In our baseline environment we set $\mu_s = 0$. We show that in such a case, the corporate interest rate schedule is equivalent to that with direct lending.\footnote{By direct lending we mean a situation in which corporations sign debt contracts directly with international creditors.} In other words, in this case, credit barriers do not play a role in shaping the business cycle. We label this environment the \textit{non-externality case}, because corporate interest rates depend exclusively on borrower’s characteristics. Then, we introduce foreign debt liquidation cost and study its implications for the business cycle. We label this case \textit{the externality case} because now the interest rate charged to each firm depends also on the whole
structure of the corporate sector.

The numerical exercise indicates that the presence of credit barriers to corporations, together with sovereign default costs, works as an amplification/propagation mechanism, reinforcing the negative correlation between sovereign interest rate and output, and increasing the volatility of interest rates, and the response of the sovereign interest rate to adverse shocks. It also increases the persistence of the sovereign interest rate, the country risk premium and to a lesser extent, the persistence and volatility of aggregate output. Since very volatile and strongly counter-cyclical sovereign interest rates are distinguishing features of emerging economies, the findings suggest that financial barriers may play a crucial role in shaping the business cycle for developing countries.\footnote{Previous literature is able to reproduce counter cyclical interest rates; but the relationship between output and sovereign interest rate in those models is not as strong as in the data. It is also stressed in the literature that existing models perform poorly in matching the volatility of the interest rate.} The smaller impact on aggregate output is rooted in a precautionary capital formation behavior explained next.

To understand the nature of the amplification/propagation mechanism, notice that the sovereign interest rate is affected by the probability of default, which depends on the vulnerability of the corporate sector. The weaker the corporate sector, the higher the sovereign interest rate, not only because firms have less ability to pay, but also because it is more likely that the sovereign debt liquidation cost has to be paid. Now, suppose that some firms get a bad shock. In the non-externality case, the whole economy becomes riskier because these—and only these—particular firms are becoming riskier. However, in the externality case, even those firms not getting the bad shock become riskier because they now face a higher interest rate as a result of a higher probability that the liquidation cost is paid. That is, the shocks on some firms propagate to the whole system, magnifying the response of sovereign interest rate to the shocks. A similar mechanism works when negative shocks lead to firm exit and, as a consequence, to a change in the age-composition of the corporate sector. Since in our environment new/smaller firms are riskier, the higher the fraction of newly born firms is, the higher the sovereign interest rate is. In the externality case, an increase in the
fraction of new firms makes existing firms more vulnerable, which magnifies the effect of a change in the age-composition of the corporate sector on the sovereign interest rates. Notice that the sovereign interest rate determines the structure of corporate interest rates and then, the investment decisions of firms. Therefore, the stronger the response of the sovereign interest rate, the stronger the effect on investment and as a result, the more persistent the effects of shocks. In sum, the interaction between financial fragility and sovereign default costs turns out to be an amplification/propagation mechanism with key implications for economic performance; in particular, for the business cycle.

The effects on aggregate output are however smaller than those on interest rates. The explanation relies on precautionary capital formation done by mature firms in some states of the economy. More precisely, when the aggregate shock is low, firms with enough resources (mature firms) accumulate more capital in the externality case than when $\mu_s = 0$. This over-accumulation is the optimal response to the risk of being involved in expensive loans in future periods. However, when the aggregate shock is high, the risk of taking expensive loans in subsequent periods is smaller and then, firms do not accumulate significatively more capital than in the non-externality case. As a consequence, the difference between the capital levels toward the economy moves in booms and in recensions is smaller in the externality case. This tends to reduce the volatility and persistence of output, partially offsetting the effects of the spill-over mechanism.

This paper relates to the work focusing on how financial frictions work as amplification mechanisms for the business cycle. One of the most prominent examples is Kiyotaki and Moore (1997) credit cycle model, in which collateral constraints play an important role in generating large and persistent fluctuations in output. In their model, durable goods play a dual role: they work as inputs in the production function and they are used as collateral for loans. Moreover, default does not exist because the debt is fully collateralized. In their model, borrowing credit limits are affected by the price of the collateral which is affected by the size of the credit limits. They find that the interaction between asset prices (prices of durable goods) and credit limits works as an amplification mechanism.\footnote{A posterior study concludes that the quantitative importance of the credit constraint in the}
In contrast, we build a model of corporate default where firms have limited access to international credit markets in which the amplification/propagation mechanism is related to the interaction between financial fragility and the cost of sovereign default.

The paper also relates to the literature on firm dynamics models as in Cooley and Quadrini (2001). Our model departs from existing models on firm dynamics by assuming that corporations can only borrow from a domestic lender. As commented, this introduces spill-over effects among firms if sovereign liquidation costs exist.

The paper relates to the literature on business cycles for small open economies started by Mendoza (1991) and to the literature on sovereign default along the lines of Arellano (2006). However, most of this work deals with a representative firm environment and hence, cannot analyze the spill-over effects studied here.

Last but not least, the paper relates with Arellano and Kocherlakota (2008). They also introduce a model in which sovereign default arises from internal debt crises. Their two-period model relies on some key assumptions: (1) the realization of entrepreneurs’ productivity is private information, (2) the contract specifies how much capital each firm has to liquidate under any realization of the productivity shocks, (3) there exists an upper bound to the overall capital that can be liquidated in the economy, and (4) the entrepreneur—but not the lender—derives utility from the consumption of the non-liquidated capital. Arellano and Kocherlakota (2008) focus on how these sort of internal/external debt crises may emerge from non-fundamental shocks that are used by entrepreneurs to coordinate on default.\footnote{In their environment, it is possible that a firm with a high return finds it optimal to default if the other firm is also defaulting. They refer to this situation as a coordinated default equilibrium. The intuition is that if the upper bound for capital liquidation is low, the efficient level of liquidation per firm is small when there are massive corporate defaults. This increases the value of defaulting under massive defaults because a larger fraction of the capital is consumed by the entrepreneur when defaulting.}

Instead, we introduce a model with symmetric information to study an amplification/propagation mechanism emerging from the interaction between firms’ distress and sovereign default costs in a model where corporations face barriers to access international credit markets.

Kiyotaki-Moore setup is small, see Arias (2003)
3.2 Environment

We model a small open economy populated by a corporate sector and a domestic intermediary. Domestic firms have no direct access to international credit markets. They can only sell one-period bonds to the domestic intermediary who interacts with international lenders in order to clear the domestic bond market.\footnote{12}{A one-period bond is a promise of delivering one unit of good next period. In this environment, there is an equivalence between firms’ bond issuing and firms’ borrowing. In fact, a contract in which a firm sells to the intermediary $b$ units of bonds at a price $q$, is equivalent to one in which the firm takes a loan whose size is $bq$ and whose interest rate is $1/q - 1$. Henceforth, we use the bond issuing notation as is standard in the default literature.}

The corporate sector is composed of a state dependent number of incumbents, $N$, and $\overline{N} - N$ newly born firms or potential entrants. That is, the number of firms in the market is bounded above by $\overline{N}$.\footnote{13}{This restriction is imposed for computational convenience. We conjecture that the qualitative properties of the amplification mechanism in this simple environment remain the same in a model in which the size of the corporate sector is endogenous. The key aspect of the mechanism is that the corporate interest rates depend on the whole structure of the corporate sector, which is independent on how the size of the corporate sector is determined.}

Each period, newly born firms decide whether to enter the market. We assume that there is no entry cost; therefore a potential entrant always enters the market. Hence, the total number of firms equals $\overline{N}$ at any period.\footnote{14}{Even with entry cost, a newly born firm is always willing to enter the market. This is the case because the entry cost is totally financed by debt issuing and because the firm can always freely default.} Each firm chooses its level of physical capital, $k_i$, and leaves the market if it is not able to pay back its debt and cover its fixed cost. Capital depreciates at a constant rate, $\delta$, and it is the only input required for production.

We assume that firms face further financial frictions in addition to limited access to international credit markets. First, they cannot finance their production plan by issuing new shares, but by either retaining profit or by issuing an one-period bond.\footnote{15}{This does not represent a serious limitation given the fact that existing firms issue new share only occasionally. Smith (1977) finds that US firms raise more than 80% of equity from internal resources. The importance of firms’ internal resources is even higher in emerging economies.} In addition, firms may exit; in such a case, the domestic intermediary liquidates the firm and recovers only a fraction of total debt. The firm liquidation process involves a cost, $\mu_f(1 - \delta)k$. That is, some fraction of the non-
depreciated capital is lost in the liquidation process. The liquidation cost reduces the expected recovery rate and increases the interest rates and the likelihood of sovereign default.\footnote{\textsuperscript{16}}

Under those frictions, the Modigliani-Miller theorem does not apply and the size (equity) of the firm becomes a relevant state in the firm’s problem. We label the equity of the i-firm as $x_i$.

There exist two sources of uncertainty, a common autocorrelated shock, $Z$, and a firm-specific i.i.d. shock, $\varepsilon$. The shocks are additive; that is, if a firm gets an idiosyncratic shock, $\varepsilon_i$, its total productivity shock is $Z + \varepsilon_i$. The shock $Z$ is distributed according to $\Pi_z(Z'/Z)$, while $\varepsilon$ is distributed according to $\Pi_\varepsilon(\varepsilon)$. Both shocks are revealed in the current period. We assume that firms choose their capital levels one period in advance; hence, firms face uncertainty about both shocks when deciding their production plans.

The firm specific shocks induce firm heterogeneity which allows for different intensities of corporate distress. In absence of firm specific shocks, the framework collapses into the representative firm environment with no spill-over effects among firms.

The state dependent structure of the corporate sector—the crucial objet in our environment—is encoded in the variable $\mathcal{CS} = \{N, \{x_i\}_1^N\}$, while the state of the economy is represented by $S = (\mathcal{CS}, Z)$. Since the firm specific shock is not persistent, the vector $(\varepsilon_1, \ldots, \varepsilon_N)$ does not need to be included as a state variable.

