Essays on Macroeconomics with Heterogeneous Agents

by

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Biographical Sketch

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Abstract

Chapter 1 develops a heterogeneous-agent general equilibrium model that incorporates both intensive and extensive margins of labor supply. A nonconvexity in the mapping between time devoted to work and labor services distinguishes between extensive and intensive margins. We consider calibrated versions of this model that differ in the value of a key preference parameter for labor supply and the extent of heterogeneity. The model is able to capture the key features of the empirical hours worked distribution, including how individuals transit within this distribution. We then study how the various specifications influence labor supply responses to temporary shocks and permanent tax changes, with a particular focus on the intensive and extensive margin elasticities in response to these changes. We find important interactions between heterogeneity and the extent of curvature in preferences.

Chapter 2 builds a model of family labor supply in which individuals choose between full-time work, part-time work, and nonemployment. The model is calibrated to replicate the movements of both male and female workers among these states. The willingness to substitute hours over time (the so-called intertemporal elasticity of labor supply) is critical for many economic analysis. A common strategy for uncovering the value of this willingness is to carry out structural estimation on micro panel data. One general issue in this estimation exercises using micro data is that misspecification of the constraints that individuals face is likely to influence inference about preference parameters. In the model economy, although the individual labor supply problem is a discrete choice problem, individuals are able to adjust hours along the intensive margin by moving between part-time and full-time work. Intuitively, adjustment along the intensive margin potentially allows one to estimate the true value of the underlying curvature parameter describing the utility from leisure. We explore the extent to which standard labor supply methods can achieve this in our setting. Although these methods deliver precise estimates that are significantly different from zero, the estimates are ef-
fectively unrelated to the true underlying values. These methods also deliver elasticity estimates for women, even when the underlying preference parameters are the same for men and women.

Chapter 3 investigates the optimal progressive tax code in an incomplete-market economy in which households are linked intergenerationally by altruism and earning ability. The model economy is calibrated to that of the US with the progressive tax code suggested by Gouviea and Strauss (1994). First, I compute the equilibrium with the optimal progressive tax code. Second, I investigate the extent to which the size of government welfare programs affects the optimal progressivity of the income tax code. I find that the optimal tax code for an economy populated with altruistic households is approximately equivalent to a proportional tax of 23.1% with a fixed deduction of approximately $17,000 in 1990 US dollars. For an economy populated with non-altruistic households, however, these numbers are 18.8% and $12,000 respectively. This result implies that inequality is more severe in an economy with intergenerational links so that the policy maker requires a more progressive tax system to provide insurance. Additionally, I find that when the size of the government welfare program is chosen carefully, the additional insurance benefits from the progressive income tax code disappear.
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Chapter 1

Individual and Aggregate Labor Supply in a Heterogeneous Agent Economy with Intensive and Extensive Margins

1.1 Introduction

Connecting individual and aggregate labor supply has long been controversial. An early controversy revolved around the fact that stand-in household models such as Kydland and Prescott (1982) assumed much larger Frisch labor supply elasticities than those estimated by researchers such as MaCurdy (1981) and Altonji (1986) from micro data. Heckman (1984) argued that this controversy was somewhat misdirected given that these studies abstracted from labor adjustment along the extensive margin. Subsequent work has addressed this deficiency, and led to a few general points of consensus. First, aggregate labor supply responses depend upon adjustment along both the intensive and extensive margins. Second, the individual preference parameter estimated by MaCurdy, Altonji and others is closely related to aggregate responses along the intensive (i.e.,
hours per worker) margin. And third, the extent of adjustment along the extensive margin is heavily influenced by the amount of heterogeneity, or more specifically, the mass of workers that are close to a “reservation wage curve”.

Chetty et al. (2011) argues that further headway in understanding labor supply requires connecting macro models with microeconomic estimates of elasticities for both intensive and extensive margins, in both steady state and business cycle settings. We agree with this call for future research, but note that this call for research points to a key void in the literature: a systematic analysis of how preference parameters and the extent of heterogeneity interact to affect the connection between individual and aggregate labor supply in aggregate models that allow for both intensive and extensive margins. Many papers that consider adjustment along the extensive margin abstract entirely from adjustment along the extensive margin, and models that feature both margins often abstract from heterogeneity. This paper seeks to fill this void by developing and analyzing a model that features heterogeneity in addition to adjustment along the intensive and extensive margins. Our model merges the framework of Chang and Kim (2006, 2007) with that of Prescott et al. (2009) and Rogerson and Wallenius (2009).

We consider several specifications of the model that differ in the extent of heterogeneity and the value of the preference parameter that dictates curvature in the disutility of work function. All of the specifications are calibrated so as to match the average employment rate and the average level of hours per worker. We assess the ability of these specifications to account for various steady state observations, including the distribution of hours of work across individuals, the transition of individuals in the hours of work distribution over time, and the distribution of labor earnings and wealth. Although our model is parsimonious, it is able to account for many of the salient features found in the data. There are two novel aspects to our steady state calibration exercise that we think are worth noting. First, we argue that the cross-sectional distribution of hours of work can serve as useful information regarding the extent of

\[1\] Hansen (1985) is the first of many papers that study business cycles in a setting with only adjustment along the extensive margin and no heterogeneity. Chang and Kim (2007) introduced heterogeneity but still did not allow for an intensive margin. Cho and Cooley (1994) and Kydland and Prescott (1991) for early studies that feature adjustment along both margins of labor supply in a general equilibrium framework, but without heterogeneity.
heterogeneity in the data. Interestingly, based on this measure, we need a degree of heterogeneity that is roughly double the amount captured by estimates of idiosyncratic wage shocks. As we note later on, it turns out that the extent of heterogeneity has important implications, so the development of simple procedures for assessing the appropriate degree of heterogeneity within an aggregate model is important. Second, to our knowledge ours is the first aggregate analysis to address how individuals transition within the hours worked distribution. Previous analyses have instead focused on how individuals transition between the states of employment and non-employment.

We then consider the implications of these different specifications for two standard types of analyses: business cycle fluctuations and steady state tax analyses. For ease of exposition and transparency we focus on specifications that are common in the literature. In the business cycle analysis we consider aggregate shocks to productivity as the driving force behind business cycles, and for the steady state tax analysis we consider a lump-sum transfer program financed by a proportional tax on labor earnings. Several findings emerge from our analysis. First, in contrast to the apparent consensus that has emerged, we find that fluctuations along the intensive and extensive margin are both functions of the underlying curvature parameter in preferences and the extent of heterogeneity. That is, intensive and extensive elasticities are not independent of each other and so must be considered jointly. Second, we find that the extent of heterogeneity has very different effects on extensive margin responses in the two settings: in the business cycle setting we find that increased heterogeneity leads to less response along the extensive margin, whereas in the case of permanent changes to tax and transfer programs, we find that increased heterogeneity leads to more response along the extensive margin. Third, in the business cycle context we find that the presence of an intensive margin dampens the effects of increased heterogeneity in an important way. It follows that abstracting from the intensive margin is a serious issue for analyses that seek to understand the effect of heterogeneity on the magnitude of aggregate fluctuations. Fourth, we find that in the context of permanent tax changes, the preference parameter plays effectively no role in influencing the change in aggregate hours when the extent of heterogeneity is relatively low, but does play a role as the extent of heterogeneity increases. That is, the extent of heterogeneity interacts with the preference parameter
in a quantitatively important manner.

Our paper is related to several in the literature. Relative to the business cycle analysis of Chang and Kim (2007), we add an intensive margin. Relative to the tax analysis of Rogerson and Wallenius (2009), we consider a richer environment in terms of heterogeneity, and allow for uncertainty and incomplete markets. The paper that is closest to ours is Erosa et al. (2011). Like us, they build a model that features heterogeneity and incomplete markets and allows for labor supply adjustment along the intensive and extensive margin. They adopt a life cycle structure, consider a richer environment in terms of sources of heterogeneity, make more effort to isolate a definitive calibration. We carry out a more extensive analysis of business cycle fluctuations and consider a wider range of specifications in order to assess the implications of variation in the key preference parameter and the extent of heterogeneity. We view these two pieces of work as complementary.

The paper is organized as follows: Section 2 specifies the model. Section 3 calibrates the different specifications of the model economy and Section 4 considers the steady state properties of the various specifications. Section 5 studies the business cycle properties of the model, while Section 6 considers the effects of permanent tax changes. Section 7 concludes.

1.2 Model

The model is essentially the indivisible labor incomplete markets model of Chang and Kim (2007) extended in the spirit of Prescott et al. (2009) to allow for adjustment along the intensive margin. The details follow. There is a unit measure of ex-ante identical infinitely lived individuals. Each individual has preferences over streams of consumption \((c_t)\) and hours of work \((h_t)\) given by:

\[
\sum_{t=0}^{\infty} \beta^t \left[ \log c_t - B \frac{h_t^{1+1/\gamma}}{1+1/\gamma} \right]
\]

where \(0 < \beta < 1\), \(B > 0\) and \(\gamma > 0\).

There is an aggregate Cobb-Douglas production function that produces output
using inputs of labor services \((L_t)\) and capital services \((K_t)\) and is subject to TFP shocks \((\lambda_t)\):

\[
Y_t = \lambda_t L_t^\alpha K_t^{1-\alpha}.
\]

The aggregate productivity \(\lambda_t\) evolves with a transition probability distribution function \(\pi_\lambda(\lambda'|\lambda) = \Pr(\lambda_{t+1} \leq \lambda'|\lambda_t = \lambda)\). In our quantitative analysis we will assume that \(\lambda_t\) follows an AR(1) process in logs:

\[
\ln \lambda_{t+1} = \rho_\lambda \ln \lambda_t + \varepsilon_\lambda, \quad \varepsilon_\lambda \sim N(0, \sigma_\lambda^2).
\]

Output can be used for either consumption or investment, and capital depreciates at rate \(\delta\).

Two features influence the mapping from time devoted to work to labor services. The first is that individuals are subject to idiosyncratic productivity shocks, denoted by \(x_t\). The stochastic evolution of \(x_t\) is described by the transition probability distribution function \(\pi_x(x'|x) = \Pr(x_{t+1} \leq x'|x_t = x)\). In our quantitative work we will also assume that \(x_t\) follows an AR(1) process in logs:

\[
\ln x_{t+1} = \rho_x \ln x_t + \varepsilon_{xt}, \quad \varepsilon_{xt} \sim N(0, \sigma_x^2).
\]

The second feature is a non-convexity associated with such factors as set-up costs, supervisory time and/or the need to coordinate with other workers. If an individual with idiosyncratic productivity \(x_t\) devotes \(h_t\) units of time to market work, this will generate \(x_t g(h_t)\) units of labor services. Following Prescott et al (2009) and Rogerson and Wallenius (2009), we assume that \(g(.)\) takes the following simple form:

\[
g(h_t) = \max \left\{ 0, h_t - \bar{h} \right\}, \quad h_t \in [0, 1].
\]

Following Bewley (1986), Huggett (1993), and Aiyagari (1994) we assume that markets are incomplete in the sense that there are no markets for insurance and the only asset is physical capital. Individuals trade claims to physical capital, and these claims are denoted by \(a\). Additionally, there is an exogenous borrowing constraint that
limits the amount of debt that an individual can acquire:

\[ a_t \geq \bar{a} \]

In each period \( t \) there is a market for units of labor services, with price \( w_t \), and a rental market for capital services, with price \( r_t + \delta \), so that \( r_t \) is the rate of return to capital. When a worker of productivity \( x_t \) devotes \( h_t \) units of time to market work, the resulting labor earnings are \( w_t x_t g(h_t) \). The government taxes labor earnings at a flat rate of \( \tau \) and rebates all the revenue with a lump-sum transfer amount, \( Tr \), to all households.

Our model assumes that all (exogenous) heterogeneity occurs along one dimension—that of productivity in market work. More generally, one could also imagine that individuals differ along a second dimension, which could be thought of as the return to non-market work, either in the form of differences in the value of leisure time or in the productivity of non-market work. From the perspective of market labor supply choices, what really matters is the relative return to market work. While we could have introduced a second idiosyncratic shock, to maintain parsimony we have opted for a single idiosyncratic shock that we will interpret as a composite shock. It will be relevant to keep this in mind when we discuss calibration.

We formulate equilibrium recursively. The individual state variables are beginning of period assets \( (a) \) and current idiosyncratic productivity \( (x) \), and the aggregate state variables will be the current aggregate productivity shock \( (\lambda) \) and a measure \( \mu \) over the individual state variables \( (a, x) \). Prices are functions of the aggregate state: \( w(\lambda, \mu) \) and \( r(\lambda, \mu) \), and the equilibrium law of motion for \( \mu \) is given by \( \mu' = T(\lambda, \mu) \).

The value function for a worker, denoted by \( V \), is:

\[
V(a, x; \lambda, \mu) = \max_{c, a', h} \left\{ \log(c) - B \frac{\mu^{1+\frac{1}{\gamma}}}{1 + \frac{1}{\gamma}} + \beta E \left[ V(a', x'; \lambda', T(\lambda, \mu)) \right] | x, \lambda \right\}
\]

s.t. \[
c = (1 - \tau)w(\lambda, \mu)x \max\{0, h - \bar{h}\} + (1 + r(\lambda, \mu))a - a' + Tr
\]
\[
c \geq 0, \ a' \geq \bar{a}, \ 0 \leq h \leq 1
\]
An equilibrium consists of a value function $V(a, x; \lambda, \mu)$, individual decision rules $c(a, x; \lambda, \mu)$, $a'(a, x; \lambda, \mu)$, $h(a, x; \lambda, \mu)$, aggregate inputs $\{K(\lambda, \mu), L(\lambda, \mu)\}$, factor prices $\{w(\lambda, \mu), r(\lambda, \mu)\}$, and a law of motion $T(\lambda, \mu)$ such that

1. Individuals optimize:

   Given factor prices, individual decision rules solve value function.

2. The representative firm maximizes profits: For all $(\lambda, \mu)$

   \[
   w(\lambda, \mu) = F_1(L(\lambda, \mu), K(\lambda, \mu), \lambda) \\
   r(\lambda, \mu) = F_2(L(\lambda, \mu), K(\lambda, \mu), \lambda) - \delta
   \]

3. The goods market clears: For all $(\lambda, \mu)$

   \[
   \int \{a' + c\} \, d\mu = Y + (1 - \delta) K
   \]

4. Factor markets clear:

   \[
   L(\lambda, \mu) = \int x g(h(a, x; \lambda, \mu)) \, d\mu \\
   K(\lambda, \mu) = \int a d\mu
   \]

5. Government balances its budget:

   \[
   \int \{\tau w(\lambda, \mu)x \max\{0, h - \bar{h}\}\} \, d\mu = Tr
   \]

6. Individual and aggregate behaviors are consistent:

   \[
   \mu'(A^0, X^0) = \int_{A^0, X^0} \left\{ \int_{A, x} 1 \left\{ a' = a'(a, x; \lambda, \mu) \right\} \frac{d\pi_x(x'|x)}{d\mu} \right\} \, da \, dx'
   \]

   for all $A^0 \subset A$, $X^0 \subset X$. 
1.3 Calibration

Our key objective is to examine how different values of the preference parameter $\gamma$ and the amount of heterogeneity influence the magnitude and nature of responses in aggregate hours to business cycle shocks and permanent fiscal policy changes. A simple way to vary the amount of heterogeneity in the economy is to vary the standard deviation of the innovations to the idiosyncratic shock process, and this is the approach that we follow. Accordingly, we will consider specifications in which $\gamma$ takes on values in the set $\{0.5, 1.0, 1.5\}$ and $\sigma_x$ takes on values in the set $\{0.165, 0.2475, 0.330\}$. Motivation for these values is given below.

