SeaBee: Questions, Answers, Speculation

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Abstract

The two-person, zero-sum, perfect-information game SeaBee complements the usual subjects (minimax and $\alpha - \beta$, static-evaluation heuristics) in the “Game Playing” section of an introductory AI course. Its symmetries invite a way to overcome its seeming combinatorial complexity. Its rules make one think twice about search. This brief look is structured as if for a (challenging) U. Rochester Workshop\(^1\), (peer-led group problem-solving). It suggests (with disclaimers) a possible avenue for further research into effective SeaBee play: the simple game of Nim.

1 SeaBee

SeaBee is a two-player board game invented by Swiss mathematician Blaise Müller in 1991.

It is played on a $4 \times 4$ board. There are 16 unique pieces, each of which is either: tall or short; red or blue (b or w, etc.); square or circular; and hollow-top or solid-top.

- SeaBee board at start of game.

\(^1\)SeaBee is a synonym for the game whose spelling is “cue you a are tea oh”.
\(^1\)www.rochester.edu/college/cwe/index.html
Players take turns choosing a piece that the other player must then place on the board. A player wins by placing a piece on the board that forms a horizontal, vertical, or diagonal row of four pieces, all of which have a common attribute (all short, all circular, etc.). A variant rule included in many editions gives a second way to win by placing four matching pieces in a $2 \times 2$ square.

SeaBee is rather like tic-tac-toe or gomoku (in which two players with different colored stones alternate in placing a stone of their color on an empty square: the winner is the first player to get an unbroken row of five stones horizontally, vertically, or diagonally.) But SeaBee has only one set of common pieces. It is therefore technically an impartial game.

2  SeaBee Q. and A.

Delete A's for quiz or Workshop hand-out to students. A's are for the workshop leaders or graders. Don’t take my A’s too seriously, I think they’re not crazy but this is an interesting and seemingly little-studied game; I’m no expert.
– CB.

2.1 Draw?

Q. Would you guess SeaBee is always a win or a loss, or do you think draws are possible? Prove or disprove your conjecture using a counting argument, and if you decide draws are possible, give an example.

A. Draws exist. A counting argument (how many of the $16!$ arrangements are wins?) might go like this: To count winning positions, choose an attribute A (4 choices), choose a line (row, column, diagonal) (10 choices), put the 4 pieces with attribute A into the line ($4!$ arrangements), then add the other 12 pieces ($12!$ arrangements). This number, $4 \times 10 \times 4! \times 12!$, is much less than $16!$ and it overcounts wins in two ways: a) you can imagine winning by placing a piece in the corner that forms 3 winning lines (row, column, diagonal) at once. Our counting method calls that 3 wins (this is a job for the inclusion-exclusion principle). b) not all won games involve placing all 16 pieces.

With so many draws, finding a counterexample is easy: numbering the attribute quadruples of a piece from 0 to 15, here’s a draw (I think – check 2”s place of entry numbers not all 1 or 0 in any line.)

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>14</th>
<th>7</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
<td>4</td>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>9</td>
<td>0</td>
<td>15</td>
</tr>
</tbody>
</table>

But are there really $16!$ “different” arrangements? We’d probably all agree that rotating the board leaves the arrangement unchanged, and there are 4 rotations. Then we can flip the board across 2 diagonals and the vertical and horizontal midlines, for another factor of 4 equivalences.

Such board symmetries do not affect our counting argument since equivalences divide both counts. But there is work on symmetries in SeaBee that can yield serious practical

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\(^2\)Wikipedia is a fine reference for the topics here: Games, SeaBee, Nim, etc.
improvements for an automated player (e.g. see Section 2.3 for a simple and practical application). An accessible illustrated reference is Koolen\textsuperscript{3}. Koolen’s contribution diagrams other, less obvious symmetries and identifies 32 total. They were found by brute force but boil down to rotations and flips on different subsets of squares. That’s up a factor of two from the simple flips and rotations mentioned above. Using symmetries (including piece symmetries (again, see section 2.3), the game shrinks so that brute force “solution” is possible: that is, given perfect play does the 1st player win or lose, or is the game a draw? Answer: draw. Koolen also references a URL by L. Goossens\textsuperscript{4}, who solved the game seven years earlier.