Due to firm bankruptcy and aggregate uncertainty, the domestic intermediary may be unable to pay back her total debt with international lenders. We refer to this situations as “sovereign default”. In such cases, a proportional-to-debt cost has to be paid in order to liquidate the foreign debt contract. In cases of sovereign default, the domestic intermediary transfers to the international lenders all the resources she collects from domestic contracts. Independently of who pays the sovereign default cost, it introduces a wedge between what is recovered by the domestic intermediary and what is obtained by international lenders.\footnote{\textsuperscript{17}}

\footnotesize
\textsuperscript{16} It is well documented that firm bankruptcy and liquidation procedures involve important costs in term of time as well as financial resources. These costs are particularly high for emerging economies, see IMF, Global Financial Stability Report, 2005.
\footnotesize
\textsuperscript{17} Without loss of generality, we assume that the cost is paid by the lender.
We would like to stress that this is a model of involuntary default, in contrast with alternative frameworks in which the borrower chooses whether to default even if she is able to pay. Models of voluntary default assume that the defaulter is temporarily excluded from financial markets and that the investor does not recover any fraction of the loan.\(^\text{18}\) In contrast, in our framework the defaulter pays as much as she can recover and is never excluded from the international credit market. These are desirable characteristics in a default model. As indicated, in spite of the existence of “haircuts”, most of the time investors recover a fraction of the loan. Moreover, there is not empirical evidence supporting exclusion of credit markets for defaulters.\(^\text{19}\)

Our framework has two additional differences relative to “standard” sovereign default models: (1) sovereign default and banking crises happen simultaneously; and (2) financial fragility is rooted in corporate sector distress. In other words, sovereign debt crises are caused by internal debt crises. Arellano and Kocherlakota (2008) find strong evidence for this. First, they show that sovereign default risk and corporate default risk move together in developing countries.\(^\text{20}\) They further find that cross-country evidence suggests that causality goes from private sector default to sovereign default.

We use the standard assumption that international credit markets are competitive; as a consequence, the interest rate charge to the domestic intermediary—the sovereign interest rate—satisfies the zero profit condition. That is, in any debt contract, the expected cost for the international lender equals its expected income. This condition takes into account that under some realizations of the productivity shocks, international lenders recover only a fraction of the debt. The information contained in \(S\) is sufficient to determine the expected recovery rate; thus, enough

\(^{18}\)An exception is provided in Yue (2006) where debt recovery rates are explicitly modeled.

\(^{19}\)In fact, Linder and Morton (1989) argue that during the 1930’s and again in the early 1980’s—periods when a number of countries defaulted—external credit was no more inaccessible to sovereign defaulters than to non defaulters. Likewise, Jorgensen and Sachs (1989) find that in the two decades following the 1930’s crises, access to international capital market for Latin American countries was severely restricted for previous non defaulters as well as for defaulters.

\(^{20}\)In particular, they show that the dollar spread on international sovereign bonds correlates to the dollar spread charged to domestic borrowers. They also find that episodes of sovereign default coincide with episodes of large domestic private default. In their sample of emerging economies, 19 out of 22 debt crises involve internal debt crises.
to determine the sovereign interest rate, $r^s$.

The sequence of events unfolds as follows. Each period starts with a set of debt contracts and capital levels brought from the previous period, $(\{b_i\}, \{k_i\})$. Periods are sub-divided into two stages. In the first stage, the debt-liquidation phase, the stochastic shocks, $(Z, \{\varepsilon_i\})$, are realized and production takes place. In addition, debt contracts from the previous period are honored and firm exit-liquidation occurs. The second stage, the decision-making phase, starts after firms exit. At the beginning of this phase, the state of the economy is summarized in $S$. In this phase, the following actions are taken: (i) if $N < \overline{N}$, the $(\overline{N} - N)$ potential entrants decide about entry and their next period capital level, which implies certain level of debt (ii) the $N$ incumbents decide their debt levels and their next period capital levels, which implies certain dividends payments,(iii) the domestic intermediary signs debt contracts with international lenders in order to clear the domestic bond market. The next period starts with the realization of the shocks and with the levels of debt/capital that correspond to those chosen in the decision-making phase.\footnote{Notice that entry and exit occur within the same period. Since newly born firms always choose to enter, the number of firms equals $\overline{N}$ at any period.}

\section{Equilibrium}

\subsection{Corporate sector}

Firms manage a production technology, $Y = (Z + \varepsilon)F(k)$. The function $f$ is assumed to be strictly increasing and strictly concave. Each firm chooses its debt, $b^f$, and next period capital, $k^f$, in order to maximize the expected discounted value of its flow of dividends. Firms discount future at a factor $\beta$. We assume that firms accumulate resources only in form of physical capital. This assumption is present in similar firm dynamics models —see for instance Pratap and Urrutia (2004). We do not consider it a strong restriction for developing countries where asset tangibility for corporations is significantly high.\footnote{Asset tangibility is the ratio fixed-asset to total-asset. The average asset tangibility for Latin America is 0.79 for the period 1993-2003 (Worldscope).}
In order to operate, the firm has to pay a fixed cost, $FC$. The i-firm net worth after paying the fixed cost is

$$x'_i = (Z' + \varepsilon')F(k'f) + (1 - \delta)k'f - b'f - FC. \quad (3.1)$$

An existing firm with net worth $x_i$ solves the following functional problem

$$V(x_i, S) = \max_{(k'_f, b'_f) \in R^2_+} x_i + q^f(x_i, b'_f | S)b'_f - k'_f + \beta E[\tilde{V}(x'_i, S')]$$

Subject to

$$d_i(S) = (x_i + q^f(\cdot)b'_i | S)b'_f - k'_f \geq 0,$$

$$S' = H(S). \quad (3.2)$$

Where the functions $\tilde{V}$ is specified as

$$\tilde{V}(x'_i, S') = \begin{cases} 
0 & \text{if } x'_i + FC = (Z' + \varepsilon')F(k'f) + (1 - \delta)k'f - b'f < 0, \\
x'_i + FC & \text{if } x'_i < 0 \text{ but } x'_i + FC > 0, \\
V(x'_i, S') & x'_i \geq 0.
\end{cases}$$

In (3.2), $V$ is the value of a firm whose net worth at the beginning of the decision making phase (second stage), equals $x_i$ when the aggregate state of the economy is $S$. Notice there exists the possibility that the firm pays back its debt but still leaves the market because it is not able to meet the fixed cost requirement. In this case, the continuation value for the firm equals its remaining resources after debt liquidation, $x'_i + FC = (Z' + \varepsilon')F(k'f) + (1 - \delta)k'f - b'f$. The function $H$ is the transition law for the state of the economy while $q^f$ is the corporate bond price schedule which is taken as given by the firm when solving its problem. Notice that this bond price is indexed by firm size, loan size, and by the state of the economy. The fact that $q^f$ depends not only on firm’s factors, $(x_i, b_i)$, but also on the whole structure of the corporate sector, encoded in $S$, is what introduces spill-over across firms. In problem (3.2), dividend payments, $d$, are restricted to be positive as a consequence of our assumption that firms do not issue shares.
The solution for the incumbent is summarized in two policy functions: one for debt issuing, \( b'(x_i, S) \), and other for capital accumulation, \( k'(x_i, S) \). These two functions induce a policy rule for dividend payments, \( d(x_i, S) = x_i + q'(\cdot)b'(x_i, S) - k'(x_i, S) \geq 0 \).

The exit rule is defined as,

\[
Exit_i(Z', \varepsilon', |S) = \begin{cases} 
1 & \text{if } x'_i < 0, \\
0 & \text{otherwise.}
\end{cases}
\] (3.3)

A newly born firm begins the period with zero equity. When entering the market, it takes a loan (i.e. issues bonds) to purchase the stock of capital to produce next period.

Let \( V^e(S) \) be the value for a newly born firm when the state of the economy is \( S \). We can write

\[
V^e(S) = \max_{k'^e > 0} \beta E[\tilde{V}_i(S')].
\]

Subject to

\[
x'_i = (Z' + \varepsilon')F(k'^e) - (1 - \delta)k'^e - b'^e - FC,
\]

\[
q^e(b'^e|S)b'^e = k'^e,
\]

\[ S' = H(S). \] (3.4)

The entrant’s problem is summarized in two policy functions: one for debt issuing, \( b'^e(S) \), and one for capital accumulation, \( k'^e(S) \). As well as an incumbent, the entrant may leave the market next period if its resources are not enough to pay back its total debt and the fixed cost.

As mentioned, if a firm is unable to pay its debt, it is liquidated and the domestic intermediary recovers the firm’s liquidation value given by

\[
L_i(Z, \varepsilon_i, k_i) = (Z + \varepsilon_i)F(k_i) + (1 - \delta)k_i - \mu_i(1 - \delta)k_i. \] (3.5)
3.3.2 Foreign debt’s conditions

Foreign debt’s conditions crucially depend on the state of the economy $S$. The less vulnerable the economy is, the more attractive the terms of the foreign contract are; in particular, the sovereign interest rate. In order to explicitly state the foreign debt’s conditions, define $\Upsilon'$ as the vector of firm specific shocks; that is, $\Upsilon' = (\varepsilon'_1, ..., \varepsilon'_{N'})$.

**Definition 3.1.** A sovereign debt contract, $C$, is a collection of functions that, in each particular state $S$, specifies a sovereign bond price schedule, $q^*(B'|S)$, and repayment rate, $\rho(Z', \Upsilon', B'|S)$ for each realization of $(Z', \Upsilon')$.