As is standard in the business cycle literature we assume that each period corresponds to one quarter. Many of our parameters are standard in the literature and so we set them to be in line with previous studies. In particular, the labor-income share, $\alpha$, is set to 0.64, the depreciation rate, $\delta$, is set to 0.025, and for the aggregate technology shocks we set $\rho_\lambda = 0.95$, and $\sigma_\lambda = 0.007$. For the benchmark economy, there is no tax and transfer: $\tau = 0$ and $T_T = 0$.

We noted earlier that our idiosyncratic shock is best thought of a composite shock that reflects the net effect of idiosyncratic shocks on the relative return to working in the market versus not working. A reasonable lower bound on the size of these shocks is provided by the literature that estimates idiosyncratic shocks to wages. A sizeable literature has done this for prime age males, including, for example, Card (1994), Floden and Linde (2001), French (2005), Chang and Kim (2006), and Heathcote et al. (2007). While there is some variation across studies, the consensus is that these shocks are large and persistent. Guided by these empirical studies, for one of our specifications we set $\rho_x = 0.975$ and $\sigma_x = 0.165$. We assume that the individual and the aggregate shock processes are orthogonal.

As noted above, we view this as a reasonable lower bound on the extent of heterogeneity induced by idiosyncratic shocks. As an upper bound we consider a specification

\footnote{This is reasonable as long as other idiosyncratic shocks are not perfectly negatively correlated with idiosyncratic wage shocks.}

\footnote{Note that all of the papers previously cited estimated shocks based on annual data, so that our benchmark values need to be converted to annualized values when comparing them to the literature. Our values correspond to the estimates in Floden and Linde (2001).}
with the same persistence, i.e., $\rho_x = .975$, but double the standard deviation of the innovations so that $\sigma_x = .330$. As we document later, the reason we view this is a reasonable upper bound is that it generates a dispersion in hours worked that exceeds what is found in the data. We also consider one intermediate value ($\sigma_x = .2475$).

Motivation for the set of values considered for $\gamma$ comes from the implied range of Frisch elasticities when we run a standard labor supply regression for workers with positive hours using micro data generated from the model. Recent work by Chetty (2010) argues that an empirically reasonable value for this elasticity is in the range of $0.40 - .50$. As discussed in Keane and Rogerson (2011), there are additional factors that Chetty abstracts from that would suggest higher values. Our set of values for $\gamma$ leads to a range of estimated Frisch elasticities that run from around $0.25$ to $1.00$, which we think of as a reasonable range.

There are four additional parameters to calibrate. For a given value of $\gamma$ and $\sigma_x$, we choose values of the discount factor, $\beta$, disutility of work parameter, $B$, the borrowing limit, $a$, and the fixed hours cost, $h$, so that in the steady state the (quarterly) rate of return to capital is $1\%$, the employment rate is $70\%$, the borrowing limit is equal to two quarters’ earnings of a worker with productivity equal to the mean of all employed workers, and average hours (conditional on working) is $1/3$.

For purposes of comparison it is of interest for us to also have an economy that does not feature an intensive margin. We do this by fixing $h$ to be $1/3$ for all workers, and removing the parameter $\tilde{h}$ from the calibration exercise. All other aspects of the calibration exercise remain the same. In particular, the value of $B$ will be adjusted so that steady state employment rate in this economy will also equal $70\%$. This alternative economy is a version of Chang and Kim (2007) and we will denote it as the Extensive Economy, abbreviated as “Ext”.

\[\text{With a quarterly employment rate of 70\%, the average annual employment rate in our model (i.e., fraction of individuals who work at least one quarter during a year) is 76.7\%. This corresponds to the average annual employment rate in the PSID over the period 1968-1998 for household heads and spouses with ages between 18 and 65. We use a cutoff of 240 annual hours as the threshold for employment, i.e., we treat individuals with less than 240 annual hours worked as not employed.}\]

\[\text{There is one exception to note. When } \sigma_x = .330 \text{ the natural borrowing constraint for the economy with only an extensive margin is tighter than the borrowing constraint used previously, so in this case the borrowing limit is set to zero. The borrowing constraint is tighter in an economy without an intensive margin since it precludes people in low productivity states with high debt from working more hours as a way to generate more income in these states.}\]
Table 1.1 summarizes the parameter values that are held constant across specifications as well as those that vary across specifications.

1.4 Properties of Steady State

In this section we consider some of the properties of the steady state equilibrium. There are two main objectives of this section. The first is to demonstrate that although our model is highly stylized, it is able to capture many features of the heterogeneity in wealth, earnings and hours worked found in the data. The second objective is to examine how different values for $\gamma$ and $\sigma_x$ influence the model’s ability to account for these features in the data.

1.4.1 The Hours Worked Distribution

As noted in the calibration section, all of our model specifications are calibrated so as to generate the same fraction of people employed and the same average hours for workers conditional on employment. In this subsection we examine the extent to which the model can account for the distribution of hours worked among workers, and how this distribution varies across the various model specifications. We begin with Table 1.2 which reports standard deviations of the steady state distribution of annual hours of work, conditional on working. We compute this measure at the annual level because it is not available at the quarterly level in the PSID. Our sample is all household heads and spouses between the ages of 18 and 65 during the period 1968-1997. In the data there are some individuals who work very few hours during the year. We therefore classify a worker as employed if he or she works at least 240 hours per year, and treat those with less than 240 annual hours as having zero hours. In the model, an individual is classified as employed if he or she has positive hours for at least one quarter during the year. While for some of the subsequent analysis we will utilize the panel nature of the PSID, this feature is not essential to this calculation, and so as a robustness check.

---

$^6$This adjustment affects relatively few individuals and our results are not very sensitive to variation in this cutoff.
we also include a measure based on the CPS\footnote{Annual Hours for the CPS data is obtained by multiplying “Average Hours per Week” and “Number of Weeks Worked”}. We normalize annual hours in the CPS and PSID so that average annual hours is the same as in the economy with $\gamma = 1$ and $\sigma_x = .165$\footnote{Average annual hours do not vary much across model specifications, so we do not renormalize for comparison with each specification}.

The standard deviations in the PSID and CPS are fairly comparable – .418 in the former and .454 in the latter. Acknowledging that there is likely to be some measurement error in hours, the true dispersion in hours will be less than indicated by these values. The standard deviations for the 9 model specifications range from .301 to .480. Consistent with intuition the hours dispersion is increasing in $\gamma$ and in $\sigma_x$. When $\sigma_x = .165$, the model cannot generate sufficient dispersion in hours even with a relatively large value of $\gamma$. However, when $\sigma_x = .330$ the model is able to match the dispersion found in the PSID as long as $\gamma$ is around 1.0 or larger. As noted earlier, this motivates our choice of $\sigma_x = .330$ as a reasonable upper bound on the extent of heterogeneity in our model\footnote{Table 2 might lead one to consider even higher values for $\sigma_x$ combined with smaller values for $\gamma$. However, as we show in a later subsection, this leads to an estimated Frisch elasticity that is well below .40, and so we do not consider this region of parameter space.}.

To examine the implications for the distribution of hours worked in somewhat more detail, we next look at the average hours worked at various percentiles of the hours distribution. Figure 1 plots these values for two of the model specifications (the high and low values of $\sigma_x$ with $\gamma = 1$) as well as the corresponding values for the PSID.

Consistent with the message based on looking simply at the standard deviations, we see that the specification with $\gamma = 1$ and $\sigma_x = .330$ does a much better job of tracking the empirical hours distributions than does the specification with the lower value of $\sigma_x$. In fact, this specification tracks the distribution in the PSID quite well.

### 1.4.2 Employment and Hours Transitions

Having assessed the model’s ability to account for the distribution of individuals between employment and non-employment as well as the distribution of hours among employed workers, we next examine the model’s ability to account for the movement
of individuals within the hours worked distribution, including transitions into and out of employment.

Our data on transitions comes from the PSID and so are based on annual measures. We begin by looking at transitions into and out of employment. Table 1.3 shows the distribution of individuals across different combinations of employment states in consecutive years for the PSID and one of our model specifications ($\gamma = 1$ and $\sigma_x = .165$). We only report statistics for one of the model specifications because it turns out that these statistics are virtually identical across all 9 specifications.

The model does a good job of accounting for transitions into and out of employment, though employment status is somewhat less persistent in the model than in the data, as evidenced by the fact that the model has greater share of workers changing employment status across consecutive years than does the PSID. However, this difference is relatively small. For example, in the PSID, the persistence of employment (i.e., the probability of being employed next year conditional on being employed this year) is .74 whereas this probability is .73 in the model.

Next we consider the transition matrices for annual hours worked between years $t$ and $t + 1$. Table 1.4 reports the transition rates between quintiles of the hours worked distribution (and non-employment) from the PSID and the two model specifications in which $\gamma = 1$ and the two extreme values of $\sigma_x$. The transition matrices for different values of $\gamma$ holding $\sigma_x$ constant turn out to be quite similar, so in the interest of space we do not include them. As we will see, although changes in $\sigma_x$ do produce some quantitative differences, most notably in the degree of persistence, the basic patterns are also not affected much by changes in $\sigma_x$.

We start by noting three prominent features in the data. First, annual hours of work exhibit a significant degree of persistence, especially for those workers with high hours of work. All of the diagonal elements for workers with positive hours are greater than 40%, and for the highest quintile this value exceeds 60%. Second, conditional on working in both periods and switching quintiles, the transition probabilities are monotone decreasing in the distance of the destination quintile from the originating quintile. Third, individuals who adjust along the extensive margin between years $t$ and $t + 1$ are disproportionately from the lowest quintile of the hours distribution.
Specifically, for those individuals who work in year $t$ but not in $t + 1$, roughly two-thirds of them have annual hours of work in the lowest quintile in year $t$. Similarly, for those individuals who did not work in year $t$ but did work in year $t + 1$, roughly three-quarters of them have annual hours in the lowest quintile in year $t + 1$.

Next we consider how the model fares in terms of reproducing this features. As the table shows, both model specifications also generate considerable persistence, though less in the economy with $\sigma_x = .33$. The average of the diagonal elements for those with positive hours in both periods is 49.48 in the data, 46.13 in the economy with $\sigma_x = .165$ and 38.48 in the economy with $\sigma_x = .33$. While both model specifications come close to matching the persistence in the highest quintile, the lowest quintile displays quite a bit more persistence in the data than in either model—49.45 versus 32.33 and 32.66. One possible explanation for this is the existence of a group of workers in the data who desire part-time work on a more permanent basis than captured by the idiosyncratic shocks in our model.

For the most part the model also matches the second observation noted above. Specifically, for the specification with $\sigma_x = .33$ the model has the same monotonicity property found in the data, whereas for the other specification though there are a couple of values which violate the pattern.

Finally, the model does a good job of matching the nature of adjustment along the extensive margin. For both of the model specifications shown in the table, roughly three quarters of all transitions along the extensive margin involve workers who are in the lowest quintile of the hours distribution.

1.4.3 Micro Frisch Elasticity

Next we examine what one would conclude about the Frisch elasticity in the steady state of our model based on running a standard labor supply regression. To do this we generate an artificial panel data from our model economy (50000 workers with 100 quarters), and aggregate it to the annual level. As is standard in the empirical labor supply literature, and in the spirit of Altonji (1986), we run the following regression:

$$\log h_{it} = b_0 + b_w \log w_{it}^h + b_c \log c_{it} + \varepsilon_{it}$$
using all observations with positive hours, where the individual wage rate is defined as earnings divided by hours, $w_{it}^h = w_{it} x_{it} g(h_{it}) / h_{it}$, conditional on working ($h_{it} > 0$). The resulting parameter estimate $b_w$ is the so-called micro Frisch labor supply elasticity. If a worker with preferences as in our model were to make optimal choices facing a linear budget constraint, the first order condition would imply that $b_w$ is equal to $\gamma$, so that under these circumstances this regression can uncover the true value of the preference parameter $\gamma$. As is known from previous work, a modification of the standard labor supply model that dispenses with a linear budget constraint will break the close link between preference parameter $\gamma$ and observed labor supply elasticity $b_w$ (see, e.g., Blomquist (1983), Moffit (1983), Rogerson and Wallenius (2009), Chang et al. (2011)). Table 1.5 reports the results of running this regression in the various specifications of our model.

While the estimated values of $b_w$ are increasing in the value of the preference parameter $\gamma$, there is a substantial discrepancy between the two. In the economies with $\sigma_x = .165$ the value of $b_w$ is slightly less than 60% as large as $\gamma$. When $\sigma_x = .330$ the estimated values of $b_w$ are roughly two-thirds of the value of $\gamma$. The two key messages from this table are first, that in this setting with non-linear wages, the estimated value of $b_w$ is substantially lower than the underlying preference parameter $\gamma$, and second, that the extent of this discrepancy is significantly affected by the value of $\sigma_x$. As we have noted earlier, to the extent that a reasonable lower bound for this elasticity is in the interval $.40 - .50$, we think that $.50$ is a reasonable lower bound for values of $\gamma$.

### 1.4.4 Distributions of Wealth and Earnings

Chang and Kim (2007) and An et al. (2009) have previously shown that a model with idiosyncratic productivity shocks calibrated to micro data, incomplete markets and indivisible labor captures many quantitative features of the wealth and earnings distribution. It turns out that adding an intensive margin to the analysis has little impact along this dimension. As a result, when $\sigma_x = .165$ our wealth and earnings distributions look very similar to those in An et al. (2009). When we consider the

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10This result is similar to the finding in Rogerson and Wallenius (2009), though their result was based on a slightly different labor supply regression and their calibration was somewhat more stylized.
specification with a much greater degree of heterogeneity, $\sigma_x = .330$, we get similar patterns qualitatively, but the model generates too much dispersion in earnings. In this section we document these properties.

Given that we calibrate our model using employment data from the PSID we think it is preferable to compare our model to data that is also based on the PSID. For this reason our primary source of information on the cross-sectional wealth and earnings measures are based on the 1984 PSID. As a robustness check we also report comparable figures for properties of the wealth distribution from the work of Diaz-Gimenez et al. (1997) that is based on the Survey of Consumer Finances (SCF). For the measures that we focus on the two data sets provide very similar answers, so this does not seem to be a major issue.