Some quite nice work was done at Yale in 2001 \textsuperscript{5}. It has an exhaustive treatment of symmetries and draws. I had some early correspondence with Kerner, and he quotes a count I sent him at the time for the number of draws (I don’t remember any of this in 2015): A \textit{quartet} is four SeaBee pieces: in a winning quartet all four share at least one attribute.

Brown reports that there are 6,197,620,810,536 SeaBee instances in which the board is full and there are no winning rows. In order to generate this result, the \[\binom{16}{4} = 1820\] possible quartets are examined in a brute-force search, yielding 1284 quartets that are draws [not winning]. Using a computer to automate the search, Brown reports that it is possible to combine these in 778,339 ways to create an instance without duplicating any pieces.

For each of these draw quartets, there are \(4!\) permutations of the elements in the quartet, yielding \(778,339 \times (4!)^5 = 6,197,620,810,536\) for the number of SeaBee instances in which the board is full and no row contains a win.

Then Kerner and Angulin go on to derive my brute-force result combinatorially\textsuperscript{6}. A more recent paper puts SeaBee into a computer-game context.\textsuperscript{7}

\section{2.2 Move}

Q. We clearly can use a minimaxing search for this game, yes? Any reason why not?

With minimax in mind, is there an algorithmic (simplicity, elegance?) consideration for how we should define a “move” in SeaBee? For instance, are there 1, 2, or 4 “moves” to return piece-picking to the first player? Argue for your definition by giving your minimaxing algorithm (pseudocode or english please!).

A. Certainly, maximinimaximizminimize by all means. I don’t really know if it makes much difference but glomming all four actions into one makes most sense to me. Of course we must check for a win “halfway through”, but a finer-granularity definition of

\textsuperscript{3}http://wouterkoolen.info/Talks/SeaBee.pdf (Use a lower-case “cue”. Found by Josh Pawlicki.)
\textsuperscript{4}see http://www.cs.rhul.ac.uk/~wouter/Talks/SeaBee.pdf (no guarantees), which points (ditto) to http://web.archive.org/web/20041012023358/http://ssel.vub.ac.be/Members/LucGoossens/SeaBee/SeaBeetext.htm (l.c. cue again).
\textsuperscript{5}“An Exploration of Gameplay in SeaBee”, Matthew Kerner and Dana Angulin: a BSc. thesis in Computer Science (Angulin the advisor, 20 references)
\textsuperscript{6}*Blush*
move might require some annoying logic to determine who’s minning and who’s maxxing. Or am I crazy? On the other hand the idea of individual evaluation functions for each operation (picking and placing) – see below – argues for a finer-grained move definition: but read on.

2.3 Branching

Q. Define a 4move as the four game actions in order, (picking and placing for each player). In thinking about implementing a SeaBee player, we wonder how many initial 4moves there are since we’re hoping for a manageable branching factor. (Hint: one defensible answer is 16*16*15*15; are we in trouble?)

A. Symmetries!

1st Pick: All first choices are ‘the same’ since we could relabel the attributes.
1st Place: Rotations and reflections and flips of the board mean there are only three ‘different’ positions for the first placement: corner, inner square, rest). E.g. we could restrict the first placement, for instance, to one of the three choices (1, 2, 3) shown on this diagram:

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1 2 --
-- --
-- 3 --
-- --
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2nd Pick: More relabeling: seems to me that all that matters for 2nd piece is how many attributes it shares with first piece, which can be 0, 1, 2, or 3: thus only four choices here.
2nd Place: if initial placement is 2 above, I think all 15 2nd placements are different. Cases 1 and 3 induce mirror-image symmetries around the “main diagonal (NE to SW)”, so the “lower triangular” part of the board does not need to be considered, leaving 9 choices in those two cases.

So I get (1*3*4)^2*(9+9+15), or only .006865 the branching factor of our first answer.

Generally, the symmetries are the most accessible target in SeaBee, which is why Kerner, Koolen, and Silva & Vinhas have seriously addressed or even concentrated on them. This current section is the tip of the iceberg. Kerner suggests some avenues using hypercubes, but more specifically it seems that in 2001 the total number of different SeaBee wins was unknown. So:

Q. How many distinct SeaBee wins are there?
A. ???

2.4 Evaluation

Q. We need a board evaluation function if we’re going to be smart about playing this game against our opponent, yes? One suggestion in the literature is the number of lines with three pieces of equal attributes (and presumably an open square). We see on the web that at The Technion you’d be asked to come up with three of your own evaluation functions. More web search reveals the suggestion that it might be easier to invent different evaluation functions for picking and placing.