In particular, we study debt contracts establishing the following repayment rate

$$\rho(\cdot) = \min \left\{ 1, \frac{\sum_{j=N+1}^{N} [b^e_j (1 - \text{Exit}_j) + \text{Exit}_j L_j] + \sum_{i=1}^{N} [b^I_i (1 - \text{Exit}_i) + \text{Exit}_i L_i]}{B'} \right\}.$$  

(3.6)

In other words, the domestic intermediary repays the total debt only if she collects enough resources from her own debt contracts with domestic firms. In case of sovereign default—partial repayment of total foreign debt—the domestic intermediary pays as much as she is able to collect.

As mentioned, we assume that under sovereign default, there exists a proportional liquidation cost, $\mu_s$. This cost introduces a wedge between what is paid by the domestic intermediary, $\rho(\cdot)$, and what is recovered by the international lenders, $\tilde{\rho}(\cdot)$. We can write

$$\tilde{\rho}(\cdot) = \rho(\cdot) - \mu_s I(\rho(\cdot) < 1),$$  

(3.7)

where $I(\cdot)$ is an indicator function returning 1 if its argument is true and zero otherwise.\(^{23}\)

\(^{23}\)This liquidation cost may reflect state dependent financial intermediation cost, if the intermediation cost is assumed to be higher under default. For instance, suppose that the international lender pays an intermediary cost, $\mu_l$, if there is not default but an intermediary cost, $\mu_h > \mu_l$, under default. Then, the recovery rate can be written as $\tilde{\rho}(\cdot) = \rho(\cdot) - \mu_l (\mu_h - \mu_l) I(\rho(\cdot) < 1)$. Normalizing $\mu_l$ to zero and re-labeling $\mu^s = \mu_h - \mu_l$ we get equation (3.7). For a model dealing with costly financial intermediation in an explicit banking model see Díaz-Gimenez et al. (1992).
To pin down the sovereign bond price from the recovery rate, $\tilde{\rho}(\cdot)$, we use the standard assumptions that international lenders are risk neutral and that the international bond market is competitive. As a consequence, the sovereign bond price satisfies the zero profit condition; that is,

$$E_{Z', Y'}[\tilde{\rho}(Z', Y', B'|S)] = q^*(B'|S)(1 + r). \quad (3.8)$$

Again we stress that this is not a traditional sovereign default model as those in which the country chooses to default even when having enough resources to pay back its debt. Instead, the debt contract specifies contingent payments characterized by full repayment under some states and partial repayments in others. Partial repayments arise due to country’s inability to pay. The contract is completely enforceable and there are no information asymmetries. Therefore, we assume that under partial repayment there is not any sort of punishment like in the sovereign default literature. For recent work on sovereign default models due to unwillingness to pay see Arellano (2006), and Aguiar and Gopinath (2006).

### 3.3.3 Domestic bond’s prices

The only role of the domestic intermediary is to satisfy corporations’ demand for funds; in doing so, we assume that she charges actuarially fair interest rates for each firm. That is, taking into account the sovereign interest rate, the repayment rate, the size of the firm and the size of the loan, the corporate bond price implies zero profit in expectation.\(^{24}\)

Define the sovereign interest rate as $r^s = 1 - \frac{1}{q^s}$ and let $q^f(x, b^f|S)$ be the bond price function for existing firms. It is indexed by firm’s equity and by loan size because these factors determine the bankruptcy probability of the firm. It is indexed by the state of the economy because $S$ determines the sovereign interest rate and the expected repayment rate. Expected zero profit condition requires

---

\(^{24}\)By using this assumption, we do not have to explicitly model the problem of the domestic intermediary. As commented, in many developing countries the bank industry is dominated by state banks. It can be argued that the main motive of the government is to allocate resources instead of exert monopoly power to maximize profits.
\[ E_{Z', \mathcal{T}}[q^f(x_i, b'^f|S)b'^f(1 + r^s)\rho(\cdot)] = E_{Z', \mathcal{T}}[b'^f(1 - Exit_i) + Exit_i L_i] \]  
(3.9)

Similarly, let \( q^e \) be the bond price function for entrant, then

\[ E_{Z', \mathcal{T}}[q^e(b'^e|S)b'^e(1 + r^s)\rho(\cdot)] = E_{Z', \mathcal{T}}[b'^e(1 - Exit_i) + Exit_i L_i]. \]  
(3.10)

The left hand side of (3.9)-(3.10) is the domestic intermediary’s expected cost (payment) associated with this particular loan. Notice that it takes into account that in some states, the domestic intermediary does partial payment—whenever \( \rho < 1 \). The right hand side is the domestic intermediary’s expected income associated with this particular loan.

Using (3.7)-(3.8), we can write the expected repayment rate conditional on being on state \( S \) as

\[ E[\rho(\cdot|S)] = E[\tilde{\rho}(\cdot|S)] + \mu_s \text{Prob} (\rho < 1|S) = \frac{1 + r}{1 + r^s} + \mu_s \text{Prob} (\rho < 1|S). \]

Using the previous equation we can rewrite the functions determining corporate bond prices \( (q^f, q^e) \) as follows

\[ q^j(1 + r^s)b'^j \left[ \frac{1 + r}{1 + r^s} + \mu_s \text{Prob} (\rho < 1|S) \right] = E[(1 - Exit_i)b'^j + Exit_i L_i], \text{ with } j \in \{f, e\}. \]  
(3.11)

This equation has important implications. In first place, it shows that if \( \mu_s = 0 \), the corporate bond price schedule is identical to the one with direct lending; that is, when firms interact directly with international creditors.\(^{26}\) Therefore, with no cost of default in international debt contracts, whether firms have limited access to international credit markets or not, does not affect the economy in any sense. However, if \( \mu_s > 0 \), each firm interest rate depends on the whole structure of

---

\(^{25}\)In this environment with no entry cost, the bond prices between an entrant and an incumbent with no equity \((x = 0)\) are the same.

\(^{26}\)With direct lending zero profit condition implies \( q b(1 + r) = E[(1 - Exit_i)b' + Exit_i L_i] \).
the corporate sector which introduces externality effects among firms. In this case, the difference in domestic bond price schedules relative to direct lending is associated with the term \((1+r^s)\mu_s \text{Prob}(\rho < 1)\). Notice that as the corporate sector becomes weaker as a whole, all firms start paying a higher interest rate, not only because \(r^s\) is higher, but also because the probability of partial repayment rises. The higher \(\mu_s\) is, the more significant the externality effect becomes. In section 5, we quantitatively evaluate the role played by this parameter in affecting the macroeconomics performance.

### 3.3.4 Equilibrium

We numerically solve for an equilibrium in this small open economy with a corporate sector and a domestic intermediary. The equilibrium is defined as follows:

**Definition 3.2.** A *recursive equilibrium* is a collection of functions specifying an equilibrium sovereign bond price, \(q^s(S)\), domestic bond price schedules for existing firms and new entrants, \(\{q_f, q_e\}\), policy functions for existing firms and new entrants \(\mathcal{P} = \{b'_f, k'_f, b'_e, k'_e\}\), and a law of motion for aggregate states, \(S' = H(S)\), such that:

1. Given the bond prices schedules, \(\{q_f(b', x|S), q_e(b'|S)\}\), the functions \(\{b'_f(x, S)\}\) and \(\{k'_f(x, S)\}\), solve the problem for existing firms and the functions \(\{b'_e(S)\}\) and \(\{k'_e(S)\}\) solve the problem for new entrants.

2. The bond market clears; that is,

\[
q^s(B'; S)B' = (N - N)q^e(b^e; S)b^e(S) + \sum_{i} q^f(x_i, b'^f_i; S)b'^f_i(x_i; S).
\] (3.12)

3. Given the policy functions, \(\mathcal{P}\), the bond market clearing condition (3.12), and the stochastic processes for the shocks, \((\Pi_z, \Pi_\varepsilon)\) the sovereign equilibrium bond price, \(q^s(S)\), is defined by (3.6),(3.7), and (3.8).

4. Given the policy functions, \(\mathcal{P}\), the repayment rate, \(\rho\), is given by (3.6).
5. Given $q^s$ and $\rho$, the corporate bond prices functions, $\{q^f(b, x|S), q^e(b'|S)\}$, are defined by (3.9) and (3.10) respectively.

6. The law of motion for states, $H$, is determined by the stochastic processes for the shocks, $(\Pi_z, \Pi_{\varepsilon})$, by the exit rule, by the policy rules, and by equation (3.1).

Combining the bond prices functions with the market clearing condition, we obtain the following equation.

$$E[\rho(\cdot|S)]B'(S) = (N - N)E[b^e(1 - \text{Exit}) + L\text{Exit}] +$$

$$\sum_{i=1}^{N} E[b^f_i(1 - \text{Exit}_i) + L_i\text{Exit}_i].$$

That is, equilibrium prices imply that, in expectation, the domestic lender recovers as much as she is expected to pay back to the international lenders.

Before the quantitative exercise, we introduce an assumption regarding to the firm’s discount factor and discuss one of its implications. In particular, we set $\beta = \frac{1}{1+r} \leq \frac{1}{1+r_s}$.

This assumption implies that firms do not pay dividends and issue debt simultaneously. This is formally stated in the next lemma.

**Lemma 3.1.** In equilibrium, firms do not pay dividends when borrowing. That is $b^f(x, S) > 0 \Rightarrow d(x, S) = 0$.

**Proof.** [See Appendix] \hfill \square

### 3.4 Quantitative Results

In this section, we numerically solve for the equilibrium previously defined. Then, we show how the magnitude of the externality effect—associated with the param-
eter $\mu_s$—influences the macroeconomic performance; in particular, the behavior of the output and the sovereign interest rate over the business cycle, as well as some aspects of the capital accumulation behavior of the firms.

The algorithm for solving the equilibrium is presented in the appendix. It is composed of two cycles, an outer cycle updating the bond prices given the firm’s policies; and a inner cycle solving for the firm’s policies given the bond prices.