Table 1.6 reports the Gini coefficients for both the wealth and earnings distributions in the eight of the different model specifications that we consider, as well as their corresponding values in the PSID and SCF.

A few patterns are evident. First, given a value for $\sigma_x$, the Gini coefficients for both the wealth and earnings distributions are (weakly) increasing in the value of $\gamma$. (Note that the Extensive Only case can be thought of as the limiting case as $\gamma$ goes to zero.) This effect is intuitive; a higher value of $\gamma$ leads to greater intertemporal substitution of labor supply, so that individuals work more when productivity is high and less when productivity is low, thereby amplifying the direct effect of productivity on earnings. Given that individuals accumulate assets to smooth consumption in the face of fluctuations in earnings, greater fluctuations in earnings leads to greater dispersion in assets. Although the qualitative effects are intuitive, the main message from Table 1.6 is that the quantitative importance of these effects are quite small. While moving from $\gamma = 1.0$ to the extensive only case does generate modest changes in both Gini coefficients, the effect of changes in $\gamma$ inside the interval $[.5, 1.5]$ is of second order importance for each measure.

It is also intuitive that higher values of $\sigma_x$ would similarly lead to increases in the Gini coefficients for both wealth and earnings distributions. However, in contrast to

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11 This is not the case if one focuses on the extreme upper tail of the wealth distribution, as it is well known that the PSID undersamples the upper extremes of the wealth distribution. However, as noted before, given our emphasis on labor supply, this extreme upper tail is not of primary concern.
the previous case, changes in $\sigma_x$ for a given value of $\gamma$ do generate first order effects on both measures, with the effect on the earnings Gini being almost twice as large as the effect on the wealth Gini.

Comparing the values in the various model specifications with their counterparts in the data, all of the model specifications seem to capture much of dispersion in the wealth and earnings distributions. If anything, the models generate too much dispersion in earnings, especially in comparison to what is found in the PSID. To look a bit deeper into the nature of the wealth and earnings distributions, Table 1.7 shows the wealth and earnings shares by quintiles of the wealth distribution. Because variation in $\gamma$ turns out to be not very important quantitatively in terms of these distributions, we only report results for the two model specifications with the extreme values of $\sigma_x$ and with $\gamma = 1$.

The basic message from Table 7 is that in addition to doing a reasonable job of accounting for the absolute amount of dispersion in wealth and earnings as captured by the Gini coefficient, the model also does a good job of accounting for the shape of these distributions. The specification with $\sigma_x = .165$ does a very good job of capturing the earnings shares, whereas consistent with Table 2, the specification with $\sigma_x = .330$ generates excessive concentration of earnings in the upper quintile of the wealth distribution. However, the specification with the higher value of $\sigma_x$ is better able to capture the amount of wealth concentrated in the upper quintile. Analyzing the wealth shares by quintiles of the wealth distribution hides the extreme concentration of wealth at the very top of the distribution. It is well known (see, for example, Diaz-Gimenez et al (1997)) that the model is not able to account for the concentration found in say the upper 1% of the wealth distribution. However, from the perspective of labor supply, accounting for the likes of Bill Gates is probably not of first order importance, and so we do not focus on the extreme upper part of the wealth distribution.

In summary, this subsection shows that all of the model specifications generate significant dispersion in earnings and wealth relative to the data. If anything, some of the model specifications generate too much dispersion. The nature of the dispersion is also empirically reasonable, in terms of matching earnings and wealth shares by quintiles of the wealth distribution. We conclude that adding an intensive margin of
labor supply to the previous analyses of Chang and Kim (2006, 2007) does not have first order effects on the earnings and wealth distributions.

1.5 Business Cycles

In this section we study the business cycle properties of our model economies. We solve the equilibrium of the model using the method proposed by Krusell and Smith (1998). In this section we focus on aggregate statistics, and since aggregate data is available at quarterly frequency, we also compute model statistics using quarterly data. As is standard, we take logs and then HP filter (with the smoothing parameter of 1,600) the simulated series before computing statistics.

Table 1.8 reports the properties of output and labor market variables from our models. Aggregate hours worked, employment (extensive margin), hours per worker (intensive margin) and aggregate efficiency units of labor services are denoted as $H$, $E$, $h$, and $L$ respectively. By definition, $H = E \times h$. Because the behavior of consumption and investment is basically the same as in standard real business cycle style exercises we do not report statistics for these variables. When reporting statistics from the US data we report measures based on both the Establishment Survey (ES) and the Household Survey (HS). The statistics for the US economy are taken from Prescott and Cooley (1998).

We start by noting two properties from the data. First, total hours, whether measured by the household or the establishment survey, are almost as volatile as output. Second, fluctuations in total hours are dominated by fluctuations in employment. According to the Establishment Survey, employment is almost three times as volatile as hours per worker, whereas according to the household survey this ratio is roughly two. It is relevant to note that the intensive and extensive margins from the establishment and household surveys are not necessarily comparable. The establishment survey is based on payroll positions, whereas the household survey is based on individuals. To the extent that some individuals hold multiple jobs these surveys will provide different measures of employment and hours volatility. We do not take a stand on which of these is “preferable”; we will simply use as a reference point for the fact that employment is
two to three times as volatile as hours per worker.

Next we turn to the properties of fluctuations in the models. We emphasize that in all of these model economies the driving force behind aggregate fluctuations is identical, i.e., the parameters of the technology shock process are held constant across all specifications. Hence, to the extent that fluctuations are different in the various model economies, it is the result of how the different models lead to different propagation of these shocks. While our focus is on labor market variables, we first note that consistent with many previous exercises, the technology shocks that we feed into our model generate output fluctuations that are between two-thirds and three-quarters of observed fluctuations in output. We will say more about the nature of these differences below, when we examine the nature of labor market fluctuations in more detail.

Rather than systematically walking the reader through the results in Table 1.8, we think it is useful to organize our discussion around several messages that we want the reader to take away from the table. The first message concerns the determinants of fluctuations along the intensive and extensive margins. In particular, we start from the notions, implicit in much of the recent literature, that fluctuations along the intensive margin are determined mostly (if not exclusively) by the value of the preference parameter \( \gamma \), whereas fluctuations along the extensive margin are determined mostly (if not exclusively) by the extent of heterogeneity, which in our model is reflected in the value of \( \sigma_x \). Intuitively, fluctuations along the intensive margin are increasing in the value of \( \gamma \), while fluctuations along the extensive margin are decreasing in the value of \( \sigma_x \). Additionally, intuition based on simple models would suggest that fluctuations along the intensive margin are proportional to the value of \( \gamma \).

While the results in Table 1.8 provide partial support for both of these notions, they also reveal that these notions reflect an oversimplification that is potentially very misleading from a quantitative perspective. For example, consistent with the first notion above, we see that for a given value of \( \sigma_x \), increases in \( \gamma \) lead to greater fluctuations along the intensive margin. However, the notion that \( \gamma \) is the dominant, let alone the exclusive factor that determines this response is strongly contradicted by the results in Table 8. Specifically, starting from the specification in which \( \gamma = 0.5 \) and \( \sigma_x = .165 \), we see that increasing \( \gamma \) to 1.5 leads to almost a tripling of the response along the intensive
margin. However, this same effect occurs if we keep the value of $\gamma$ fixed at 0.50 but instead increase the value of $\sigma_x$ to .330. To the best of our knowledge, we are the first to point out that the aggregate fluctuations along the intensive margin are affected in a quantitatively critical way by the amount of heterogeneity in the economy.

Similarly, starting from the specification $\gamma = .5$ and $\sigma_x = .165$, we note that moving to the specification with $\sigma_x = .33$ leads to roughly a 50% reduction in fluctuations along the extensive margin. However, if we instead kept the value of $\sigma_x$ unchanged at .165 and instead increased the value of $\gamma$ to 1.5, we would still have a decrease in fluctuations along the extensive margin of almost 20%.

The key message that emerges from this exercise is that fluctuations along the intensive and extensive margin are jointly determined in a quantitatively important fashion by both the value of the preference parameter $\gamma$ and the extent of heterogeneity, as captured here by the value of $\sigma_x$. It follows that one should not think of “intensive margin elasticities” and “extensive margin elasticities” as two independent parameters.

A second and related message is that pure indivisible labor models are a poor guide for assessing the impact of heterogeneity on fluctuations in aggregate hours. Table 1.8 shows that the drop in the volatility of aggregate hours in the extensive only economies as $\sigma_x$ increases is much greater than in the economies with $\gamma = 0.5$.

The third message concerns the implications of $\gamma$ and $\sigma_x$ for fluctuations in aggregate hours. We begin by looking at how the results vary as we change $\gamma$ holding the value of $\sigma_x$ fixed at .165. Keeping in mind that the aggregate shocks are constant across all specifications, and that each specification matches both the average employment rate and the average level of hours per worker, the first striking result is the huge variation in the volatility of aggregate hours across these four specifications, ranging from .67 to .93. However, despite these large differences in the volatility of aggregate hours, Table 8 also shows that differences in the volatility of labor services are roughly an order of magnitude smaller, ranging from .85 to .89. Because of this, differences in output fluctuations are also relatively minor. Before we explain the reason for these results, we note that a key message is that the split of aggregate fluctuations into intensive and extensive margins has first order implications for fluctuations in total hours but not necessarily for output.
To understand the patterns just discussed, it is useful to start with the observation that as we increase $\gamma$ holding $\sigma_x$ constant, one would intuitively expect an increase of the elasticity of aggregate labor supply and therefore an increase in the volatility of hours. This intuition does show up in the results if we apply it to labor services, i.e., the volatility of labor services increases as we increase $\gamma$. This raises the question of why hours do not exhibit the same behavior? The key to understanding this discrepancy lies in understanding the role of heterogeneity in individual productivity and how this matters for aggregate fluctuations. To answer this question, we start with the following exercise. Consider the response of the economy with $\gamma = 1.5$ and $\sigma_x = .165$ to a positive aggregate shock, and consider how this response changes as we decrease $\gamma$. Intuitively, decreasing $\gamma$ leads to smaller increases along the intensive margin. One piece of intuition for the results comes from asking what would happen to fluctuations in aggregate hours of work if we were to compensate for this decrease in fluctuations along the intensive margin with sufficiently greater fluctuations along the extensive margin so as to keep the total increase in labor services constant. To the extent that the employed are on average of higher productivity than the non-employed, when we substitute increases along the extensive margin for increases along the intensive margin, the hours are on average less productive. It follows that it would take a larger increase in hours along the extensive margin in order to perfectly compensate for the smaller increase in hours per worker. In fact, intuition suggests that the overall increase in labor services should be lower as we decrease $\gamma$. To see why, note that if the change in labor services is unchanged, then so is the change in the wage. But in order to get an additional increase along the extensive margin when we lower $\gamma$ will require a larger increase in the wage rate, i.e., a smaller increase in labor services.

The net prediction is that the increase in labor services should decrease as we decrease $\gamma$, while the change in hours is ambiguous since there are two opposing effects. However, to the extent that the change in the response of labor services is not too large, we would expect that the net effect is for the change in hours to increase. To pursue this a bit further, the extent to which labor services increase by a smaller amount is dictated by the density of individuals who are willing to move into employment in response to small increases in the wage. The more compressed is the distribution of
individual heterogeneity, the more likely it is that a large mass of individuals are willing
to enter employment in response to small increases in the wage.

With this intuition in mind, we next turn to the results for the case of $\sigma_x = .330$. While the basic economics are the same for the different values of $\sigma_x$, the higher value of $\sigma_x$ is associated with a less compressed distribution of (the exogenous component of) individual heterogeneity. Based on the above intuition, we would expect this to show up as a more pronounced effect of $\gamma$ on the magnitude of fluctuations in labor services. Indeed, we observe this in Table 1.8, with the range now being $.46 - .61$. And, given the larger effects on fluctuations in labor services, we also see larger differences in hours.

The other pattern that is evident is the decrease in fluctuations in both aggregate hours and labor services as we increase the value of $\sigma_x$ for a given value of $\gamma$. This is consistent with the intuition expressed earlier—the greater dispersion in individual heterogeneity leads to lower responses along the extensive margin. Although this results in greater fluctuations along the intensive margin, the curvature in the disutility of work function implies that this effect only partially offsets the lower fluctuations on the extensive margin.

There are some additional interesting patterns across the specifications. For example, fixing $\sigma_x = .165$, we see that changes in the value of $\gamma$ within the interval $[.5, 1.5]$ has virtually no impact on the magnitude of fluctuations in aggregate hours, though going to the extreme case of no adjustment along the intensive margin leads to a very large increase in the standard deviation of aggregate hours. That is, for these specifications, a tripling of the willingness of individuals to substitute work effort over time, in the form of an increase in $\gamma$ from $.5$ to $1.5$, has virtually no impact on the actual fluctuations in aggregate hours of work. However, Table 1.8 shows that this result breaks down if we consider a specification with substantially more heterogeneity. Specifically, in the specifications with $\sigma_x = .330$ we see that increasing $\gamma$ from $.50$ to $1.5$ results in roughly a twenty percent increase in the size of fluctuations in aggregate hours.

Lastly, we assess the extent to which the various specifications are able to account for the fact that over the business cycle, employment fluctuates between two and three
times as much as hours per worker. With the lower degree of heterogeneity, i.e., \( \sigma_x = 0.165 \), achieving this ratio would require a value of \( \gamma \) of at least 1.5, since this implies that employment is three times as volatile as hours per worker and this ratio is decreasing in \( \gamma \). When \( \sigma_x = 0.165 \) and \( \gamma = 0.50 \), this ratio exceeds ten. With the higher degree of heterogeneity, this range is achieved for \( \gamma \) somewhat less than 0.50, since when \( \gamma = 0.50 \) and \( \sigma_x = 0.330 \), employment fluctuations are a bit less than twice those of hours per worker.

### 1.6 Taxes

In this section we consider the effect of steady state changes in fiscal policy in the various specifications. Given our focus on labor supply responses, we choose to focus on a single fiscal policy for which labor supply responses are known to be of first order importance: a proportional tax on labor earnings used to finance a lump-sum transfer. Specifically, in each of the models we solve for the effect on steady state of instituting a 10% proportional tax on labor earnings used to finance a lump-sum transfer (\( \tau = 0.1 \)).

Recalling that in all of our economies the initial steady state equilibrium has the same values for both the employment to population ratio and the average hours of work per employed worker. Table 1.9 shows the effects on various labor market outcomes.

There are two patterns worth noting. One concerns the effects of increases in \( \sigma_x \) for a fixed value of \( \gamma \), while the second concerns the effects of increases in \( \gamma \) for a fixed value of \( \sigma_x \). Before presenting these patterns it is useful to articulate two key mechanisms at work in steady state equilibrium that lay behind optimal individual labor supply decisions.