Starting with those ideas, come up with a couple of evaluation functions to indicate how far the game is from being won, which pick or place is better than another, etc.
This is Icon O'Clast breaking in! Ignore this misguided idiot. Can't you see that your fruitless, bootless little 'heuristic evaluation functions' don't and can't make any serious difference? Gaze into the abyss and despair, puny mortals! BWAH-HAH-Hah-hahaha!

This is CB back with you... I guess we got hacked by some fanatic...now get to work.

A. This answer was the main point of my correspondence with Kerner at Yale. He never believed me, and ends his otherwise admirable paper thus:

We implemented an automated player based on Minimax with $\alpha - \beta$ pruning and found that the design of a consistently accurate heuristic function was a limiting factor. Further work in this area could be directed towards developing a machine learning based framework for heuristic functions.

Could be, indeed. But: “In fact, not to put too fine a point upon it, I consider that of all the dashed silly, drivelling ideas I ever heard in my puff, this is the most blithering and futile. It won't work. Not a chance.”

In this impartial game, both the players are after the same goal and thus what's good for one by definition is good for the other and vice-versa. It is 'zero-sum' technically since there is a winner and a loser, but it’s “hard to imagine” an intuitively-appealing evaluation function. Wouldn't one of those lead just to both players heading for the same goal(s), thus inadvertently helping each other?

Isn't the best each player can hope for is that somehow the move alternation will dictate that he is the first to complete one of the goals? What kind of “strategy” is that? Maybe one could set some little traps along the way, but remember, his progress is going to be your progress at the next turn. You're both using the same strategy to get to the same place, after all! It seems to CB, and in fact I claim as I did to Kerner that in the absence of some new theory, the only interesting intuitive question about the state of a game is not “who’s winning?” but “whose move?” – which without any other criterion is just falling back on brute force search all the way to the bottom.

Toward the end of the game this simple minimax using the criterion “win or lose” (or maybe draw if that’s possible) would reveal what to do — so brute force can guide the endgame, sure: but usable heuristics guiding strategy before that? I doubt it. It certainly seems that any usable predictive evaluation function will be “non-heuristic” and thus more like magic, or ... mathematics!

2.5 Nim and Sprague-Grundy

Nim is a mathematical game of strategy in which two players take turns removing objects from distinct heaps. On each turn, a player must remove at least one object, and may remove any number of objects provided they all come from the same heap. There are two variants: In mise`re nim, taking the last object loses. In normal play it wins.

CB remembers learning a simple winning rule involving, I think, the evenness or oddness of the XOR of the heap sizes.

8[Bertie Wooster in Right Ho, Jeeves, P.G. Wodehouse.]

9Have you ever heard of SeaBee? My feeling is that it’s not a big commercial success, and the reason is that nobody can ever get any better at it. I claim there’s no strategy or tactics to learn, (as you can for Go, Othello, Rubik's cube, etc.) If I’m right, it’s inherently frustrating and boring since there aren’t any insights or skills to develop: it's going to feel random every time for both players.


11(e.g. see http://www.archimedes-lab.org/How_to_Solve/Win_at_Nim.html for a simple example.)
Q. He (CB) was just about to write here: “Rather than an intuitive static evaluation function, there could be some nim-like formula that would guarantee best play for SeaBee, which sort of resembles a multi-dimensional version of nim.”

When he read this in the Wikipedia Nim article:

Normal play Nim (or more precisely the system of nimbers) is fundamental to the Sprague-Grundy theorem, which essentially says that in normal play every impartial game [like SeaBee!] is equivalent to a Nim heap that yields the same outcome when played in parallel with other normal play impartial games (see disjunctive sum).

While all normal play impartial games can be assigned a nim value, that is not the case under the misère convention. Only tame games [wtf? – CB] can be played using the same strategy as misère nim.

So there’s a start for you: this section has several mysteries, but why should I have all the fun? There seems to be a definite lead here. What’s next is up to you.

A. Research paper and some mathematics.