**Parametrization**

In order to solve the problem numerically, we need to provide an explicit functional form for the production function, $F$. We also need to define the processes governing both shocks, the common shocks, $Z$, and the firm specific shock, $\varepsilon$. Finally, we need to specify the maximum number of firms, $\overline{N}$, the world interest rate, $r$, the firm’s discount factor, $\beta$, the capital depreciation rate, $\delta$, the sovereign default cost, $\mu_s$, the firm bankruptcy cost, $\mu_f$, and the fixed cost, $FC$.

The general parametrization strategy goes as follows. We set $\mu_s = 0$ for the benchmark parametrization. As explained, this represents the case of a financially integrated economy; then, we calibrate the rest of the parameters to a small open economy like Canada. Some of the parameters are taken from previous literature; while other are set such that the model closely matches some moments of the Canadian economy.

We set the time period to 1 year. As in Pratap and Urrutia (2004), we set the risk free interest rate to 5%, consistent with the average postwar U.S. treasury bills rates, and $\beta = \frac{1}{1 + r} = .95$. We want to stress that “standard” sovereign default models rely on high impatience—i.e. a low $\beta$—to generate sufficient level of default and hence, to capture the counter-cyclicality of sovereign interest rates.\footnote{For instance, Aguiar and Gopinath (2006), use $\beta = .8$ while Arellano (2006), sets $\beta = .84$. In both cases the time period is set to be a quarter.} Our environment of involuntary default, does not rely on a low $\beta$ to generate strongly counter-cyclical interest rates. Following Mendoza (1991), we set the depreciation rate to .1. This value is commonly used in the business cycle literature. We set $\overline{N} = 2$; this allows us to represent the structure of the corporate sector in a very tractable way. This simple environment is rich enough to study how the spill-over
effects among firms affect the macroeconomic performance. We understand that it represents a restriction that may not be innocuous from a quantitative point of view; however, we believe that the same interdependency studied here will emerge in a more general environment.

The common productivity shock, $Z$, is parameterized as a 2-state Markov chain, that is $Z \in \{Z_l, Z_h\}$. We normalize $Z_l$ to $.5$. The conditional distribution governing the process for $Z$ is given by the following transition matrix

$$\Pi_Z = \begin{pmatrix} \pi_z & 1 - \pi_z \\ 1 - \pi_z & \pi_z \end{pmatrix}.$$ 

Therefore, this conditional distribution can be represented by a single number, $\pi_z \equiv \text{Prob}(z' = z_l | z = z_l)$. Notice that in the stationary distribution of $Z$, it is the case that $\text{Prob}(Z = Z_l) = \text{Prob}(Z = Z_h) = .5$.

The support for the firm specific i.i.d shock, $\varepsilon$, is defined so that the lowest total productivity shock, $Z_l + \varepsilon_0$, is zero. In particular,

$$\varepsilon \in \{-Z_l, -2Z_l, 0, 2Z_l, 4Z_l\}.$$ 

We set $\Pi_{\varepsilon_0} \equiv \text{Prob}(\varepsilon = -Z_l) = .1$. Then, the unconditional probability that the total shock is zero equals 5%.

Conditional on $\varepsilon \neq -Z_l$, the firm specific shock follows the following distribution,

<table>
<thead>
<tr>
<th>$\varepsilon$</th>
<th>-$0.4Z_l$</th>
<th>$-0.2Z_l$</th>
<th>0</th>
<th>$0.2Z_l$</th>
<th>$0.4Z_l$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>0.1</td>
<td>0.2</td>
<td>0.4</td>
<td>0.2</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Notice that conditional on being different from $-Z_l$, the firm specific shock is symmetrically distributed around zero.

The production technology is defined as a concave function of capital. In particular, $F(k) = k^\alpha$. We set $\alpha = .33$. This value is also used in Yoshida (2005) for a model calibrated to Japan, and it is similar to the one obtained in Pratap and Urrutia (2004) when calibrating a model to Mexico.
Firm liquidation cost is a fraction, $\mu_f$, of the un-depreciated capital. That is, when the domestic intermediary liquidates a firm with capital $k$, it recovers $L = (Z + \varepsilon)F(k) + (1 - \mu_f)(1 - \delta)k$.

For the U.S economy, Altman (1984) estimates the cost of bankruptcy at about 20% of the assets of the firm, while Alderson and Betker (1995), using data from chapter 11 procedures, estimate an average cost equal to 36% of the assets of the firm. We set $\mu_f = .25$ which equals the value used in Carlstrom and Fuerst (1997).

It remains to define three parameters. The fixed cost paid by the firm in order to operate, $FC$, the parameter determining the persistence of the common shock, $\pi_z$, and the highest value of the common shock, $Z_h$. We calibrate $(FC, \pi_z, Z_h)$ such that the model closely matches three moments of the Canadian economy: (1) the autocorrelation coefficient of GDP, (2) the firm exit rate, and (3) the capital output ratio.

Mendoza (1991) reports an auto correlation for GDP equal to .615 while Bartelsman et al. (2004) indicates an exit rate of 8% for firms in the Canadian manufacturing sector. Finally, we try to match a capital-output ratio of 2.3. A similar value is targeted in Cooley and Quadrini (2001) and in Amaral and MacGee (2002).

This strategy yields to $(FC, \pi_z, Z_h) = (.37, .91, .8)$. Under this parametrization, the artificial economy reproduces an exit rate of 7.70%, an output autocorrelation coefficient of .621, and a capital output ratio of 2.234.

Table 3.2 summarizes the benchmark parametrization.

**Results**

In this subsection, we numerically solve for the equilibrium previously defined under the parametrization presented in table 3.2. Then, we generate paths and compute business cycle statistics for the variables of interest; in particular, for aggregated output and for the sovereign interest rate. We repeat this process for different values of the sovereign default cost, keeping constant the remaining parameters. Specifically, in addition to the benchmark case, we set $\mu_s \in \{.15, .25, .30, .35\}$. By this, we analyze how the spill-over effects, associated with the level of $\mu_s$, influence the economic performance.
Table 3.2: Benchmark Parametrization

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
<th>Basis and Targets</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r$</td>
<td>0.05</td>
<td>Risk free interest rate</td>
<td>US Treasury bill rates</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$\frac{1}{1+r}$</td>
<td>Firm Discount Factor</td>
<td>Standard</td>
</tr>
<tr>
<td>$\delta$</td>
<td>.1</td>
<td>Capital depreciation rate</td>
<td>Mendoza (1991)</td>
</tr>
<tr>
<td>$N$</td>
<td>2</td>
<td>Maximum number of firms</td>
<td>Simplification</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>.33</td>
<td>Production function parameter</td>
<td>Yoshida (2005)</td>
</tr>
<tr>
<td>$Z_l$</td>
<td>.5</td>
<td>Lowest value for $Z$</td>
<td>Normalization</td>
</tr>
<tr>
<td>$\mu_f$</td>
<td>.25</td>
<td>Firm liquidation cost</td>
<td>Carlstrom and Fuerst (1997)</td>
</tr>
<tr>
<td>$\mu_s$</td>
<td>0</td>
<td>Sovereign default cost</td>
<td>Financially integrated economy</td>
</tr>
<tr>
<td>$\pi_z$</td>
<td>.91</td>
<td>$Prob(Z_{t+1} = Z_t</td>
<td>Z_t = Z_i)$</td>
</tr>
<tr>
<td>$Z_h$</td>
<td>.8</td>
<td>Highest value for $Z$</td>
<td>To match K/Y</td>
</tr>
<tr>
<td>$CF$</td>
<td>.37</td>
<td>Fixed Cost</td>
<td>To match exit rate</td>
</tr>
</tbody>
</table>

To illustrate how the spill-over effects work, consider the case in which $\mu_s = .3$. Figure 3.1 shows the equilibrium sovereign bond price in the case when the number of surviving firms equals 2. Notice that if both firms are sufficiently large, there is no sovereign default risk and the bond price equals $\frac{1}{1+r}$. This is represented by the flat surface in Figure 3.1. However, if at least one of the firms is sufficiently small, the default probability becomes non-zero, and the sovereign bond price drops dramatically due to the fact that it is more likely that the sovereign default cost has to be paid. That is, whenever the equity of either firm falls below a threshold, the risk of default becomes non-zero and the sovereign interest rate jumps. If the default cost were zero, the increase in the default rate would imply a smaller increase in the sovereign interest rate. Figure 3.2 shows the sovereign interest rate path for a particular realization of the history shocks, $\{\Upsilon_t, Z_t\}_{0}^{T}$, for the benchmark case, $\mu_s = 0$, and for the case with $\mu_s = .3$. It is clear that the presence of such costs magnifies the effect of adverse shocks on the sovereign interest rate, increasing its volatility.

In a environment with costly sovereign default, together with firms’ barriers to international capital market, each firm’s interest rate depends on the whole structure of the corporate sector. This introduces spill-over effects whose magnitude is directly related to the parameter $\mu_s$. In order to illustrate how these spill-over effects affect the economic performance, we compute the business cycle statistics
for different values of the sovereign default cost. Table 3.3 summarizes the results.