One key mechanism at work at the individual level in the steady state equilibrium of this economy is intertemporal substitution of labor supply. With complete markets and no uncertainty, individuals would arrange their labor supply so that they work only when productivity is above some threshold, and hours when working would be increasing in productivity above this threshold. Both the threshold for working versus not working and the slope of the hours profile reflect intertemporal substitution. Intuitively, by having a steeper slope to the profile of hours versus productivity the
individual would generate more income while working and could therefore increase the productivity threshold for working. The intertemporal trade off between changes in hours of work while working and the fraction of periods in employment is influenced both by the preference parameter $\gamma$ and the mass of states that are near the reservation productivity level.

However, the presence of uncertainty and incomplete markets adds another aspect to the analysis. Specifically, because the individual cannot perfectly insure against idiosyncratic shocks, optimal choices will entail an individual working in some low productivity states as a way to help smooth consumption. In these states, the individual’s labor supply problem is effectively a static problem, and in a static setting we know that a tax and transfer system decreases the incentives to work. The lower the level of productivity, the greater is the income effect of the transfer payment, making it more likely that low productivity individuals choose to not work. Intuitively, it turns out that this phenomenon is more present in the economies that feature greater heterogeneity, since greater an increase in $\sigma_x$ implies greater uncertainty, thereby making it harder for individuals to self-insure.

With these mechanisms in mind, we turn to the patterns in Table 1.9. Note first that increasing the value of $\sigma_x$ for a fixed value of $\gamma$ leads to a greater response along both the extensive and intensive margins, and therefore a greater response in aggregate hours. This pattern is in contrast to our findings in the business cycle context and at first pass may seem counterintuitive. Specifically, in the previous section we argued that an increase in heterogeneity led to less mass in the neighborhood of the reservation wage curve and therefore a smaller response along the extensive margin. The second mechanism noted above—the presence of low productivity workers with positive hours—is key to understanding the employment response. More specifically, the greater the dispersion in idiosyncratic shocks creates a larger mass of individuals who are working despite having low productivity, thereby creating a large negative response in employment in response to an increase in the income transfer.\footnote{The reader may wonder why this mechanism was not evident in the business cycle analysis. Key to understanding the difference is that the two driving forces are different. As noted above, these individuals effectively solve a static problem. In a static model with balanced growth preferences, an increase in the wage has no effect on labor supply, whereas an increase in the size of a tax and transfer program has a negative effect on labor supply.}
Why is there also a greater decrease along the intensive margin as we increase the value of $\sigma_x$? This response is basically what one would predict from intertemporal substitution considerations. A tax and transfer system basically has the government carrying out part of the intertemporal substitution that the individual wants to carry out, in the sense that the net transfer from the government is relatively high when individual productivity is low. As a result the individual will do less intertemporal substitution. This can happen along the extensive margin or the intensive margin. With more heterogeneity there is less scope for intertemporal substitution along the extensive margin, similar to what we found in the analysis of business cycles. As a result, we get a greater response along the intensive margin as we increase the value of $\sigma_x$.

The second pattern concerns the effect of an increase in the value of $\gamma$ for a given value of $\sigma_x$. Rogerson and Wallenius (2009) effectively considered an experiment of this sort, and found that the first order effect was to increase the response along the intensive margin, decrease the response along the extensive margin, but to leave the change in aggregate hours relatively unchanged. Rogerson and Wallenius carried out their exercise in a setting with a relatively small degree of heterogeneity and no uncertainty. The results in Table 1.9 show that these results do not necessarily carry over to settings with higher degrees of heterogeneity.

In particular, when $\sigma_x = .165$ our results exactly mirror those found in Rogerson and Wallenius, in that we find that the change in aggregate hours is almost independent of the value of $\gamma$. Interestingly, however, for higher values of $\sigma_x$ the aggregate response is not independent of $\gamma$ as $\gamma$ increases from .5 to 1.5. To the extent that increases in $\gamma$ represent a “more elastic” labor supply, one would intuitively expect that increasing $\gamma$ would lead to a greater response in aggregate hours, which is what happens as we increase $\gamma$ from .5 to 1.5. Additionally, the size of this effect is increasing in the value of $\sigma_x$. These effects can be understood through the by the manner in which $\sigma_x$ influences the relative efficacy of intertemporal substitution along the intensive and extensive

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13 Note that we found a similar result for the case of business cycle fluctuations. Interestingly, for higher degrees of heterogeneity, the business cycle and steady state tax effects are of opposite signs, with increases in $\sigma_x$ leading to smaller fluctuations in hours in the business cycle setting but greater changes in the steady state tax and transfer setting.
margins. To see why it is useful to begin with an analysis of the $\sigma_x = .165$ case. When $\sigma_x = .165$, we see this tradeoff at work as we move to higher values of $\gamma$, in the sense that the greater decrease along the intensive margin is accompanied by a smaller decrease on the extensive margin. Moreover, the two effects are roughly offsetting. As we consider higher values of $\sigma_x$ we see the same pattern for the intensive margin, in that the responsiveness of hours per worker increases as we increase $\gamma$. However, what differs as we consider higher values of $\sigma_x$ is that the opposing effect on employment becomes much smaller. The reason that the employment response becomes smaller is precisely due to the effects that we have emphasized earlier: with a higher value of $\sigma_x$ there is less mass of “states” near the reservation curve and as a result there is less scope for intertemporal substitution along the extensive margin. In fact, when $\sigma_x = .33$, we see that the response along the extensive margin is effectively the same for all three values of $\gamma$. One additional factor that influences the statistics in this case is composition effects on average hours associated with changes on the extensive margin. Specifically, since individuals who are on the margin of working versus not-working for purposes of intertemporal substitution are working fewer hours, when these individuals drop out there is an added increase in hours per worker for employed individuals. This further explains why the drop along the intensive margin increases as the employment drop decreases.

We note two additional points from Table 1.9. First, the employment response in the extensive only model is quite similar to the employment response in the $\gamma = .50$ economies. Second, it is striking that the variation in the response of aggregate labor services is much smaller than the variation in aggregate hours.

In summary, we again find that in general responses along both the intensive and extensive margin are influenced in a non-trivial way by both the preference parameter $\gamma$ and the extent of heterogeneity as captured by the parameter $\sigma_x$. For relatively large values of $\sigma_x$ we did find that the response along the extensive margin was basically unaffected by the value of $\gamma$. However, changes along the intensive margin are affected by both parameters.
1.7 Conclusion

Recent advances in modeling aggregate labor supply have emphasized the importance of accounting for adjustment along the intensive and extensive margins. Adjustment along the extensive margin has also been shown to depend on the extent of heterogeneity. In this paper we build a model in which individual labor supply adjusts along both the intensive and extensive margins in an environment that features heterogeneity and incomplete markets. We believe that this is the appropriate benchmark model for understanding the joint determination of adjustment along the two margins. We consider a family of specifications of this model that differ along two key dimensions: the value of the preference parameter that influences curvature of utility in hours of work, and the standard deviation of innovations in the idiosyncratic shock process, which in turn influences the extent of heterogeneity in the invariant distribution for idiosyncratic shocks.

We consider the ability of the various specifications of the model to account for key features of employment and hours worked in the cross-section. We then use this model to consider labor supply responses to temporary shocks and permanent tax changes, along both the intensive and extensive margins. Three key findings emerge. First, extensive and intensive margin elasticities are jointly determined by both the preference parameter and the extent of heterogeneity. That is, one cannot speak of intensive and extensive margin elasticities as independent parameters of the economic environment. Second, the effect of increased heterogeneity on extensive margin elasticities is of opposite sign in the two contexts: in the business cycle setting increased heterogeneity leads to less adjustment along the extensive margin, whereas in the case of permanent changes in the size of the tax and transfer system we find that increased heterogeneity leads to more adjustment along the extensive margin. Third, in terms of fluctuations in aggregate hours, we find that abstracting from the intensive margin can be very misleading about the effect of heterogeneity. Fourth, although the various specifications that we consider generate very large differences in responses in aggregate hours, the associated responses in aggregate labor services is considerably smaller.
Table 1.1: Calibrated Parameter Values

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<th><strong>A. Values Held Constant Across Specifications</strong></th>
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<th><strong>B. Values That Vary Across Specifications</strong></th>
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Notes: 'Ext' denotes the model specification with the extensive margin only.

Table 1.2: Standard Deviation of Annual Hours Conditional on Working

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<th><strong>σ_x = .165</strong></th>
<th><strong>σ_x = .2475</strong></th>
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<td><strong>Models</strong></td>
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Notes: Annual hours in the CPS and PSID are normalized so that average annual hours is the same as in the economy with γ = 1 and σ_x = .165.
### Table 1.3: Distribution of Employment Transitions

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<th>Shares in Model</th>
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*Notes:* Employment status at time $t$ ($E_t$), is denoted by 1 (working) or 0 (not working).
Table 1.4: Transition Probability of Annual Hours

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<th>2nd</th>
<th>3rd</th>
<th>4th</th>
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<td>45.77</td>
<td>18.27</td>
<td>11.15</td>
<td>4.63</td>
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<tr>
<td>( t )</td>
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Table 1.5: Estimated Values of $b_w$ based on the labor supply regression

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Table 1.6: Gini Coefficients for Wealth and Earnings Distributions

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Table 1.7: Wealth and Earnings Shares by Wealth Quintiles

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Table 1.8: Business Cycle Fluctuations

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*Note:* Total hours ($H$) = Employment ($E$) $\times$ Hours per worker ($h$). The variable $L$ denotes labor hours in efficiency units. “ES” and “HS” denotes the data based on the Establishment Survey and Household Survey, respectively.
Table 1.9: Effects of Taxes on Steady State

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*Note: Total hours ($H$) = Employment ($E$) × Hours per worker ($h$). The variable $L$ denotes labor hours in efficiency units.*
Notes: Annual hours in the PSID are normalized so that average annual hours is the same as in the economy with $\sigma_x = .165$. 

Figure 1.1: Distribution of Hours Worked
Chapter 2

Interpreting Labor Supply
Regressions in a Model of Full-
and Part-Time Work

2.1 Introduction

The canonical model of life cycle labor supply (see, e.g., MaCurdy (1981)) assumes that at each age individuals can vary their hours continuously and face a wage per unit of time that is independent of hours worked. In this setting, one can in principle use the response of hours worked to (exogenous) changes in wages to estimate key preference parameters such as the intertemporal elasticity of substitution. At the other extreme, many aggregate models (e.g., Hansen (1985) and Rogerson (1988)) assume that workers face a binary choice with regard to working: they can either work a fixed amount that corresponds to full time work, or they can not work. In this setting, individual panel data on wages and hours of work provides no information about the individual preference parameter that dictates intertemporal elasticity of substitution.

While both of these two extremes capture elements of reality, it seems reasonable to think that reality probably lies in between the two extreme cases. In this paper we consider the empirically appealing intermediate case in which individuals who decide to
participate in the labor market can choose between full-time work and part-time work. We allow for the possibility that part-time work carries a wage penalty relative to full-time work. The question we ask is to what extent standard labor supply methods based on continuous choice and linear earnings schedules will uncover the true individual preference parameter. As this model allows for some choice along the intensive margin, it is not clear a priori what to expect. Because the prevalence of part-time work varies significantly across males and females, we consider estimates for both male and female labor supply, since the results may be sensitive to the use of part-time work. We therefore model households as consisting of two members and consider the joint labor supply problem of the household in a dynamic setting. The model is essentially the family labor supply model of Chang and Kim (2006), extended to allow for part-time work. In the spirit of Bewley (1983), Huggett (1993) and Aiyagari (1994), individuals are subject to uninsurable idiosyncratic shocks to the return to market work, and households use labor supply and asset accumulation to respond to these shocks. We calibrate our model so as to match the movements of workers between full-time work, part-time work and non-employment.

Several interesting findings emerge. First, in contrast to what one would find in the case in which full time work and no work are the only options, we find that standard labor supply regressions on individual panel data do generate non-zero estimates for labor supply elasticities at the individual level. However, we find that the estimated values are virtually unrelated to the underlying preference parameter that the methods are supposed to uncover. Additionally, we find that the estimates are higher for women than for men, even if the underlying preference parameters are identical. We conclude that standard methods do not correctly identify individual preference parameters in our intermediate model. While we focus on one specific economy, our results point to the importance of properly modeling the choices of hours and earnings that individuals actually face and using methods that can uncover the underlying preference parameters given the nature of these choice sets.

One may ask whether it is of interest to try to estimate the individual preference parameter. Related models of labor supply include Cho and Rogerson (1988), Floden and Linde (2001), Pijoan-Mas (2005), Chang and Kim (2007), An et al. (2009) and Alonso-Ortiz and Rogerson (2010).
parameter that determines curvature in the utility from leisure function in a model that features discrete choice. As argued in both Rogerson (2010) and Chetty et al (2010), if the discrete choices are dictated by the need to coordinate, then the values of these choices will respond to changes in the aggregate economic environment, and in this case the curvature parameter will play a key role.

Our paper is related to several papers in the literature that consider that discuss how the issue of estimating individual preference parameters if workers are not free to choose hours, or more generally can choose among hours and wage bundles. Biddle (1988) argues for using standard labor supply methods but on a sample that is restricted to self-employed workers that are free to adjust hours. In addition to possible selection biases, a shortcoming of this sample is that self-employed workers are likely to face stochastic employment opportunities as opposed to being able to choose hours at a fixed wage. Altonji and Paxson (1988, 1992) consider the case in which workers face a continuous hours choice but face a wage that is contingent on hours. French (2005) and Rogerson and Wallenius (2009) consider a similar setting. Chetty et al (2010) consider a model in which each firm offers a fixed workweek and search frictions prevent workers from being able to instantaneously adjust hours of work by switching to another firm. Our paper is the first to explore the performance of standard labor supply methods in a context that features multiple but discrete choices for hours of work and tied hours-wage bundles.

2.2 The Model

There is a continuum of measure one of ex-ante identical households. Each household consists of one male and one female member. We assume a unitary household that has

\[^2\] See, for example, Kahn and Lang (1991), Bell and Freeman (2001) and Sousa-Poza and Henneberg (2003) for evidence that workers are not always able to work their desired number of hours. On the issue of tied hours and wage bundles, see for example, Moffitt (1984), Keane and Wolpin (1991) and Aaronson and French (2004).

\[^3\] See also the papers by Lundberg (1985), Biddle and Zarkin (1989), and Dickens and Lundberg (1993), and the discussion in Rogerson (2010).
preferences over streams of (household) consumption and individual leisure given by:

$$\max_{\{c_t,h_{mt},h_{ft}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ 2 \ln(0.5c_t) + B_m \frac{(1 - h_{mt})^{1-1/\gamma}}{1 - 1/\gamma} + B_f \frac{(1 - h_{ft})^{1-1/\gamma}}{1 - 1/\gamma} \right\}$$  \hspace{1cm} (2.1)

where $c_t$ is household consumption, $h_{mt}$ is hours of market work by the male member and $h_{ft}$ is hours of work by the female member. There are only three possible choices for hours of market work by an individual: full time work, denoted by $h^F$, part time work, denoted by $h^P$, or zero. Individuals are subject to idiosyncratic shocks that affect their productivity in market work. Letting $x_{it}$, $i = m, f$ denote the idiosyncratic productivity for male and female household members respectively, we assume:

$$\log x'_{it+1} = (1 - \rho_i) \log \bar{x}_i + \rho_i \log x_{it} + \epsilon_{it}, \hspace{1cm} \epsilon_{it} \sim N(0, \sigma_i^2)$$  \hspace{1cm} (2.2)

The innovations for these markov processes are i.i.d. across individuals, households and time.