We use standard notation; $E$, $\sigma$ and $\rho$ represent the mean, the standard deviation, and the coefficient of correlation respectively.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\mu_s = 0$</th>
<th>$\mu_s = .15$</th>
<th>$\mu_s = .25$</th>
<th>$\mu_s = .30$</th>
<th>$\mu_s = .35$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E(r^s)$</td>
<td>5.3%</td>
<td>5.5%</td>
<td>5.7%</td>
<td>5.8%</td>
<td>5.9%</td>
</tr>
<tr>
<td>$Max(r^s)$</td>
<td>8.8%</td>
<td>11.4%</td>
<td>13.50%</td>
<td>15.0%</td>
<td>15.5%</td>
</tr>
<tr>
<td>$\frac{\sigma(r^s)}{E(r^s)}$</td>
<td>.170</td>
<td>.284</td>
<td>.364</td>
<td>.405</td>
<td>.444</td>
</tr>
<tr>
<td>$\rho(r^s, r^s)$</td>
<td>.678</td>
<td>.796</td>
<td>.816</td>
<td>.821</td>
<td>.827</td>
</tr>
<tr>
<td>$\frac{\sigma(y^s)}{\sigma(y)}$</td>
<td>.381</td>
<td>.383</td>
<td>.384</td>
<td>.385</td>
<td>.386</td>
</tr>
<tr>
<td>$\rho(y^s, r^s)$</td>
<td>.62</td>
<td>.626</td>
<td>.629</td>
<td>.630</td>
<td>.631</td>
</tr>
<tr>
<td>$\rho(r^s, y)$</td>
<td>-.238</td>
<td>-.263</td>
<td>-.278</td>
<td>-.285</td>
<td>-.292</td>
</tr>
</tbody>
</table>

The existence of sovereign default costs has important implications. First, when $\mu_s$ changes from 0 to 0.3, the average risk premium the country pays, $E(r^s) - r$, increases by 10% approximately. Moreover, it almost doubles the response of the sovereign interest rate to severe adverse shocks represented in $Max(r^s)$. It is well known that the volatility of the interest rates in emerging markets is higher than the one in developed countries. In our model, the coefficient of variation of the sovereign interest rate is doubled when increasing the sovereign default cost from 0 to .15. The table also suggests that the effect of shocks on the sovereign interest rate becomes more persistent, in fact, the auto correlation coefficient increases in 21% when increasing the sovereign default cost from 0 to .3. Finally, the interest
rate becomes more counter-cyclical: the absolute value of $\rho(r^s, y)$ increases in 20% when changing $\mu_s$ from 0 to .3. For a set of developing countries, Neumeyer and Perri (2005) report an average coefficient of correlation of output-sovereign interest rate equal to -54. In their sample, this correlation ranges from -.38 for Brazil up to -.70 for Korea.

In sum, the externality works as an amplification mechanism, increasing the volatility and the response of interest rates to adverse shocks, reinforcing the correlation between sovereign interest rate, and output and making more persistent the interest rates. The mechanism works as follows. Suppose that some firms get an adverse shock, it reduces the next period expected repayment rate and also increases the probability of partial repayment. These two components interact together with the liquidation cost $\mu_s$, increasing the sovereign interest rate; and as a consequence, the interest rates paid by all firms, even those firms not having the adverse shocks. Then, the interest rates jump not only because it is more likely that the sovereign default cost has to be paid, but also because all firms are becoming relatively riskier.

However, notice that the persistence in output, as well as its volatility, increases only marginally. How is it possible that these externality effects have little impact on aggregate output performance? What explains that the impact in interest rates does not translate in similar effects on output? The answers to these puzzling questions relies on the existence of precautionary capital formation. In the presence of default costs, a mature firm accumulates more capital—relative to the case with $\mu_s = 0$— in those states from which it is possible to transit to a high-interest-rate state next period. This over-accumulation is carried out in order to minimize the likelihood of taking big loans at a significantly high interest rate in the future.

More precise, let $k(x_1, x_2, Z)$ be the capital chosen by firm with equity $x_1$ when the other firm’s equity equals $x_2$ and the aggregate shock is given by $Z$. Define

$$K^*(Z, x_2) = \max_{x_1} k(x_1, x_2, Z).$$

---

29 Aguiar and Gopinath (2007) remark that there are not significant differences in output persistence between developed and emerging economies.
It is the case that, whenever $x_1 > K^*(Z, x_2)$, the firm chooses capital $K^*$, pays dividends $x_1 - K^*$, and does not borrow (recall lemma 3.1). When building this critical capital, the firm takes into account the probability of transiting to a state in which the firm wants to borrow and the externality effect triggers a sudden increase in the interest rates. Figure 3.3 shows the function $K^*$ for the benchmark case, and for the case with $\mu_s = 0.3$.

Figure 3.3: Precautionary Capital Formation.

Some aspects are remarkable about $K^*$. First, $K^*_{\mu_s}(Z_h, x_2) > K^*_{\mu_s}(Z_l, x_2)$ for both values of the sovereign default cost. This is the case because when the common shock is higher, the future expected returns of capital are higher due to the expectation that the high shock remains. Second, $K^*_{\mu_s}(Z, x_2)$ is independent of $x_2$ when $\mu_s = 0$. This is true from the fact that in this case, the firm’s interest rate depends exclusively on idiosyncratic factors. Third, $K^*_{\mu_s=0}(Z_l, x_2) > K^*_{\mu_s=0}(Z_l, x_2)$. This effect is what we refer as precautionary capital formation. When $\mu_s > 0$, loans become very costly when there exists risk of default. Then, the firm has stronger incentive—relative to the case where $\mu_s = 0$—to accumulate capital in those states from which it is likely to transit to a high-interest-rate state next period. Fourth, $K^*_{\mu_s=0}(Z_l, x_2)$ is independent of $x_2$. This is the case because when $Z = Z_h$ and the firm is choosing $K^*$, there is virtually no risk that next period the firm has to take an expensive loan. Finally, $K^*_{\mu_s=0}(Z_l, x_2)$ is decreasing in
This is the case because under $Z_l$, there exists a possibility that the firm is involved in an expensive loan next period, but this probability decreases with $x_2$. However, from certain point on, $K^*_{\mu_s = .3}(Z_l, x_2)$ becomes independent of $x_2$, and almost equals to $K^*_{\mu_s = .0}(Z_l, x_2)$.

What implication for output does this precautionary accumulation of capital behavior have? Suppose that the state of the economy is given by $(Z_l, x_1, x_2)$, where $x_1 \in \text{argmax}$ of the problem (3.14), indicating that firm 1 is big enough, and $x_2$ correspond to the level indicated by the vertical dotted line in figure 3.3. By definition, the firm having $x_1$ chooses the capital level denoted as point (1a), if $\mu_s = 0$, and the capital level and in point (1b), if $\mu_s = .3$. Notice the precautionary capital formation; that is, $K^*_{\mu_s = .3} > K^*_{\mu_s = .0}$. Now suppose that the economy has a good common shock, i.e $Z' = Z_h$. If the firm is big enough, it will choose the capital level associate with the superior horizontal line, as in point (2) of figure (3.3). This will be the capital level chosen by firm 1 independently of the size of the other firm, $x_2$, and for both values of $\mu_s$. If the firm does not have enough resources, its capital will not jump to (2) immediately, but will eventually converge to that level in absence of further changes in $Z$. In sum, in this example the capital tends to move from (1a) toward (2), if $\mu_s = 0$, and from (1b) toward (2), if $\mu = .3$. That is, this precautionary capital formation behavior—arising when $\mu_s > 0$—tends to reduce the variation in capital, and hence in output.

A similar argument can be used when the initial state is $(Z_h, x_1, x_2)$, and the capital for the firm having $x_1$ is as indicated in point (2). If the economy switches to $Z' = Z_l$ and $x_1'$ is still in the argmax of problem (3.14), the capital for the $x_1$-size firm will change from (2) to a point in the dotted decreasing line if $\mu_s = .3$, or to a point in the the inferior horizontal line if $\mu_s = 0$. Again, if $x_1'$ is not big enough, the capital will not move to those levels immediately. The smaller capital level associated with $\mu_s = 0$, reduces the expected next period output with respect to the one with $\mu_s = .3$, which leads to a higher output persistence. In other words, the precautionary capital formation practice—arising when $\mu_s > 0$—tends to reduce the persistence of output.

In sum, these precautionary capital formation forces partially offset the interdependency forces that work as a amplification/propagation mechanism for
3.5 Concluding Remarks

We study the business cycle implications of corporations’ limited access to international capital market together with sovereign default costs. If sovereign default cost were zero, each firm’s interest rate would depend exclusively on idiosyncratic factors. Moreover, the corporate interest rates coincide with the corporate interest rates under direct lending. However, with non zero sovereign default costs, each firm’s interest rate depends not only on idiosyncratic factors but also on the whole structure of the corporate sector. As a result, externality effects among corporations arise implying that the shocks in some firms may affect the interest rate charged to all firms. Our environment allows one to study the business cycle implications of financial liberalization—achieved either by promoting direct lending or by reducing the cost associated with sovereign default.

The spill-over effects work as an amplification mechanism, making the sovereign interest rate more counter-cyclical, more volatile and more persistent. Since these are distinguishing features of developing economies, the results suggest that financial barriers and sovereign default costs may play a role in shaping the business cycle of small emerging economies. The results therefore indicate a channel by which opening the financial system would lead to less persistent and less volatile interest rates.

This spill-over mechanism affects aggregate output to a less extent. The reason is that the presence of sovereign default costs also introduces a precautionary capital formation behavior in mature firms, in order to avoid taking expensive loans. The two opposite effects works in different domains of the state space. When the firm is small, it is more likely that its investment decision, and hence its output, is influenced by the spill-over (magnification/propagation) mechanism. In contrast, when the firm is big, it is more likely that its investment-production path is dominated by precautionary capital formation.

Future research can be targeted at confronting the model to the data. The most direct testable implication is that the cost of capital paid by individual firms growing firms.
should also depend on different moments of the firm-size distribution, and not only on individual firm’s characteristics. Moreover, the model also suggests that small firms tend to under-accumulate capital while big firms tend to over-accumulate capital in economies that are not financially integrated.
Bibliography


Appendix

A-1 More on chapter 1

A-1.1 Proof of Lemma 1.1

Lemma 1.1 Consider problem (1.5), $e > 0 \Rightarrow U^+ > U^-$. Under effort observability and concavity of $V^{z,h}(U)$ with respect to $U$, in the optimal contract the firm fully insures the worker against the effort-dependent productivity shock; that is, $U' \equiv U^+ = U^-$. 

Proof. If effort is not observable, (ICC) must hold. Since $e > 0 \Rightarrow v'(e) > 0$, the right hand side of (ICC) must be positive which requires $U^+ > U^-$. Under observable effort, the problem is equivalent to (1.5) but (ICC) is not required. Assume by contraction that the optimal solution, $(e, w, U^+, U^-)$, for a particular match, $(h, z, U)$, satisfies $U^+ \neq U^-$. Consider the case in which $U^+ > U^-$, the opposite case follows the same arguments. To ease the notation write $U^{*,\tilde{z},h}$ as $U^{*,\tilde{z}}$. One of the following situations must happen

1. $U^+ \notin \{U^{*,\tilde{z}}\}_{\tilde{z}=1}^N$ and $U^- \notin \{U^{*,\tilde{z}}\}_{\tilde{z}=1}^N$,
2. $U^+ \in \{U^{*,\tilde{z}}\}_{\tilde{z}=1}^N$ and $U^- \notin \{U^{*,\tilde{z}}\}_{\tilde{z}=1}^N$,
3. $U^+ \notin \{U^{*,\tilde{z}}\}_{\tilde{z}=1}^N$ and $U^- \in \{U^{*,\tilde{z}}\}_{\tilde{z}=1}^N$,
4. $U^+ \in \{U^{*,\tilde{z}}\}_{\tilde{z}=1}^N$ and $U^- \in \{U^{*,\tilde{z}}\}_{\tilde{z}=1}^N$.

• CASE 1. Consider the alternative candidate $(e, w, \tilde{U}^+, \tilde{U}^-)$, where $(e, w)$ are set as in the optimal solution, while $\tilde{U}^+ \equiv U - \varepsilon^+$ and $\tilde{U}^- \equiv U^- + \varepsilon^-$. The scalars $(\varepsilon^+, \varepsilon^-)$ are positive and will be defined such that the alternative candidate satisfies the promised keeping constraint. Define $z_j$ as the highest $z$ satisfying $U^+ \geq U^{*,z_j}$, and $z_i$ as the highest $z$ satisfying $U^- \geq U^{*,z_i}$. It is the case that:

- $z_j \geq z_i$, 
\(- U^{*;z_{j+1}} > U^+ > U^{*;z_j} \text{ and } U^{*;z_{i+1}} > U^- > U^{*;z_i} \text{ and,} \)

\(- \text{ For sufficiently low } (\varepsilon^+, \varepsilon^-) \text{ it is the case that} \)

\[ U^{*;z_{j+1}} > \tilde{U}^+ > U^{*;z_j} \text{ and } U^{*;z_{i+1}} > \tilde{U}^- > U^{*;z_i} . \]  

\((15)\)

Both solutions satisfying the promised keeping constraint implies

\[ \varepsilon^+ = \left( \frac{(1 - \pi^s) + \pi^s \text{Prob}(z \leq z_i)}{(1 - \pi^s) + \pi^s \text{Prob}(z \leq z_j)} \right) \frac{1 - \pi(e)}{\pi(e)} \varepsilon^- \]  

\((16)\)

The values for \((\varepsilon^+, \varepsilon^-)\) satisfy conditions \((15)-(16)\).

Let \(V^*\) be the value of the firm from the allocation assumed to be the solution, and \(\tilde{V}\) the value associated with the alternative feasible candidate.

We have that

\[ \tilde{V} - \tilde{V}^* = \beta(1 - \lambda)\psi\{[aV(\tilde{U}^+) + bV(\tilde{U}^-)] - [aV(U^+) + bV(U^-)]\} \]  

\((17)\)

where

\[ a = [\pi^s + (1 - \pi^s) \text{Prob}(z \leq z_j)]\pi(e) > 0 \]

\[ b = [\pi^s + (1 - \pi^s) \text{Prob}(z \leq z_i)](1 - \pi(e)) > 0 \]

Let \(\alpha = \frac{a}{a+b}\), then \(\tilde{V} - \tilde{V}^* > 0\) reduces to

\[ \alpha V(\tilde{U}^+) + (1 - \alpha)V(\tilde{U}^-) > \alpha V(U^+) + (1 - \alpha)V(U^-) \]  

\((18)\)

Notice that by construction

\[ \alpha \tilde{U}^+(1 - \alpha)\tilde{U}^- = \alpha U^+(1 - \alpha)U^- . \]

To see this notice that

\[ \alpha \tilde{U}^+ + (1 - \alpha)\tilde{U}^- = \alpha U^+ + (1 - \alpha)U^- - \alpha \varepsilon^+ + (1 - \alpha)\varepsilon^- \]
and,

$$-\alpha \varepsilon^+ + (1 - \alpha) \varepsilon^- = -\frac{a}{a + b} \varepsilon^+ + \frac{b}{a + b} \varepsilon^- = \frac{a}{a + b} \left[ \frac{b}{a} \varepsilon^- - \varepsilon^+ \right]$$  \hspace{1cm} (19)

\[ = 0 \text{ by condition (16)} \]

Hence, given that $U^+ - U^- > \tilde{U}^+ - \tilde{U}^-$, condition (18) holds, and $U^+ \neq U^-$ cannot be optimal.

- **CASE 2.** Define $(z_i, z_j, \tilde{U}^-, \tilde{U}^+)$ as in case 1. We have that:
  - $z_j \geq z_i$,
  - $U^*,z_{j+1} > U^+ = U^*z_j$ and $U^*,z_{i+1} > U^- > U^*,z_i$, and
  - For sufficiently low $(\varepsilon^+, \varepsilon^-)$ it is the case that

$$U^*,z_{j+1} > U^* > U^*,z_{j-1} > \tilde{U}^- > U^*,z_i. \quad (20)$$

Both solutions satisfying promised keeping constraint implies

$$\varepsilon^+ = \frac{(1 - \pi^s) + \pi^s \text{Prob}(z \leq z_i)}{(1 - \pi^s) + \pi^s \text{Prob}(z < z_j)} \frac{1 - \pi(e)}{\pi(e)} \varepsilon^- \quad (21)$$

The values for $(\varepsilon^+, \varepsilon^-)$ satisfy (20)-(21).

Let $V^*$ be the value of the firm from the allocation assumed to be the solution, and $\tilde{V}$ the value associated with the alternative feasible candidate. Recall that the incumbent keeps the worker if the outside competitor is not willing to offer a higher promised utility than the one it will deliver. Then we have,

$$\tilde{V} - V^* = \beta(1 - \lambda) \psi \left[ a' V(\tilde{U}^+) + b V(\tilde{U}^-) \right] + \int_{\varepsilon \geq 0} \left[ V(U^*,z_j) I(z > z_j) - V(U^+) \right] - \left[ a' V(U^+) + b V(U^-) \right]$$

where $a' = [\pi^s + (1 - \pi(e)) \text{Prob}(z < z_j)] \pi(e) > 0$, and $b$ is defined as in
case 1. Let $\alpha' = \frac{a'}{a + b}$. Recall that in this case, $U^+ = U^{*,z}$, and hence the term $c$ is non negative. Then $\tilde{V} - \tilde{V}^* > 0$ reduces to

$$\alpha V(\tilde{U}^+) + (1 - \alpha)V(\tilde{U}^-) > \alpha V(U^+) + (1 - \alpha)V(U^-)$$  \hspace{1cm} (23)$$

Notice that by construction

$$\alpha' \tilde{U}^+ (1 - \alpha') \tilde{U}^- = \alpha' U^+ (1 - \alpha') U^-.$$

Hence, given that $U^+ - U^- > \tilde{U}^+ - \tilde{U}^-$, condition (23) holds and $U^+ \neq U^-$ cannot be optimal.

- **CASE 3.** Here, $(\epsilon^+, \epsilon^-, V^*, \tilde{V})$ are defined as in case 1. Then, $\tilde{V} > V^*$ and $U^+ \neq U^-$ cannot be optimal.

- **CASE 4.** Here, $(\epsilon^+, \epsilon^-, V^*, \tilde{V})$ are defined as in case 2. Then, $\tilde{V} > V^*$ and $U^+ \neq U^- \neq U^-$ cannot be optimal.

\[\square\]

### A-1.2 Robustness Check

Here I set two alternative values for the parameter $\Delta_A$ and recalibrate the parameters $(h_2, \rho, \Delta z)$ in order to match the targets. In particular, I set $\Delta_A = 0.45$ and $\Delta_A = 0.25$. I refer to these two alternative calibrations as high-$\Delta_A$ and ligh-$\Delta_A$ respectively. In the two scenarios I study what is the impact of removing the informational friction in wage dispersion. Table 4 shows the results.

<table>
<thead>
<tr>
<th>Measure</th>
<th>Target</th>
<th>MH</th>
<th>NHM</th>
<th>MH</th>
<th>NHM</th>
</tr>
</thead>
<tbody>
<tr>
<td>College Premium</td>
<td>1.784</td>
<td>1.784</td>
<td>1.908</td>
<td>1.778</td>
<td>1.908</td>
</tr>
<tr>
<td>Ratio $w_{80} - w_{20}$</td>
<td>3.1</td>
<td>3.005</td>
<td>3.225</td>
<td>3.164</td>
<td>3.25</td>
</tr>
<tr>
<td>Ave Incentive Comp.</td>
<td>5%</td>
<td>5.012%</td>
<td>0</td>
<td>5.02%</td>
<td>0</td>
</tr>
<tr>
<td>Coefficient of Variation</td>
<td>x</td>
<td>0.471</td>
<td>0.511</td>
<td>0.466</td>
<td>0.51</td>
</tr>
</tbody>
</table>

Table 4: Robustness Check
The low-$\Delta_A$ yields $(h_2, \rho, \Delta_z) = (2.09, 4.25, 0.19)$ while the high-$\Delta_A$ yields $(h_2, \rho, \Delta_z) = (2.34, 2.31, 0.263)$. In both cases the (moral hazard) model closely match the targets. As can be seen, the main result holds under these alternative scenarios. In both cases, removing the informational friction leads to greater wage dispersion.