We assume that there is a penalty associated with part-time work. Additionally, to capture the fact that men and women have differential labor supply across occupations and that this penalty may differ across occupations, we allow the penalty to be gender specific. In particular, if $w_t$ is the wage rate per unit of labor services in period $t$, and an individual of gender $i$ has idiosyncratic productivity $x_{it}$, then he or she will receive labor earnings of $x_{it}w_t h^F$ if working full time and $(1 - \lambda_i)x_{it}w_t h^P$ if working part time. It follows that the wage rate per hour of market work that an individual of gender $i$ faces is a function of both his or her own idiosyncratic productivity shock and choice of work hours. We denote this by $\bar{w}_{it}(x_{it}, h_{it})$. There are no markets for insurance against idiosyncratic productivity shocks, and the only asset available to households is claims to physical capital. Letting $a_t$ denote the capital that the household carries into period $t$, and $r_t$ the rate of return on capital net of depreciation, the one period budget constraint faced by the household is given by:

$$a_{t+1} = \bar{w}_{mt}(x_{mt}, h_{mt})h_{mt} + \bar{w}_{ft}(x_{ft}, h_{ft})h_{ft} + (1 + r_t)a_t - c_t,$$
We assume that capital holdings cannot go below zero, i.e., $a_t \geq 0$ for all $t$.

A representative firm that produces output according to a constant-returns-to-scale Cobb-Douglas technology in capital ($K_t$) and efficiency units of labor ($L_t$): $Y_t = K_t^{\alpha}L_t^{1-\alpha}$. Capital depreciates at rate $\delta$.

We focus on a steady state equilibrium in which the aggregates $K_t$ and $L_t$ as well as the two prices $r_t$ and $w_t$ are constant. The household’s optimization problem can be conveniently re-formulated in the recursive form. The individual state variable for a household will be the triplet $(a; x_m; x_f)$. Let $V_{jk}(a; x_m; x_f)$ denote the value to a household when the male member’s labor market state is $j$ and the female member’s is $k$. Due to indivisibility of labor as assumed above, $j, k \in \{F, P, N\}$ where $F$ stands for full-time work, $P$ part-time work and $N$ not-working. Hence a household has nine types of value functions associated with its two members’ labor market states. For example, the value to a household in which both male and female members work full time is defined by the following Bellman equation:

$$V_{FF}(a; x_m; x_f) = \max_{a' \in A} \left\{ u(c, h^F, h^F) + \beta E\left[V(a', x'_m, x'_f| x_m, x_f)\right] \right\}$$

subject to

$$c = w^F_m x^F_m h^F + w^F_f x^F_f h^F + (1 + \tau)a - a', \text{ and } a' \geq 0.$$ 

Other value functions, $V_{FP}, V_{FN}, V_{PF}, V_{PP}, V_{PN}, V_{NF}, V_{NP}, V_{NN}$, are be defined in a similar way. Dropping the arguments of the value functions for notational convenience, a household’s joint decision for labor supply can be characterized as:

$$V(a; x_m; x_f; \mu) = \max \left\{ V_{FF}, V_{FP}, V_{FN}, V_{PF}, V_{PP}, V_{PN}, V_{NF}, V_{NP}, V_{NN} \right\}.$$  

(2.4)

### 2.3 Quantitative Analysis

#### 2.3.1 Calibration

We set a time period equal to one year. As shown in Chang and Kim (2006), some interesting implications arise when allowing for a shorter time period and then time
aggregating to generate panel data at annual frequency. Because we want to focus on different issues, we set a time period equal to one year and so abstract from these time aggregation effects.

Our main goal is to assess how the relationship between the preference parameters $\gamma_m$ and $\gamma_f$ and the estimates from standard labor supply regressions is affected by the presence of restricted hours choices. For this reason we will consider five different economies that differ in their values for these two labor preference parameters. For simplicity we will consider the symmetric case in which the two parameters are the same, and denote the common value by $\gamma$. The five economies that we consider correspond to $\gamma = .2, .4, .6, .8$, and 1.0. Given these assumed values for $\gamma$, the rest of the model’s parameters will be calibrated as follows.

There are many papers in the literature that estimate idiosyncratic wage and/or earnings shocks, though almost exclusively for prime aged males that are employed full time. The consensus from this literature is that wage shocks are very persistent and large. For our benchmark analysis we again impose symmetry and use the same values for both male and female workers: $\rho_i = .92$ and $\sigma_i = .21$ for $i = m, f$. This particular choice corresponds to the estimates of Floden and Linde (2001) for prime aged males. The value of $\bar{x}_m$ is set to zero as a normalization, while $\bar{x}_f$ is set so that the unconditional average productivity for females is 35% less than that of males.

The labor share, $\alpha$, is set to .64, and the annual depreciation rate, $\delta$, is set to .08. The discount factor $\beta$ is chosen so that in steady state we have $r = .05$. At work, individuals supply $h = 0.4$ and $h = 0.2$, as a full-time and part-time worker, respectively.

This leaves four parameters: the two utility parameters $B_m$ and $B_f$, and the two part-time wage penalty parameters, $\lambda_m$ and $\lambda_f$. For each value of $\gamma$ we choose the values of these parameters so as to match four employment targets: the fraction of the male

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5We incorporate this feature so as to generate a gender wage gap in equilibrium, though this feature plays no role in our results. Because the preference parameters $B_i$ are allowed to be gender specific, the value of $B_f$ plays a similar role as $\bar{x}_f$ in terms of influencing labor supply. Moreover, the model has some difficulties in matching the data along this dimension. For example, in the equilibrium with $\gamma = .40$, this results in a 25% gender wage gap. However, for full-time workers the wage gap is only 7.7%, reflecting the presence of strong selection effects in labor-market participation for females.
and female population that are employed full time and part time. Based on data from the PSID for the years 1968-2002 for married couples with the age of the household head between 25 and 55, the target values for the fraction of individuals employed full time is 0.891 and 0.402 for males and females respectively, while the corresponding values for part time employment are 0.057 and 0.226.

Table 2.1 reports the implied values for the part-time penalties.

As the table indicates, the required penalty is decreasing in $\gamma$, and ranges from zero to as much as 50%. The fact that the penalty is decreasing in $\gamma$ is intuitive. We are holding the productivity process constant and seeking to match the same behavior over time in terms of fraction of time spent in full-time and part-time work. As $\gamma$ is decreased, individuals are less willing to intertemporally substitute leisure, implying that they are less willing to move between part-time and full-time in response to a given change in compensation. Increasing the size of the part-time penalty implicitly leads to a greater variation in effective wages, thereby increasing the willingness of individuals to intertemporally substitute leisure by moving between part-time and full-time work. Several issues make it difficult to measure this penalty in the data. However, there is a small literature that provides some estimates this penalty, including, for example, the early contribution of Moffitt (1984) and more recent papers by Keane and Wolpin (2001) and Aaronson and French (2004). Based on these papers it seems that a reasonable upper bound for the size of the penalty is around 20%. From this perspective it seems that all of our specifications with the exception of $\gamma = 0.2$ are within the reasonable range. The fact that the $\gamma = 0.2$ specification requires what appears to be an excessively large part-time penalty suggests that this specification is not reasonable, but we will continue to consider it below.

The model is calibrated so as to exactly match the distribution of both males and females across the three different levels of work. However, the model also does a good job of capturing the joint distribution of male and female employment status across households. In the interest of space we only present results for the $\gamma = 0.4$, but as shown in Table 2.2, the model does a good job of capturing the distribution of household across

---

6These values reflect the following mapping between annual hours of work and employment status: annual hours greater than 1800 corresponds to full time work, annual hours between 400 and 1800 corresponds to part time work and annual hours less than 400 corresponds to zero work.
the nine different labor supply configurations.

Note that because all households in our model are ex ante identical, the steady state distribution of households across types is also the distribution of a given household over states over time. In particular, since females are much more evenly distributed across the three employment states than men, it follows that females in our model have much more variable labor supply over time. It is interesting to note that we obtain this result despite the fact that we impose that $\gamma_f = \gamma_m$ and $\sigma_m = \sigma_f$. That is, although the standard deviation of the underlying shocks and the willingness to substitute intertemporally is the same for both males and females, females nonetheless display more fluctuations in labor supply over time. The key feature that generates this difference in volatility is the difference in mean productivity levels in conjunction with the family structure. Because women have lower productivity on average, their labor supply decision plays a greater role in responding to shocks.

Moreover, as shown in Chang et al (2010), the model also does a reasonable job of capturing the transitions of males and females across the three different employment states.

In summary, our model does a reasonable job of matching both the distribution of males and females across the three employment states as well as the transitions among the three states.

2.3.2 Results

Our main goal is to assess how standard methods for estimating labor supply elasticities assuming a continuous choice of working hours and a linear earnings schedule (i.e., earnings are linear in time devoted to work at a particular point in time) fare in our setting. Assuming a continuous choice and a linear earnings schedule, the period utility function implies that the following condition for male and female labor supply must hold in any period in which hours are positive:

$$\log(1 - h_{it}) = A - \gamma \log w_{it} + \gamma \log c_t$$

\footnote{The part-time wage penalty is also different across men and women, but this plays a less important role in accounting for the differential volatility.}
where \( A \) is a constant and \( c_t \) is household consumption in period \( t \).

To begin our analysis we use the steady state equilibrium decision rules to generate a panel data set consisting of 5000 households with 100 years of observations and then run the above regression using all observations with positive hours.\(^8\) Later on we will consider how various selection criterion influence the estimates. The results are in Table 2.3. In running this regression we impose that the coefficients on \( \log w_{it} \) and \( \log c_t \) are equal in absolute value and of opposite sign, though the estimates are effectively unchanged even if we do not impose this condition.

Several interesting results emerge. First, whereas in a pure indivisible labor model in which full time work is the sole option for working, one would necessarily find a zero estimate for the labor supply elasticity parameter (given that we abstract from time aggregation effects), here we obtain positive values for both male and females in all cases. Second, the estimated preference parameter is always larger for women than for men. Third, the estimated values \( \hat{\gamma}_i \) bear essentially no relationship to the true preference parameters. For men we find that the estimated value of \( \hat{\gamma}_m \) is effectively equal to .10 independently of the true value of \( \gamma \). A similar result arises for the estimated preference parameter for women. Here we find that when \( \gamma = .20 \) the estimated value \( \hat{\gamma}_f \) is substantially lower than for the other cases, but for \( \gamma \) between .40 and 1.00 we see that there is relatively little change in \( \hat{\gamma}_f \) despite a more than doubling of \( \gamma \).

Next we show how the estimates are affected by various sample selection criterion. In the literature on male labor supply it is common to impose sample selection criterion that eliminate men who have an observation for hours that falls below some threshold, thereby biasing the sample toward men who are employed full time. As a way to illustrate the effect of this type of sample selection criterion we carry out the following exercise. For each individual in our sample, we construct an index of work over the 100 year period, with each year of full time work contributing 1 and each year of half time work contributing .5. We then run the same regression as above but for two different samples. The first sample consists of highly attached workers, those with an index of at least 95. The second sample consists of a sample of individuals with low attachment.

\(^8\)For the results we report we imposed that the coefficients on log wages and log consumption are the same, though the results are effectively the same if this is not imposed.
those with an index less than 50. The results are shown in Table 2.4.

Many of the same patterns appear. Specifically, the regression coefficient is higher for females than males, and is largely unaffected by the underlying value of \( \gamma \). The new result that appears is that one obtains a much higher estimate from the less attached group than for the highly attached group. We have also run the same exercise with a sample panel that lasts for ten years, and obtained similar results.

The above regressions have used the log of leisure as the left hand side variable, since this is consistent with our specification of preferences. We have also run the same regressions using log of hours worked as the left hand side variable. The results are effectively identical, though as expected the estimated values of \( \gamma_i \) are now larger since the labor supply elasticity of labor supply for hours worked is greater than the labor supply elasticity for leisure. Specifically, based on the 100 year sample, the estimates are roughly xx and xx for men and women respectively.

2.4 Conclusion

We built a model of family labor supply in which individuals choose between full-time work, part-time work and non-employment. The model is calibrated so as to replicate the movements of both male and female workers among these states. Although the individual labor supply problem is a discrete choice problem, individuals are able to adjust hours along the intensive margin by moving between part-time and full-time work. Intuitively, adjustment along the intensive margin allows one to estimate the true value of the underlying curvature parameter describing the utility from leisure. We explore the extent to which standard labor supply methods can achieve this in our setting. Although these methods deliver precise estimates that are significantly different from zero, the estimates are effectively unrelated to the true underlying values. These methods also deliver elasticity estimates for women even when the underlying preference parameters are the same for men and women.
Table 2.1: Part-time Wage Penalties

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>$\lambda_m$</th>
<th>$\lambda_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>.2</td>
<td>.503</td>
<td>.482</td>
</tr>
<tr>
<td>.4</td>
<td>.222</td>
<td>.196</td>
</tr>
<tr>
<td>.6</td>
<td>.120</td>
<td>.009</td>
</tr>
<tr>
<td>.8</td>
<td>.064</td>
<td>.035</td>
</tr>
<tr>
<td>1.0</td>
<td>.032</td>
<td>.005</td>
</tr>
</tbody>
</table>
Table 2.2: Household Decomposition by Labor Supply

<table>
<thead>
<tr>
<th></th>
<th>PSID</th>
<th>Model ($\gamma = .4$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Female</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Full-Time</td>
<td>Part Time</td>
</tr>
<tr>
<td>Female</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Full-Time</td>
<td>35.98</td>
<td>19.94</td>
</tr>
<tr>
<td>Male</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Part-Time</td>
<td>2.59</td>
<td>1.61</td>
</tr>
<tr>
<td>Not-working</td>
<td>1.84</td>
<td>0.80</td>
</tr>
</tbody>
</table>
Table 2.3: Estimates of $\gamma$

<table>
<thead>
<tr>
<th></th>
<th>$\gamma = .2$</th>
<th>$\gamma = .4$</th>
<th>$\gamma = .6$</th>
<th>$\gamma = .8$</th>
<th>$\gamma = 1.0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{\gamma}_m$</td>
<td>.10</td>
<td>.09</td>
<td>.09</td>
<td>.09</td>
<td>.09</td>
</tr>
<tr>
<td>$\hat{\gamma}_f$</td>
<td>.25</td>
<td>.34</td>
<td>.37</td>
<td>.37</td>
<td>.38</td>
</tr>
</tbody>
</table>
Table 2.4: Estimates of $\gamma$ for High and Low Attachment Samples

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>$\gamma = .2$</th>
<th>$\gamma = .4$</th>
<th>$\gamma = .6$</th>
<th>$\gamma = .8$</th>
<th>$\gamma = 1.0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{\gamma}_m$</td>
<td>.06</td>
<td>.05</td>
<td>.05</td>
<td>.05</td>
<td>.04</td>
</tr>
<tr>
<td>$\hat{\gamma}_f$</td>
<td>.13</td>
<td>.12</td>
<td>.11</td>
<td>.11</td>
<td>.11</td>
</tr>
<tr>
<td>High Attachment Sample</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| $\hat{\gamma}_m$ | .17            | .20            | .20            | .20            | .20            |
| $\hat{\gamma}_f$  | .26            | .39            | .43            | .45            | .46            |
| Low Attachment Sample |
Chapter 3

Implications of Intergenerational Links for the Optimal Progressive Income Tax Code

3.1 Introduction

Progressive income tax codes, which have been adopted by most countries, can play potentially beneficial roles. First, they promote an equal distribution of income by mitigating differences in earning ability. Second, they provide insurance against labor income uncertainty.