\section*{A-2 More on chapter 2}

\textbf{Proof of lemma (2.1)}

\textit{Proof.} First notice that that optimality requires

\[ m_{t+1} = m_t = e \Rightarrow U_t(W; h^t) = U_{t+1}(W; h^{t+1}). \]  

(24)

Now, suppose by contradiction that at any arbitrary period of time $T$, the constrained efficient contract $W^* = (P^*, I^*)$ delivers a promised utility $U_T(W^*; h^T) < \underline{U}$. We prove this violates incentive compatibility by constructing a feasible deviation for any possible labor status $m_T$ providing a utility higher than $\underline{U}$.

- If $m_T = u$, consider the following feasible deviation $\bar{P}$, with $\bar{a}_t(h^t), \bar{s}_t(h^t)) = (0, s^*)$ if $m_t = u$, and $\bar{a}_t(h^t), \bar{s}_t(h^t)) = (0, 0)$ if $m_t = e$. Clearly, $\bar{P} \in A$ and $U_T((\bar{P}, I^*); h^T) = \underline{U} > U_T((P^*, I^*); h^T)$. Hence, the contract $W^*$ is not incentive compatible and, therefore, cannot be the constrained efficient contract.

- If $m_T = e$, define $\hat{T}$ as the first period of that employment spell. By property (24), $U_{\hat{T}}(W^*; h^\hat{T}) = U_T(W^*; h^T)$. Now consider the situation in $\hat{T} - 1$. By definition, $m_{\hat{T}-1} = u$. Moreover, since $m_{\hat{T}} = e$, it has to be the case that the contract prescribes $a_{\hat{T}-1}(\cdot) > 0$. However, if $U_T(W^*; h^T) < \underline{U}$, it is the case that $a_{\hat{T}-1}(\cdot) > 0$ is not compatible incentive because a deviation with $\hat{a}_{\hat{T}-1}(\cdot) = 0$ implies higher utility than $\underline{U}$. To see this notice that since $v$ is
strictly increasing,

\[
\begin{align*}
\mathcal{U}_{\tilde{T}}^{-1}(W^*; h^{\tilde{T}} - 1) &= u(c_{\tilde{T}} - 1) - v(s_{\tilde{T}} - 1) + \beta[p(a_{\tilde{T}} - 1)\mathcal{U}_{\tilde{T}}(W^*; h^{\tilde{T}}|m_{\tilde{T}} = e) + (1 - p(a_{\tilde{T}} - 1))\mathcal{U}_{\tilde{T}}(W^*; h^{\tilde{T}}|m_{\tilde{T}} = u)] \\
&< u(c_{\tilde{T}} - 1) - v(s_{\tilde{T}} - 1) + \beta[p(a_{\tilde{T}} - 1)\mathcal{U}_{\tilde{T}}(W^*; h^{\tilde{T}}|m_{\tilde{T}} = e) + (1 - p(a_{\tilde{T}} - 1))\mathcal{U}_{\tilde{T}}(W^*; h^{\tilde{T}}|m_{\tilde{T}} = u)].
\end{align*}
\]

We showed in the first part of this proof that during unemployment, promised utility must be at least \( \mathcal{U} \); i.e.,

\[
\mathcal{U} \leq \mathcal{U}_{\tilde{T}}^{-1}(W^*; h^{\tilde{T}}|m_{\tilde{T}} = u).
\]

Then if \( \mathcal{U}_{\tilde{T}}^{-1}(W^*; h^{\tilde{T}}|m_{\tilde{T}} = e) < \mathcal{U} \), it is the case that

\[
\mathcal{U}_{\tilde{T}}^{-1}(W^*; h^{\tilde{T}} - 1) < u(c_{\tilde{T}} - 1) - v(s_{\tilde{T}} - 1) + \beta\mathcal{U}_{\tilde{T}}(W^*; h^{\tilde{T}}|m_{\tilde{T}} = u).
\] (25)

The last expression indicates that \( W^* \), which prescribes \( a_{\tilde{T}} - 1 > 0 \), is not incentive compatible because a deviation with \( a_{\tilde{T}} - 1 = 0 \) provides higher utility to the worker. Hence, \( W^* \) cannot be the constrained efficient contract.

\[
\Box
\]

**Proof of lemma 2.2**

*Proof.* Take \( U \geq \mathcal{U} \) and consider the contract \( \widetilde{W} \). For each \( t \) and \( h_t \) it assigns \( b_t(h_t) = \tilde{b} \), \( s_t(h_t) = \tilde{s} \), \( a_t(h_t) = 0 \), and \( w_t(h_t) = \tilde{w} \). Define \( (\tilde{b}, \tilde{w}, \tilde{s}) \) by

\[
u'(\tilde{b} + \tilde{s}x) = v'(\tilde{s}),
\]

\[
U = \frac{u(\tilde{b} + \tilde{s}x) - v(\tilde{s})}{1 - \beta},
\]

\[
u(\tilde{b} + \tilde{s}x) - v(\tilde{s}) = u(\tilde{w}) - v(\tilde{a}).
\]

By construction \( u(c_t(h_t)) - v(x_t(h_t)) \) is constant across time and histories and \( \mathcal{U}_t(\widetilde{W}; h_t^t) = U \). It remains to prove that \( \widetilde{W} \in C \). Since deviations are only
possible during unemployment and a contract is incentive compatible against any possible deviation if and only if it is incentive compatible against one period deviation, incentive compatibility implies that for each $t$ and each $h^t$

$$U \geq \max_{(s,a)} u(b + s\varpi) - v(s + a) + \beta U.$$  \hspace{1cm} (29)

$$\text{s.t } a \geq 0, s \geq 0, \text{ and } s + a \leq \vartheta.$$ 

In (29) we impose the fact that for each history and period, $\mathcal{U}_t(W; h^t) = U$. Clearly, (29) is satisfied because the solution of the maximization problem implies $a = 0$ and $s = \tilde{s}$. Hence, $\tilde{W} \in \mathcal{C}(U; h^0)$.

**Proof of lemma 2.3**

*Proof.* The proof is straightforward. The first best contract is characterized by equations (2.18)-(2.22). Under observability of effort, (ICC1) is not relevant while (ICC2) is satisfied by (2.18).

**Proof of lemma 2.4**

*Proof.* Consider the following Relaxation Problem:

$$V^*_R(U) = \max_{b,e,s,U^e,U^u} -b + \beta[p(a)V^e(U^e) + (1 - p(a))V^u(U^u)]$$  \hspace{1cm} (RP)

$$\text{s.t. }$$

$$u(b + s\varpi) - v(s + a) + \beta[p(a)U^e + (1 - p(a))U^u] - U = 0$$  \hspace{1cm} (PKC)

$$\beta p'(a)[U^e - U^u] - v'(s + a) = 0$$  \hspace{1cm} (ICC1)

$$(U^e, U^u) \in \Phi^2$$  \hspace{1cm} (FPUC)

$$s \geq 0.$$ 

We need to prove that if $v$ is linear, the solution for (RP) also solves (PP). To that end, denote $D(U)$ the set of all $(b, a, s, U^u, U^e)$ satisfying (ICC1),(ICC2),(PKC), (FPUC), and non negativity for $s$. Likewise, denote $D_R(U)$ the set of all $(b, a, s, U^u, U^e)$ satisfying previous restriction but (ICC2). Clearly $D(U) \subseteq D_R(U)$. Hence,
$V^u_R(U) \geq V^u(U)$. Notice however that

$$(\hat{e}, \hat{s}, \hat{U}^e, \hat{U}^u) \in \arg\max_{D_R(U)} V^u_R(U)$$

satisfies (ICC2). To see this, take first order condition with respect to $s$ and notice that $\lambda > 0$ and $\phi \geq 0$. In consequence, $(\hat{e}, \hat{s}, \hat{U}^e, \hat{U}^u) \in D(U)$ and solves (PP); otherwise, it will not be true that $V^u_R(U) \geq V^u(U)$.

**Proof of lemma 2.5**

*Proof.* 1. First observe that $\mu_2 = 0$.\(^{30}\) To see this, notice that if $s = 0$, constraint (ICC2) does not bind and, by complementary slackness, $\mu_2 = 0$. On the other hand, if $s > 0$, equation (2.25) implies $\mu_2 u''(b + s\varpi)\varpi = 0$ and then $\mu_2 = 0$. Now, manipulating first order conditions, and letting $\xi^u = 0$, we find

$$\frac{1}{u'(c_t)} - \frac{1}{u'(c_{t+1})} = \mu_1 p'(a) p(a) > 0. \quad (30)$$

Finally, by strict concavity of $u$ we have $c_t > c_{t+1}$.

2. The lower bound may be violated only if $b < \xi$. In that case, $s = 0$ is not incentive compatible. Since participation in the hidden labor market must be optimal from the agent’s point of view, it will solve $u'(b + s\varpi)\varpi = \alpha$. This implies that whenever the unemployed worker participates in the hidden labor market, $c = b + s\varpi = \xi$.

3. Suppose by contradiction that constraint (FPUC) never binds. Then, we can write the problem (PP) without it. Moreover, by corollary (?), $s_t = 0$ for each $t$. As a consequence, the problem (PP) collapses into the problem studied by Hopenhayn and Nicolini (1997) with no hidden labor market participation and no relevant bound for promised utility. As shown in Proposition 3 in Pavoni (2007), in this environment $U_t$ falls below any bound with positive probability. This contradicts lemma (2.1).

---

\(^{30}\)This is consistent with lemma 2.4 which indicates that with linear effort-cost function incentive compatibility constraint (ICC2) is not needed to formulate the constrained planner’s problem.
Proof of lemma 2.6

Proof. 1. Let \( U = \underline{U} \), using first order condition with respect to \( U^u \) and envelope condition we get

\[
V'(U^u) - V^{u}(\underline{U}) = \mu_1 \frac{p'(a)}{(1-p(a))} - \frac{\xi^u}{\beta(1-p(a))}.
\]

Clearly \( U^u < \underline{U} \) is not possible because it violates (FPUC). Suppose then that \( U^u > \underline{U} \). This implies (FPUC) does not bind and hence \( \xi^u = 0 \). However if \( \xi^u = 0 \), the concavity of \( V^u \) implies \( U^u \leq \underline{U} \), which contradicts our assumption that \( U^u > \underline{U} \). Therefore, it must be that \( U^u = \underline{U} \).