However, providing insurance through tax or transfer policies often creates losses in efficiency (e.g., Alonso-Ortiz and Rogerson (2010)) because most tax codes are distortionary and progressive income tax codes are no exception. How, then, should a progressive income tax code be optimally designed in order to enhance a more equal distribution of income and supplement the incomplete insurance market? The answer depends on the nature and the magnitude of inequality in earning ability, the size of labor market risks, and the sources of insurance that are accessible to households.

It is well documented that earnings are unevenly distributed in the US (e.g., Diaz-Gimenez et al. (1997), Kopczuk et al. (2010)). The inequality in earnings reflects
differences in earning ability and labor market risks. The inequality in lifetime or long-term earnings is also significant (e.g., Haider (2001), Kopczuk et al. (2010)). In addition, an extensive body of literature indicates that labor market risks are sizable and persistent (e.g., Card (1994), Floden and Linde (2001), and Heathcote et al. (2010)).

Households partially insure themselves from labor market risks by saving more and/or working more, for example, and governments in many advanced countries operate a wide range of welfare programs and adopt progressive income tax codes to enhance the equity in welfare and to supplement missing insurance markets.

The transmission of human capital from parents to children is an important determinant of individual earning ability. Hugget et al. (2010) and Storesletten et al. (2007) emphasize inequality in factors such as human capital and earning ability before entering the labor market to account for approximately 50% to 70% of lifetime earnings inequality. In addition, Solon (1992) finds that the correlation of log earnings between individuals from two adjacent generations amounts to approximately 0.4, implying that earnings inequality is persistent across generations.

Savings are important sources of insurance, but altruism plays an important role in wealth inequality. Kotlikoff and Summers (1981) estimate that life-cycle wealth, defined as the accumulated net surplus of earnings over consumption, accounts for between 20% and 40% of US wealth, which, as a result, emphasizes the importance of wealth transfers. Gale and Scholz (1994) argue that intended private transfers and bequests account for at least 51% of US wealth and that the majority of transfers occur among intergenerationally linked households, mainly from parents to their children.

Consequently, the transmission of human and physical capital from parents to children is an important determinant of individual earning ability, wealth, and, in turn, welfare. A social planner should therefore take these intergenerational links into account to enhance social welfare when designing a policy affecting the entire population. Life cycle models have been widely adopted to design and evaluate various government policies (e.g., Hubbard et al. (1995), Conesa and Garriga (2006)). However, intergenerational links are often ignored despite their importance in these models.

In this paper, I first, investigate the optimal progressive tax code in an incomplete-
markets economy in which households are altruistically connected across generations. Starting from initial conditions that represent the current US economy and the tax code suggested by Gouviea and Strauss (1994), I compute the equilibrium with the optimal progressive tax code and describe the welfare effects. Second, I conduct the same policy experiment in an incomplete-markets economy in which links between generations are broken in order to highlight the extent to which the assumptions on the intergenerational links drive the results. Third, I investigate the extent to which the size of government welfare programs affects the optimal progressivity of the income tax code.

From these policy experiments, I find that the optimal tax code for an economy populated with altruistic households is approximately equivalent to a proportional tax of 23.1% with a fixed deduction of about $17,000 relative to the average income of $50,000. For an economy populated with non-altruistic households, however, these numbers are 18.8% and $12,000, respectively. These results imply that the issue of inequality is more severe in an economy with intergenerational links, so the policy maker should choose a progressive tax system that emphasizes the provision of insurance. Additionally, I find that when the social planner optimizes the size of the government welfare program, the insurance benefits from the progressive income tax code diminish.

My paper is related to several papers in the literature. Ventura (1999) studies quantitatively the general equilibrium implications of a revenue-neutral tax reform in which the current income and capital income tax structure in the US is replaced by a flat tax, as proposed by Hall and Rabushka (1995). Conesa and Kruger (2006), whose work is closely related to mine, compute the optimal progressivity of the US federal income tax code in a life-cycle model. I extend their work by considering bequests and the intergenerational earnings correlation.

Floden and Linde (2001) examine the optimal level of government redistribution in an economy where agents are subject to uninsurable, individual-specific productivity risk. In contrast to Floden and Linde (2001), I add a progressive income tax code to the policy toolbox to examine the optimal level of government redistribution.

In a dynastic environment, Erosa and Koreshkova (2007) study the effect of progressive taxation on human capital accumulation and the welfare consequences of a
flat tax reform. However, Erosa and Koreshkova (2007) do not investigate the optimal progressive tax code in their model economy.

Fuster, Imrohoroglu, and Imrohoroglu (2008) compute the welfare effects of different revenue-neutral tax reforms that eliminate capital taxation both in a dynastic model and in a life-cycle model. My motivation is similar to that of Fuster, Imrohoroglu, and Imrohoroglu (2008). By presenting a progressive tax reform, My paper contributes to a deeper understanding of the implications of intergenerational links in policy experiments.

The paper is organized as follows: In Section 3.2, I specify the model economy. In Section 3.3, I calibrate different specifications of the model economy. In Section 3.4, I define the policy experiment. In Section 3.5, I report the optimal tax codes of the model economies. In Section 3.6, I present my conclusion.

3.2 Model

I consider two different model economies: one with intergenerational links and one without. One economy is populated by households that are altruistic toward their descendants. An altruistic household values the welfare of its descendants as well as its own. I call an immediate descendant household a child household, an immediate ancestor household a parent household, and a sequence of households a dynasty. I denote this first model economy as the altruistic economy, or AE. The other economy is populated by households that only value their own welfare. I denote this model economy as the non-altruistic economy, or NE.

3.2.1 Environment

3.2.1.1 Population Dynamics

I assume that time $t$ is discrete. Each model economy is populated by a unit measure of households and an infinitely lived government. At any point in time, the households are either working or retired. In each period, working households face an exogenous

\footnote{The model environment is stationary. For simplicity, time subscripts are omitted unless they are absolutely necessary. Variables marked with a prime designation ($'$) denote period $t + 1$.}
and positive probability of retiring, \( p_R \), and retired households face an exogenous and positive probability of dying, \( p_D \). When a working household retires, it no longer takes part in the labor market.

In the AE, when a retired household dies, a newborn child household in the working stage enters the economy and inherits the assets of the parent household. In the NE, on the other hand, when a retired household dies, it is replaced by a newborn household in the working stage and the newborn household enters the economy without assets. I assume that the government collects the assets, if any, from deceased households and redistributes them to working households in a lump-sum manner.

### 3.2.1.2 Endowment and Preferences

I assume that all households are endowed with one unit of time in each period. A household in the working stage spends the time endowment by supplying labor to a competitive labor market or consuming leisure. A household in the retirement stage does not work.

I assume that households are heterogeneous in two dimensions that affect their labor productivity. First, I assume that households are born endowed with different earning abilities \( z \in Z \), where \( Z \) denotes a set of possible realizations of earning abilities, and that this earning ability does not change in a household’s lifetime. In the AE, when a retired household dies, some of its earning ability is transmitted to a newborn child household. The transmission of earning ability over generations follows a stationary finite-state Markov chain with transition probabilities given by

\[
\Pi_Z(z'|z) = \Pr(z_c = z'|z_p = z),
\]

where \( z_c \) is the earning ability of the child household and \( z_p \) is the earning ability of the parent household. In the NE, a newborn household draws its earning ability from the invariant distribution of earning abilities, denoted by \( \Pi^I_Z \).

Second, I assume that working households endowed with the same earning ability can differ in their labor market productivity because they are subject to idiosyncratic productivity shocks, \( x \in X \), where \( X \) denotes a set of possible realizations of the idiosyncratic productivity shock. The idiosyncratic productivity shocks evolve through time according to a stationary finite-state Markov chain with transition probabilities
given by $\Pi_x(x'|x) = \Pr(x_{t+1} = x'|x_t = x)$. I denote the invariant distribution of the idiosyncratic productivity shocks as $\Pi^I_X$. I assume that a newborn household draws an initial idiosyncratic productivity shock from the invariant distribution in both the AE and the NE.

When a working household of type $(z, x)$ devotes $h$ units of time to market work, this will generate $z x h$ units of labor services and the resulting labor earnings are $w z x h$, where $w$ is the market wage per efficiency unit of labor.

Each household has preferences over streams of consumption $\{c_t\}$ and hours of work $\{h_t\}$. I assume that preferences are time separable, with the constant discount factor $\beta$. The period utility function is denoted by $u(c, h)$ and I choose the following standard form:

$$u(c, h) = \frac{c^{1-\sigma}}{1-\sigma} - B \frac{h^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}}$$

where the parameter $\sigma$ is the coefficient of relative risk aversion, the parameter $\gamma$ reflects the willingness to substitute hours over time, and the parameter $B$ reflects disutility from hours of work.

I assume that a household in the AE cares about the utility of its descendants as much as it cares about its own utility. I assume that a household obtains zero utility after its death in the NE. A household, in both economies, maximizes the following standard expected utility function:

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t, h_t) \right\}$$

3.2.1.3 Government

The government runs a balanced budget pay-as-you-go social security system for retired households. The vector $(SS, \tau_s, b)$ summarizes this system. When a working household retires, it is entitled to receive social security benefits $SS$ at a replacement rate $b$ of the average labor earnings of each economy. The social security benefits are independent of the household’s earnings history. Social security benefits are financed through a proportional labor income tax and the social security tax rate $\tau_s$ is set to balance the period-by-period budget of the system.
The government also operates a transfer program for working households to guarantee a minimum level of consumption by providing them with a lump-sum transfer income $TR_A$. Note that, in the NE, the government collects assets from deceased households and redistributes them to working households. I denote this transfer as $TR_N$. In the NE, the total transfer income of a working household, $TR$, is the sum of $TR_A$ and $TR_N$. For the AE, the total transfer income of a working household is equal to $TR_A$. The government spends money on purchasing goods $G$. Government expenses excluding the social security payments ($TR_A + G$) are financed through an income tax and a consumption tax. The government is also assumed to balance this general budget period-by-period. The government levies a proportional tax on consumption expenditures at the rate of $\tau_c$.

I assume that the government cannot condition income tax rates on the source of income, i.e., it cannot tax labor and capital income separately at different rates. I employ the following parametric income tax function in my model economies:

$$\Psi(y) = \Theta(y) + \tau_m y$$

$$= a_0 \left( y - \left( y^{\alpha_1} + a_2 \right)^{-1/a_1} \right) + \tau_m y$$

where $y$ denotes taxable income, which is household income net of the social security tax bill, and $\Psi(y)$ denotes the corresponding income tax bill.

The functional form $\Theta(y) = a_0 \left( y - \left( y^{\alpha_1} + a_2 \right)^{-1/a_1} \right)$ is suggested and estimated by Gouveia and Strauss (1994). Their estimate is based on a relationship between individual effective federal income tax rates and income for US tax returns over 1979–1989. The functional form is fairly flexible in that it encompasses a wide range of progressive, proportional and regressive tax schedules. I consider only progressive and proportional tax schedules in my policy experiments and rule out regressive tax schedules. This functional form has been adopted in many quantitative studies (e.g., Castañeda et al. (2003), Conesa and Krueger (2006), Conesa et al. (2009), Erosa and Koreshkova (2009), and Cagetti and De Nardi (2009)).

My income tax function adds a linear term $\tau_m y$ to the function $\Theta$ because the government also obtains tax revenues from, for example, property and excise taxes. A
proportional tax rate $\tau_m$ reflects these sources of tax revenue.

### 3.2.1.4 Technology and Market Structure

I assume that the aggregate technology can be represented by a standard Cobb–Douglas production function:

$$Y = K^\alpha L^{1-\alpha}$$

where $K$ denotes the aggregate capital input, $L$ denotes aggregate labor input measured in efficiency units, and $\alpha$ denotes the capital share. I assume that the aggregate capital stock depreciates at the rate of $\delta$. As is standard with a constant returns to scale technology and perfect competition, I assume that there exists a representative firm running the technology.

I assume that there are no insurance markets for mortality risk, retirement risk, earning ability, or idiosyncratic productivity shocks. However, households are able to trade claims for physical capital, $a$, which yields a rate of return, $r$. Physical capital is the only asset available to households, and households face borrowing constraint: $a' \geq 0$.

### 3.2.2 The Household’s Problem

I formulate the household’s decision problem recursively. The parameter $\phi$ reflects the differences between the two economies. For the NE, $\phi$ is equal to zero and, for the AE, $\phi$ is equal to one.
3.2.2.1 A Retired Household \((R)\)

\[
V_R(a; z) = \max_{a',c} \left\{ \begin{array}{c}
    u(c, 0) + \beta (1 - p_D) V_R(a', h) \\
    + \phi \beta p_D \sum_{x'} \Pi_x' (x') \Pi_z' (z'|z) V_W(a', x', z_c)
\end{array} \right\} \\
\text{s.t.} \quad (1 + \tau_c) c + a' = (1 + r) a + SS - \Psi(y) \\
\quad y = ra + SS \\
\quad a' \geq 0, \ c \geq 0
\]

where \(V_R\) denotes the value function for a retired household whose state is \((a; z)\) and \(V_W\) denotes the value function for a working household. For the NE, \(\phi\) is equal to zero, which reflects the fact that the continuation value is zero when the household dies. For the AE, \(\phi\) is equal to one, which indicates that the continuation value is identical to the value of its child household whose state is \((a', x'; z_c)\), where \(z_c\) denotes the earning ability of its child household. The retired household’s decision rules that solve this problem is a set of functions denoted by \(f_{c_R}(a; z), a_R'(a; z)\). The measure of retired households over the state space \(A \times Z\) is denoted by \(\mu_R\).