2. By lemmas 2.5.3 and the previous results, there exist a \( t^* \) such that \( U^u = \underline{U} \) and \( c_t = \overline{c} \) for each \( t > t^* \). By lemma 2.5.2, \( \overline{c} \geq \underline{c} \) which rules out \( \overline{c} < \underline{c} \). We prove \( \overline{c} > \underline{c} \) is not a solution. Suppose by contradiction that \( \overline{c} > \underline{c} \). Since \( u'(\overline{c})\overline{w} < \alpha \), at this point \( s = 0 \). Let \( a \) be the search effort prescribed in the optimal contract. We can write

\[
\begin{align*}
u(\overline{c}) - v(a) + \beta[p(a)\underline{U} + (1-p(a))\underline{U}] &
\geq
u(\overline{c}) + \beta \underline{U} > u(\underline{c}) - v(s^*) + \beta \underline{U} \equiv \underline{U}.
\end{align*}
\]

The first inequality arises form the fact that incentive compatibility requires that the utility delivered by the contract is not lower than the one associated with any feasible deviation, in particular with \( s = a = 0 \). The second strict inequality arises form our assumption that \( \overline{c} > \underline{c} \) and from the fact that \( v \) and \( u \) are strictly increasing. However, this implies that in the contract the planner is delivering a lifetime utility strictly higher than the one promised, which is not optimal. Hence \( \overline{c} = \underline{c} \).

\[\square\]
Proof of corollary 2.1

Proof. 1. Suppose by contradiction that at certain period \( s_t \neq 0 \) but lower bound for promised utility has not been reached. By lemma (2), \( c_t = \underline{c} \), but since \( U^u > \underline{U} \), by lemma (1), \( c_{t+1} < c_t = \underline{c} \) which is a contradiction.

2. The proof is trivial. When \( U = \underline{U}, b=0 \). Then, by incentive compatibility for \( s, s = s^* \).

\( \square \)

Algorithm

1. Define a grid for promised utility: \( U = \{U_1, ..., U_N\} \). The range of \( U \) is set such that \( U_1 = \underline{U} \) and \( U_N \) is big enough to guarantee that the value functions become negative.

2. Define search effort grids for effort and participation: \( E_f = \{a_1, ..., a_M\} \) and \( S = \{s_1, ..., s_M\} \), where \( a_1 = s_1 = 0 \), and \( a_M \) and \( s_M \) are big enough to avoid grid-dependent corner solutions.

3. Make initial guess \( V^{u,0}(U) = V^{u,FB}(U) \) for each \( U \in U \), where \( V^{u,FB}(U) \) is the first best planner’s surplus. For values out of the grid, use piece-wise linear interpolation.

4. Iteration \( i \) starts. Get \( V^{u,i+1} = TV^{u,i} \) as follow:

   (a) Suppose \( s = 0 \). Use equation (2.26) to obtain \( b \). Given \( b \), for each \( a \in E_f \) use (ICC1) and (PKC) to solve for the values of \( U^e \) and \( U^u \). Choose the combination \( \{a, b, U^e, U^u\} \) maximizing (PP). Check if (ICC2) is satisfied for \( s = 0 \). If it is satisfied, the maximization ends. If not, go to the next step.

   (b) Look for the best solution \( \{a, s\} \in E_f \times S \), using (ICC1), (ICC2) and (PKC) to solve for the corresponding \( b, U^e \) and \( U^u \).

5. Check convergency. That is, given a metric \( d \), evaluate \( d(V^{u,i}, V^{u,i+1}) > \epsilon \)? If so, let \( V^{u,i} = V^{u,i+1} \) and go to (4), otherwise END.
A-3 More on chapter 3

Proof of Lemma 3.1

Proof. Denote \( \{k_t', b_t'\}_{t=1}^{T} \) as the optimal paths for capital and debt for a firm that survive up to time \( T \). Denote \( \{d_t, p_t, r^f_t\}_{t=1}^{T} \) as the corresponding paths for dividend, probability of survive, and corporative interest rate respectively. Suppose by contradiction that exists a \( j < T \) such that \( d_j \neq 0 \) and \( b'_j \neq 0 \).

Take a \( \varepsilon < b_t \) satisfying \( 0 < \varepsilon < d_t \) and consider the perturbation \( \{\tilde{k}'_t, \tilde{b}'_t, \tilde{d}_t, \tilde{p}_t, \tilde{r}^f_t\}_{t=1}^{T} \) such that

\[
\tilde{b}_t = \begin{cases} b_t - \varepsilon & \text{if } t = j \\ b_t & \text{Otherwise} \end{cases}
\]

\[
\tilde{d}_t = \begin{cases} d_t - \varepsilon & \text{if } t = j \\ d_t + b'_j (r^f_j - \tilde{r}^f_j) + \varepsilon(1 + \tilde{r}^f_j) & \text{if } t = j + 1 \\ d_t & \text{Otherwise} \end{cases}
\]

Notice that \( \tilde{k}'_t = k'_t \) for \( t = 1, \ldots, T \). Moreover, \( \tilde{p}_j > p_j \) and \( \tilde{p}_t = p_t \) for \( t \neq j \). Similarly \( \tilde{r}^f_j < r^f_j \) and \( \tilde{p}_t = p_t \) for \( t \neq j \). Denote \( \tilde{V}_j \) as the value of the firm associated with the perturbation and \( V_j \) as the value of the firm associated with original paths at period \( j \). We can write

\[
\tilde{V}_t - V_t \geq -\varepsilon + \beta(1 + \tilde{r}^f)\varepsilon \geq 0
\]

Last inequality arises because \( \beta \leq \frac{1}{1+r^s} \), and implies that there exists a feasible perturbation delivering higher value for the firm, which contradicts optimality of the original sequence.

Algorithm

Here we describe the algorithm used to compute the equilibrium. It is integrated by two cycles. One for updating bond prices \( (q^s, q^f, q^e) \) and the expected repayment rate, \( E_{\rho}(S) \), and the other for updating the values of the firms \( (V, V^e) \). In the algorithm we identify by supra-index the iteration associated with the first cycle and by sub-index the one for the inner cycle.
1. Define a grid for firms’ capital \( k = \{0, k_2, ..., h_{NS}\} \) and a grid for firms’ equity \( x = \{0, x_2, ..., x_N\} \).

2. Make initial guesses for equilibrium sovereign bond prices \( q^s_0(S) = \frac{1}{1+r} \) and for the expected domestic repayment rate \( E\rho,0(S) = 1 \). Based on \((q^s_0, E\rho,0(S))\) compute the associated corporate bond prices schedule \((q^f_0, q^e_0)\) as (3.9)-(3.10):

3. Make an initial guess value of the firm \( V_0(x, S) \).

4. Suppose you are in iteration \( t \) with bonds prices \( q_t = \{q^f_t, q^e_t, q^s_t\} \). Compute the associated value of firm \( V^*_t \) as follow. Use as initial guess the one obtained in previous cycle, that is \( V^0_t = V^*_{t-1} \)

   (a) Suppose we are in \textbf{inner} iteration \( l \) solving for the value of \( V^l_t(x, S) \).
   
   For each \( k_j \in k \) define \( b_j \) according to the following rule: if \( k_j \leq x \) \( b_j = 0 \) otherwise \( b_j q^f_t(b_j, x, S) = x - k_j \). (Here we are using lemma 3.1).

   (b) For each point \((x, S)\) compute \( V^l \) as follow
   
   \[
   V^l_t(x, S) = \max_j x_i + q^f_t(b_j, x, S)b_j - k_j + E[\tilde{V}^l_{t-1}(x', S')|x' \geq 0].
   \]

   In previous expression \( S' = H^{l-1}(S) \). That is the function \( H \), is based on policies at previous inner iteration. In addition \( \tilde{V} \) is defined as in the problem 3.2.

   (c) Evaluate convergency in value function for incumbent. That is \( V^l_t \approx V^l_{t-1} \)? If so go to (4), if not update and go to (4.a)

5. Given value function for incumbent \( V^*_t \), solves problem for a newly born firm, \( V^*_{t_e} \).

6. Check convergency in sovereign bond price and expected repayment rate as follow:
(a) Compute the *equilibrium* sovereign recovery rate in each future state by:

\[
\rho_{t+1}(Z', \varepsilon'_1, \varepsilon'_2|S) = \min \left\{ 1, \frac{\sum_{j=N+1}^{N} b'_i (1 - \text{Exit}_j) + \text{Exit}_j L_j}{B'(S)} \right\}
\]

Where \( B'(S) \) is defined by the equilibrium condition (3.13).

(b) For each \( S \), compute equilibrium sovereign bond prices and expected recovery rate according to

\[
E[\rho_{t+1}(Z', \varepsilon'_1, \varepsilon'_2|S)] = q^s_{t+1}(S)(1 + r).
\]

\[
E_{\rho,t+1}(S) = E[\rho_{t+1}(Z', \varepsilon'_1, \varepsilon'_2|S)]
\]

(c) Compute also equilibrium domestic bond prices \( q^f_{t+1}, q^e_{t+1} \) according to (3.9)-(3.10).

(d) If convergency, i.e. \( (q^{t,s}, E_{\rho,t}) \) and \( (q^{t+1,s}, E_{\rho,t+1}) \) are sufficiently close, END. Otherwise, let \( q^{t,s} = q^{t+1,s} \), \( E_{\rho,t}(S) = E_{\rho,t+1}(S) \) and go to (3).