3.2.2.2 A Worker Household \((W)\)

\[
V_W(a, x; z) = \max_{a',c,h} \left\{ \begin{array}{c}
    u(c, h) + \beta (1 - p_R) \sum_{x'} \Pi_x (x'|x) V_W(a', x'; z) \\
    + \beta p_R V_R(a'; z)
\end{array} \right\} \\
\text{s.t.} \quad (1 + \tau_c) c = (1 + r) a + (1 - \tau_S) wzxh + TR - \Psi(y) \\
\quad y = (1 - \tau_S) wzxh + ra + TR \\
\quad a' \geq 0, \ c \geq 0, \ 0 \leq h \leq 1
\]

where \(V_W\) denotes the value function for a working household whose state is \((a, x; z)\). Notice that the social security tax is levied on gross labor earnings and the remaining income is subject to the income tax. The working household’s decision rules that solve this problem are a set of functions and are denoted by \(\{c_W(a, x; z), a'_W(a, x; z), h(a, x; z)\}\).
The measure of working households over the state space $\mathcal{A} \times \mathcal{X} \times \mathcal{Z}$ is denoted by $\mu_W$.

### 3.2.3 Definition of Competitive Equilibrium

A competitive equilibrium consists of a set of value functions $\{V_R (a; z), V_W (a, x; z)\}$, a set of household decision rules $\{c_W (a, x; z), c_R (a; z), a'_W (a, x; z), a'_R (a; z), h_W (a, x; z)\}$, factor prices $\{r, w\}$, aggregate inputs $\{L, K\}$, a set of measures $\{\mu_W, \mu_R\}$ and a set of the government policies $\{\Psi, SS, \tau_S, b, TR, \tau_c, G\}$ such that

1. Households optimize:
   
   Given prices and government policies, the household decision rules solve value functions (3.1) and (3.2).

2. The representative firm maximizes profits:

   $\begin{align*}
   r &= \alpha \left( \frac{K}{L} \right)^{\alpha-1} - \delta \\
   w &= (1 - \alpha) \left( \frac{K}{L} \right)^{\alpha}
   \end{align*}$

3. Factor markets clear:

   $\begin{align*}
   L &= \int zxh_W (a, x; z) \, d\mu_W \\
   K &= \int ad\mu_W + \int ad\mu_R
   \end{align*}$

4. The goods market clears:

   $\begin{align*}
   \int c (a, x; z) \, d\mu_W + \int c (a; z) \, d\mu_R + \int a' (a, x; z) \, d\mu_W + \int a' (a; z) \, d\mu_R + G \\
   &= K^\alpha L^{1-\alpha} + (1 - \delta) K
   \end{align*}$
5. The government balances the general budget:

\[ G + TR_A = \int \Psi ((1 - \tau_S) wz x h (a, x; z) + ra + TR) d\mu_W \]
\[ + \int \Psi (ra + SS) d\mu_R \]
\[ + \tau_c \left( \int c_W (a; x; z) d\mu_W + \int c_R (a; z) d\mu_R \right) \]

\[ TR_N = p_D \int (1 + r) d\mu_R \]
\[ TR = TR_A + (1 - \phi) TR_N \]

6. The government balances the social security budget:

\[ SS \int d\mu_R = \tau_S \int wz x h (a, x; z) d\mu_W \]
\[ SS = b \int wz x h (a, x; z) d\mu_W \]

7. Individual and aggregate behaviors are consistent:

\[ \mu_R (A^0, Z^0) = (1 - p_D) \int_{A^0, Z^0} \left\{ \int_{A, Z} 1 \left[ a' = a'_R (a; z) \right] 1 \left[ z' = z \right] d\mu_R \right\} da' dz' \]
\[ + p_R \int_{A^0, Z^0} \left\{ \int_{A, X, Z} 1 \left[ a' = a'W (a, x; z) \right] 1 \left[ z' = z \right] d\mu_W \right\} da' dz' \]

for all \( A^0 \subset A \) and \( Z^0 \subset Z \)

\[ \mu_W (A^0, X^0, Z^0) = \]
\[ (1 - p_R) \int_{A^0, X^0, Z^0} \left\{ \int_{A, X, Z} 1 \left[ a' = a'_W (a, x; z) \right] 1 \left[ z' = z \right] \Pi_x (x|a) d\mu_W \right\} da' dz' dx' \]
\[ + \phi p_D \int_{A^0, X^0, Z^0} \left\{ \int_{A, Z} 1 \left[ a' = a'_R (a; z) \right] \Pi'_x (x') \Pi_x (z'|z) d\mu_R \right\} da' dz' dx' \]
\[ + (1 - \phi) p_D \int_{A^0, X^0, Z^0} 1 \left[ a' = 0 \right] \Pi'_x (x') \Pi'_x (z') da' dz' dx' \]

for all \( A^0 \subset A, X^0 \subset X, \) and \( Z^0 \subset Z \)
3.3 Calibration

In this subsection, I discuss the parameterization of the model economies that I employ in the quantitative analysis. I denote the calibrated model of the AE as the benchmark model of the AE and denote the calibrated model of the NE as the benchmark model of the NE.

3.3.1 Preference and Endowment

The idiosyncratic productivity shocks $x$ are assumed to follow an AR(1) process in logs:

$$\log (x') = \rho_x \log (x) + \varepsilon'_x, \quad \varepsilon'_x \sim N (0, \sigma_{x\varepsilon}^2)$$

Many studies have measured the labor market risks faced by prime age males, including Card (1994), Floden and Linde (2001), French (2005), and Chang and Kim (2006). While there is some variation across studies, the consensus is that these shocks are large and persistent. In my quantitative study, I adopt the estimate by Floden and Linde (2001); $\rho_x = 0.9136$, $\sigma_{x\varepsilon} = 0.2064$. The authors estimate these values based on the waves in the PSID for the years 1988 to 1992.

For the AE, the earning ability $z$ is assumed to stay constant in an individual household’s life and is transmitted to a child household following an AR(1) process:

$$\log (z_c) = \rho_z \log (z_p) + \varepsilon_z, \quad \varepsilon_z \sim N (0, \sigma_{z\varepsilon}^2)$$

where $z_c$ is the earning ability of the child household and $z_p$ is the earning ability of the parent household.

The earning ability process and the idiosyncratic productivity shock processes are assumed to be orthogonal. Given the parameter values of $\rho_x$ and $\sigma_{x\varepsilon}$, I choose the values of $\rho_z$ and $\sigma_{z\varepsilon}^2$ to simultaneously match two empirical observations. First, the intergenerational earnings persistence measured by the correlation of log-earnings between individuals from two adjacent generations is reported to fall between 0.4 and 0.5 (see Solon (1992) and Mulligan (1997)). Here, I target the value of 0.4. Second,
according to Diaz-Gimenez et al. (1997), the Gini coefficient of cross-sectional earnings of workers in the US economy is 0.43. For the NE, I use identical values of $\sigma^2$ in order to sort out the implications of the presence of intergenerational links.

I set the relative risk aversion coefficient $\sigma$ equal to 2. This value falls within the range (one to three) that is standard in the literature. The parameter $\gamma$ reflects the willingness to substitute hours over time and Chetty et al. (2012) argue that an empirically reasonable value for the Frisch elasticity falls between 0.4 and 0.5. I set $\gamma = 0.5$. I choose the value of the discount factor $\beta$ to match the aggregate capital output ratio of $K/Y = 3.0$, and the disutility of work parameter $B$ to match average hours of work equal to $1/3$ of the time endowment.

### 3.3.2 Government

Gouveia and Strauss estimate $a_0 = 0.258$, $a_1 = 0.768$, and $a_2 = 0.031$. Because the relationship was estimated based on incomes in 1990 US dollars and is not unit free, the tax formula requires normalization, which amounts to choosing $a_2$ in the model so that $a_{2Model} = a_2 \left( \bar{y}_{Model}/\bar{y}_{US1989} \right)^{-a_1}$, where $\bar{y}$ is the average household income (approximately $50,000$ for the US in 1989).

The social security system is chosen so that the replacement rate (the ratio of social security benefits to average earnings of the model economy) is 40%. The implied social security tax rate required to finance benefits is equal to $\tau_S = 0.1$. The social security tax revenues amount to 6.4% of GDP in the model\(^2\).

The sum of transfer income distributed to the working population relative to GDP is set to $\int TR_A d\mu_W/Y = 3.6\%$. This roughly matches the mandatory outlays of the US federal government excluding social security, Medicare, and $2/3$ of Medicaid\(^3\). Government spending is set to satisfy $G/Y = 20.5\%$, which is taken from Castañeda et al. (2003). The total outlays of the government in the benchmark economy including social security payments amount to 30.5% of GDP. The consumption tax rate is set to $\tau_c = 5\%$, which is in line with other studies including Conesa et al. (2009) and

\(^2\)The historical average (1981 ~ 2012) for this value is 6.2%. Table F2, The Budget and Economic Outlook (CBO).

\(^3\)Table F5, The Budget and Economic Outlook (CBO).
Domeij and Heathcote (2004). Finally, the intercept of the income tax function $\tau_m$ is calibrated to balance the government’s general budget in the model economy.

### 3.3.3 Other Parameters

Following Castañeda et al. (2003), the exogenous probability of dying, $p_D$, is set to 0.066 and the exogenous probability of retiring, $p_R$, is set to 0.022, implying that the average duration of working lives and retirements is 45 years and 18 years, respectively. The economy is composed of 75% of working households and 25% of retired households.

The capital income share, $\alpha$, is set equal to 0.36, in accordance with the long-run capital share for the US economy. The depreciation rate, $\delta$, is set equal to 0.08, implying an investment output ratio of 24%. Table 3.1 summarizes the parameter values for the benchmark calibrations of the AE and NE.

### 3.4 Policy Experiment

I define the optimal income tax code within the previously given parametric class as the tax code generating the highest ex-ante expected value of a newborn working household. The social welfare function for the policy maker to maximize is then given by

$$WF(a_0, a_1) = \int V_W(a, x; z) \, d\mu_B$$

where $\mu_B$ denotes the measure of newborn households over the state space and is obtained as follows:

$$\mu_B(A^0, X^0, Z^0) = \phi p_D \int_{A^0, X^0, Z^0} \left\{ \int_{A, Z} 1[a' = a'_R(a; z)] \Pi_x'(x') \Pi_z'(z'|z) \, d\mu_R \right\} \, da' dz'dx'$$

$$+ (1 - \phi) p_D \int_{A^0, X^0, Z^0} 1[a' = 0] \Pi_x'(x') \Pi_z'(z') \, da' dz'dx'$$

for all $A^0 \subset A, X^0 \subset X$, and $Z^0 \subset Z$.

I conduct a revenue-neutral policy change. I define the government’s problem as finding an equilibrium allocation that maximizes $WF(a_0, a_1)$, holding constant the
level of transfer payments, $TR_A$, and the level of government spending, $G$. Numerically, this experiment is conducted by finding $(a_0, a_1)$, thereby maximizing the social welfare function. The associated value of $a_2$ is determined to achieve a balanced budget \[3.4\].

### 3.5 Results

In the benchmark model of the AE and NE, newborn households draw their earning abilities from identical distributions and face identical idiosyncratic shock process. The difference between the optimal tax codes stems from the intergenerational transmission of earning ability and the altruism. Note that, in the NE, a newborn household starts its life without assets.

I calibrated the model to match the Gini coefficient for the earnings distribution of working households in both benchmark economies. It turns out that the Gini coefficient for the wealth distribution of working households is 0.69 in the benchmark model of both the AE and NE, which is close to the empirical counterpart: 0.74 (see Diaz-Gimenez et al. 1997).

To see the extent to which altruism alters households’ saving decisions, I report the aggregate savings of the retired households and their consumption. In the benchmark model of the AE, retired households collectively hold 14.92% of the capital stocks and consume 9.11% of GDP per year. In the benchmark model of the NE, these numbers are 12.69% and 9.65%, respectively. In the economy populated with altruistic households, households hold more assets and consume less in their retirement. The wealth distribution of the retired households is notably concentrated in both benchmark models: The Gini coefficient is 0.83 in the AE and 0.85 in the NE, respectively. Diaz-Gimenez et al. (1997) document that retired households hold 22.4% of wealth and the Gini coefficient of the wealth distribution for retired households is 0.69. Compared to the data, the retired households in the benchmark model economies hold less wealth and its concentration is too high. A high concentration of wealth among retired households is a common feature of incomplete-market models with a lump-sum pay-as-you-go pension

\[Note that other government policy variables (SS, \tau_S, b, \tau_c, \tau_m) are not subject to optimization by the policy maker. The level of social security benefit, SS, will be adjusted to ensure that system’s budget is balanced.\]
system (e.g., Hugget (1996)).

3.5.1 The Optimal Tax Codes

From the policy experiment, I find that the optimal tax code for the AE is described by
\[ a_0 = 23.1, \quad a_1 = 42.1, \]
which is approximately equivalent to a proportional tax of 23.1% with a fixed deduction of about $17,000 relative to the US average income of $50,000 as of 1990. For the NE, these numbers are \( a_0 = 0.188, \ a_1 = 40.0, \) yielding 18.8% and $12,000 respectively.

These optimal tax codes, which are characterized by a flat marginal tax rate with a sizeable deduction, comes close in shape to the tax reform proposal by Hall and Rabushka (1995), who suggested a constant marginal tax rate of 19% with a deduction of $22,500 (for a family of four) for the US economy.

I am not the first one to show that the optimal progressive tax code (with the functional form suggested by Gouviea and Strauss) in an incomplete-markets model is approximately equivalent to a flat marginal tax rate with a substantial deduction. Conesa and Krueger (2006) calculate that a constant marginal tax rate of 17.2% with a deduction of $9,400 is the optimal progressive tax code for their life-cycle economy. Considering the differences in specification, the result for the non-altruistic economy comes close to that of Conesa and Krueger (2006). Therefore, the results of both the AE and the NE confirm the robustness of Conesa and Krueger’s result in different economic environments.

Figure 1 shows the average and marginal tax rates implied by the optimal income tax codes for the AE and the NE, together with the values from the US economy (Gouveia and Strauss’s estimates for 1989). Compared to the benchmark economy of the AE, every household whose income is above $34,000 would pay more income tax under the optimal tax system. For the NE, I observe that households whose incomes fall between $22,000 and $125,000 would pay more tax under the optimal tax system.

Conditioning the social security benefits on the earnings history of the individual household may alleviate the problem, but its computational burden is significant when it comes to finding the optimal tax code.
but households whose incomes are above $125,000 pay fewer taxes under the system.

In order to assess the implications of the tax code for equilibrium allocations, I report the changes in equilibrium aggregate variables in Table 3.2.

In the AE, under the optimal tax system, I observe that capital stock drops substantially by 6.43% from the level of the benchmark model of the AE. Two effects contribute to this result. First, this is an immediate consequence of the increase in the tax rate faced by the majority of households, except low-income households. Second, providing more insurance through the tax code implies that the incentive to accumulate assets decreases.

The change in tax rates also induces adjustments in the labor supply. While the average hours worked drop by 2.08%, the total labor supply measured in labor efficiency units drops by 0.84%. Thus, households endowed with low earning ability drive the reductions in labor supply following the tax reform. With drops in capital stocks and labor supply, I see a drop in aggregate output of 2.83% and a drop in aggregate consumption of 2.47%.

In the NE, however, many productive worker households experience reductions in their marginal tax rate, which provides a better incentive to work and save. Thus, the impacts of the increased provision of insurance and this efficiency boost are more or less balanced, which leads to only a mild decrease in aggregate output and consumption, 0.4% and 0.34%, respectively.

Compared to the NE, I observe a much larger deduction in the optimal tax code in the AE. From the perspective of a newborn household, entering the economy endowed with low earning ability is more costly in the AE. First, newborn households with low earning ability usually inherit few assets. Second, the household’s descendants are more likely to have low earning ability. An altruistic newborn household takes this into account, and the welfare of the newborn household with low earning ability is, therefore, much lower in the AE compared to the NE. As a result, enhancing the welfare of the households endowed with low earning ability is more important to the social planner in the AE, which leads to a larger deduction in the optimal tax code.

I also observe that the tax rate is higher under the optimal tax system in the AE.

\footnote{Note that in the NE, every newborn household enters the economy without assets.}
One reason for this is that a higher deduction is provided to the low-income households in the AE. Another reason is that equilibrium aggregate output falls much more in the AE, which requires a higher tax rate to finance the government layouts \((TR + G)\) in the revenue-equivalent tax reform.

### 3.5.2 Decomposition of the Welfare Effects

In order to quantify the welfare effects of the tax reform, I use a consumption equivalent variation measure, which I denote as CEV. I quantify the welfare change of a given policy reform by asking a household: how much (in %) does consumption have to increase in all future periods and contingencies, keeping hours constant from the old steady state so that expected future utility does not change?\(^7\) The results are reported in Table 3.3.

Even though the economy shrinks sharply under the optimal tax system in the AE, the increase in welfare, measured in terms of CEV, is substantial: 2.39% of consumption in every stage of life and all contingencies. The welfare gains mostly stem from a better allocation of consumption and from a reduction in the average of hours of work. This change more than offsets the lower average level of consumption in utility terms.

In the NE, the increase in welfare as a result of the tax reform is larger: 2.62% of CEV. Note that I observe only a mild decline in aggregate output and consumption in the NE. This comes from the fact that high-income households have a greater incentive to work and save due to the reduction in the marginal tax rate under the optimal tax system.

In order to investigate the difference in welfare effects, I decompose CEV into a component, \(CEV_C\), stemming from the change in consumption, holding hours of work in the benchmark model economy constant, and a component, \(CEV_H\), stemming from the change in hours of work, keeping consumption at the steady state with the optimal tax system. The consumption component can again be divided into a part, \(CEV_{CL}\), that captures the change in average consumption, and a part, \(CEV_{CD}\), that reflects the change in the distribution of consumption across households with different earning

\(^7\)See Appendix 3.A for details.
abilities and across states of the world. \( CEV_H \) can be divided in the same way.

The welfare gain of the consumption component is much smaller in the AE: 0.74% for the AE and 1.35% for the NE. In the NE, by sacrificing 0.34% of average consumption, the policy maker achieves a 1.69% distributional welfare gain and therefore a 1.35% overall welfare gain. In the AE, however, the policy maker faces a more challenging trade-off between equity and efficiency. Notice that a newborn household in both the AE and the NE faces the identical earning ability distribution and idiosyncratic shocks. The difference between the optimal tax codes stems from altruism and the intergenerational transmission of earning ability. In addition, the inequality in initial assets, which is a consequence of the altruism, amplifies inequality in the distribution of welfare. Consequently, the policy maker in the AE places more emphasis on achieving a more equal distribution of welfare. By implementing the optimal tax, the social planner achieves a 3.19% distributional gain, but the overall gain in the consumption component is only 0.74% due to the large loss in average consumption, which amounts to 2.37%.

The welfare gain from the hours component is substantial. The distributional gain in both economies is similar in size: 0.33% for the AE and 0.27% for the NE. Because the average hours of work drops more in the AE, the level component is higher in the AE. As a result, the overall welfare gain from the hours component is higher in the AE.

### 3.5.3 Effects of the Intergenerational Earnings Correlation

In order to see the implications of the intergenerational correlation of earnings for the optimal progressive tax code, I recalibrate the altruistic economy to target a correlation coefficient 0.2 of log earnings, holding the variance of earning abilities constant. In this AE with low correlation, the retired households collectively hold 15.33% of the capital stock and consume 9.03% of GDP per year. Compared to the benchmark model of the AE, the retired households hold more wealth and consume less. The reason is that altruistic households are willing to leave more assets to their descendants due to the "generational" consumption smoothing motive. Note that a retired household with high earning ability now has a smaller probability of having a descendant with

\(^8\)I employ the decomposition method suggested by Conesa et al. (2009)
high earning ability and is therefore willing to bequeath more assets. The correlation coefficient between earning ability and inherited wealth is 0.22, compared with 0.38 in the benchmark model of the AE. As a result, a newborn household in the AE with low correlation inherits more assets and the inheritance is relatively more evenly distributed. Consequently, the social planner provides a smaller deduction under the optimal tax code. I find that the optimal tax code of the AE with weaker generational links is approximately equivalent to a proportional tax of 21.4% with a fixed deduction of about $15,500.

3.5.4 Policy Experiment 2

In the absence of insurance markets against innate earning ability and labor market risks, a well-designed progressive income tax system provides a partial substitute for these missing insurance markets. However, lowering the tax rate for low-income households comes at a cost of imposing higher tax rates for high-productivity households. Therefore, a progressive income tax system performs a dual role: to finance government outlays and to produce a more equal distribution of income and, as a result, more equal consumption. The governments in many advanced countries operate a wide range of social welfare programs to accomplish the latter. For this reason, the optimal progressivity of the income tax code depends on the size of welfare programs.

For instance, I find that when the transfer level is doubled in the AE, the optimal level of deduction falls to $14,500 from $17,000 (with a higher constant marginal tax rate to finance more transfers). Figure 2 shows the average and marginal tax rates implied by the optimal income tax codes.

Now I ask a question. When the level of transfers is optimally set, does the optimal progressive tax system (within the class considered in this paper) converge to a proportional tax system? I conduct this policy experiment including the amount of government transfers in the set of optimization subjects. I find that a proportional tax rate of 37.8% without any deduction is the optimal income tax code in this experiment. The government transfer provides social insurance, and the tax system only needs to finance government outlays. Hence, when the size of the government welfare program is chosen carefully, the insurance benefits from the progressive income tax code disappear.
3.6 Conclusion

This chapter investigates the optimal progressive tax code in an incomplete-markets economy where generations are linked by earning ability and altruism. Within the class of progressive income tax function suggested by Gouviea and Strauss (1994), the optimal tax code for an economy populated with altruistic households is approximately equivalent to a proportional tax of 23.1% with a fixed deduction of about $17,000 in 1990 US dollars. For an economy populated with non-altruistic households, on the other hand, these numbers are 18.8% and $12,000, respectively. Thus, the need for redistribution and social insurance is stronger in an economy with intergenerational links as inequality tends to persist over time. I also find that when the government also optimizes the size of the welfare program, the additional insurance benefits from the progressive income tax code disappear.

These results strongly suggest that inequality in earning ability and its transmission over generations are crucial for social welfare. In the policy experiments, the distribution of earning ability is assumed to be held constant with respect to changes in government policy. Future research should investigate how sensitive the findings are to a more detailed model of human capital formation, particularly early in life, since it may respond to the shape of the income tax code.
3.A Decomposition of the Welfare Change

Consider a stream of consumption and hours of work from $t$ onwards

$$\{c_t, h_t\}_{t=0}^\infty$$

Life-time utility from the stream, $W$, is

$$W (\{c_t, l_t\}_{t=0}^\infty) = \sum_{t=0}^{\infty} \beta^t u (c_t, h_t)$$

The measure of social welfare, $WF$, is defined as

$$WF = \int E_0 W (\{c_t, l_t\}_{t=0}^\infty) d\mu_B$$

Now, let $\{c_t^0, h_t^0\}_{t=0}^\infty$ denote a stream of consumption and hours in the benchmark economy and let $\{c_t^*, h_t^*\}_{t=0}^\infty$ denote a stream of consumption and hours in the economy under the optimal tax system.

Let $CEV_C$ and $CEV_H$ be defined as

$$WF (\{c_t^*, l_t^0\}_{t=0}^\infty) = WF (\{(1 + CEV_C) c_t^0, l_t^0\}_{t=0}^\infty)$$
$$WF (\{c_t^*, l_t^*\}_{t=0}^\infty) = WF (\{(1 + CEV_H) c_t^*, l_t^*\}_{t=0}^\infty)$$

Then we can verify that

$$1 + CEV = (1 + CEV_C) (1 + CEV_H)$$

or

$$CEV \approx CEV_C + CEV_H$$

We further decompose $CEV_C$ into a level effect $CEV_{CL}$ and a distribution effect
$CEV_{CD}:$

\[
WF\left(\left\{c_t^0, l_t^0\right\}_{t=0}^{\infty}\right) = WF\left(\left\{(1 + CEV_{CL}) c_t^0, l_t^0\right\}_{t=0}^{\infty}\right)
\]

\[
WF\left(\left\{c_t^*, l_t^0\right\}_{t=0}^{\infty}\right) = WF\left(\left\{(1 + CEV_{CD}) c_t^0, l_t^0\right\}_{t=0}^{\infty}\right)
\]

\[
\hat{c}_t^0 = \left(\frac{C^*}{C^0}\right) c_t^0
\]

where $\{c_t^0\}$ is the consumption allocation resulting from scaling the allocation $\{c_t^0\}$ by the change in aggregate consumption $C^*/C^0$. A calculation shows that the level effect equals

\[
CEV_{CL} = \frac{C^*}{C^0} - 1
\]

A similar decomposition applies to leisure:

\[
WF\left(\left\{c_t^*, \hat{h}_t^0\right\}_{t=0}^{\infty}\right) = WF\left(\left\{(1 + CEV_{HL}) c_t^*, \hat{h}_t^0\right\}_{t=0}^{\infty}\right)
\]

\[
WF\left(\left\{c_t^*, l_t^*\right\}_{t=0}^{\infty}\right) = WF\left(\left\{(1 + CEV_{CD}) c_t^*, \hat{h}_t^0\right\}_{t=0}^{\infty}\right)
\]

\[
\hat{h}_t^0 = \left(\frac{H^*}{H^0}\right) h_t^0
\]
Table 3.1: Calibrated Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>common parameter values for the benchmark model of the AE and the NE</td>
<td></td>
</tr>
<tr>
<td>probability of retiring</td>
<td>( p_R )</td>
</tr>
<tr>
<td>probability of dying</td>
<td>( p_D )</td>
</tr>
<tr>
<td>risk aversion</td>
<td>( \sigma )</td>
</tr>
<tr>
<td>intertemporal elasticity of substitution</td>
<td>( \gamma )</td>
</tr>
<tr>
<td>persistence of idiosyncratic shocks</td>
<td>( \rho_x )</td>
</tr>
<tr>
<td>standard deviation of innovation of ( x )</td>
<td>( \sigma_{x\varepsilon} )</td>
</tr>
<tr>
<td>consumption tax rate</td>
<td>( \tau_c )</td>
</tr>
<tr>
<td>progressive parameter</td>
<td>( a_0 )</td>
</tr>
<tr>
<td>progressive parameter</td>
<td>( a_1 )</td>
</tr>
<tr>
<td>replacement rate</td>
<td>( b )</td>
</tr>
<tr>
<td>capital share</td>
<td>( \alpha )</td>
</tr>
<tr>
<td>depreciation rate</td>
<td>( \delta )</td>
</tr>
</tbody>
</table>

Parameter values used to match some features in the Model Economy

<table>
<thead>
<tr>
<th>Parameter</th>
<th>AE</th>
<th>NE</th>
</tr>
</thead>
<tbody>
<tr>
<td>discount rate</td>
<td>( \beta )</td>
<td>0.954</td>
</tr>
<tr>
<td>disutility of working</td>
<td>( B )</td>
<td>37.876</td>
</tr>
<tr>
<td>intercept of income tax function</td>
<td>( \tau_m )</td>
<td>0.115</td>
</tr>
<tr>
<td>social security tax rate</td>
<td>( \tau_S )</td>
<td>0.1</td>
</tr>
<tr>
<td>persistence of earning ability</td>
<td>( \rho_z )</td>
<td>0.810</td>
</tr>
<tr>
<td>standard deviation of earning ability</td>
<td>( \sigma_z )</td>
<td>0.633</td>
</tr>
</tbody>
</table>

Notes: AE denotes the altruistic economy and NE denotes the non-altruistic economy
Table 3.2: Changes in Aggregate Variables

<table>
<thead>
<tr>
<th></th>
<th>AE</th>
<th>NE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>$Y$</td>
<td>-2.90%</td>
</tr>
<tr>
<td>Capital Stock</td>
<td>$K$</td>
<td>-6.43%</td>
</tr>
<tr>
<td>Average Hours Worked</td>
<td>$H$</td>
<td>-1.80%</td>
</tr>
<tr>
<td>Total Labor Supply</td>
<td>$L$</td>
<td>-0.87%</td>
</tr>
<tr>
<td>Aggregate Consumption</td>
<td>$C$</td>
<td>-2.50%</td>
</tr>
<tr>
<td>interest rate</td>
<td>$r$</td>
<td>4%→4.45%</td>
</tr>
<tr>
<td>wage</td>
<td>$w$</td>
<td>-2.07%</td>
</tr>
</tbody>
</table>

Notes: The table reports percentage changes in aggregate variables from implementing the optimal tax code.
Table 3.3: Decomposition of Welfare Change

<table>
<thead>
<tr>
<th></th>
<th>AE</th>
<th>NE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Change</td>
<td>CEV</td>
<td>2.39%</td>
</tr>
<tr>
<td>Consumption</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>CEVC</td>
<td>0.74%</td>
</tr>
<tr>
<td>Level</td>
<td>CEVCL</td>
<td>-2.37%</td>
</tr>
<tr>
<td>Distribution</td>
<td>CEVCD</td>
<td>3.19%</td>
</tr>
<tr>
<td>Hours of Work</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>CEVH</td>
<td>1.64%</td>
</tr>
<tr>
<td>Level</td>
<td>CEVHL</td>
<td>1.30%</td>
</tr>
<tr>
<td>Distribution</td>
<td>CEVHD</td>
<td>0.33%</td>
</tr>
</tbody>
</table>

Notes: CEV denotes the consumption equivalent variation measure.
Figure 3.1: Optimal Progressive Tax Code

Notes: US(Gouviea Strauss) denotes the estimated US effective federal income tax rate for the year of 1989. AE denotes the optimal tax code for the altruistic economy. NE denotes the optimal tax code for the non-altruistic economy.
Figure 3.2: Effect of the Size of Transfers on Optimal Progressive Tax Code

Notes: US(Gouviea Strauss) denotes the estimated US effective federal income tax rate for the year of 1989. AE denotes the optimal tax code for the altuistic economy. AE with 2xTR denotes the optimal tax code for the altuistic economy when the government doubles the levels of transfer.
Bibliography


