Spatio-Spectral Interferometric Imaging and the Wide-field Imaging Interferometry Testbed

by

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To Maria and Regina, my light and inspiration.
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Biographical Sketch

Alexander S. Iacchetta was born in Rochester, NY. He attended the University of Rochester from 2007 to 2011 graduating Magna Cum Laude with a Bachelor of Science degree in Optical Engineering with highest distinction and with a minor in Mathematics. Alex entered the graduate program at The Institute of Optics at the University of Rochester in 2011. He joined the research group of James Fienup in 2012 and began research in image reconstruction for interferometric imaging. He was eventually awarded a NASA Space Technology Research Fellowship in 2014 and began the research in this thesis on wide-field spatio-spectral interferometric imaging.

The following is a list of publications and presentations written during doctoral study:


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Abstract

The light collecting apertures of space telescopes are currently limited in part by the size and weight restrictions of launch vehicles, ultimately limiting the spatial resolution that can be achieved by the observatory. A technique that can overcome these limitations and provide superior spatial resolution is interferometric imaging, whereby multiple small telescopes can be combined to produce a spatial resolution comparable to a much larger monolithic telescope. In astronomy, the spectrum of the sources in the scene are crucial to understanding the material composition of the sources. So, the ultimate goal is to have high-spatial-resolution imagery and obtain sufficient spectral resolution for all points in the scene. This goal can be accomplished through spatio-spectral interferometric imaging, which combines the aperture synthesis aspects of a Michelson stellar interferometer with the spectral capabilities of Fourier transform spectroscopy.

Spatio-spectral interferometric imaging can be extended to a wide-field imaging modality, which increases the collecting efficiency of the technique. This is the basis for NASA’s Wide-field Imaging Interferometry Testbed (WIIT). For such an interferometer, there are two light collecting apertures separated by a variable distance known as the baseline length. The optical path in one of the arms of the interferometer is variable, while the other path delay is fixed. The beams from both apertures are subsequently combined and imaged onto a detector. For a fixed baseline length, the result is many low-spatial-resolution images at a slew of optical path differences, and the process
is repeated for many different baseline lengths and orientations. Image processing and synthesis techniques are required to reduce the large dataset into a single high-spatial-resolution hyperspectral image.

Our contributions to spatio-spectral interferometry include various aspects of theory, simulation, image synthesis, and processing of experimental data, with the end goal of better understanding the nature of the technique. We present the theory behind the measurement model for spatio-spectral interferometry, as well as the direct approach to image synthesis. We have developed a pipeline to preprocess experimental data to remove unwanted signatures in the data and register all image measurements to a single orientation, which leverages information about the optical system’s point spread function. In an experimental setup, such as WIIT, the reference frame for the path difference measured for each baseline is unknown and must be accounted for. To overcome this obstacle, we created a phase referencing technique that leverages point sources within the scene of known separation in order to recover unknown information regarding the measurements in a laboratory setting. We also provide a method that allows for the measurement of spatially and spectrally complicated scenes with WIIT by decomposing them prior to scene projection.
Contributors and Funding Sources

Professors James R. Fienup (advisor), Miguel A. Alonso, and James M. Zavislan all from The Institute of Optics, and Dr. David T. Leisawitz of NASA Goddard Space Flight Center served as the dissertation committee and supervised the production of this thesis.

Experimental data from the Wide-field Imaging Interferometry Testbed (WIIT) was provided by NASA Goddard. Matthew R. Bolcar of NASA initiated the automated collection of WIIT datasets. Roser Juanola-Parramon of NASA aided in trying to process the datasets using different methods.

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Chapter 1

Introduction

1.1 Motivation

Imaging interferometry offers the space technology necessary to achieve the angular resolution required to answer many important astrophysical questions. Optical interferometric imaging provides one or more orders of magnitude increase in resolution over any available solitary telescope by synthesizing a virtual pupil from multiple measurements. Producing a single telescope with resolution comparable to an imaging interferometer would not be feasible since it would require mirror diameters so large that they would be difficult to fabricate and test, impractical to launch, and be enormously expensive. Ground-based optical interferometers such as the Center for High Angular Resolution Astronomy (CHARA), the Navy Precision Optical Interferometer (NPOI), and Cambridge Optical Aperture Synthesis Telescope (COAST) already have a rich history in astronomical imaging. Saha [1] discusses some of the important astronomical observations from optical interferometers, including diameters and temperatures of
giant stars, stellar surfaces and envelopes, binary stars, central regions of galaxies, and other wavelength-dependent objects. Saha [1] and Monnier [2] both provide in-depth overviews of optical interferometry for astronomy. Although improvements in ground-based interferometers have made them vital for astronomical imaging, these interferometers are limited by atmospheric turbulence, which severely degrades coherency and the SNR needed for accurate phase measurements of dim or extended objects, and by atmospheric transmittance, which excludes many regions of the electromagnetic spectrum that are absorbed by the atmosphere. An obvious way to overcome the detrimental effects of the atmosphere is to deploy an imaging interferometer into space. The Space Infrared Interferometric Telescope (SPIRIT) and the Submillimeter Probe of the Evolution of Cosmic Structure (SPECS) are examples of NASA-proposed space interferometers. Such observatories would offer high resolution to answer astrophysical questions regarding the formation of planetary systems and habitable planets, exoplanet detection, galaxy mergers, and the formation of the first stars in the universe [3,4]. All of the technology necessary for these proposed missions to become a reality is not yet in place. This thesis aims to progress our understanding of spatio-spectral, or double-Fourier, interferometry and the corresponding reconstruction algorithms.

1.2 Double-Fourier Interferometry

Spatio-spectral interferometry is a method of probing the mutual coherence function at the plane of the interferometer’s apertures. Due to the propagation of the mutual coherence function, it is then possible to relate the coherence properties in the interferometer plane to source properties. In the case of an incoherent source, such as a radiative astronomical object, the measured mutual coherence function as it varies with path delay can be related to the spectral density of the source much like the van Cittert-Zernike theorem [5, Sec. 4.4.4; 6, Sec. 5.7] relates
the mutual intensity at an interferometer to the brightness distribution of the source. Measurement models for astronomical double-Fourier interferometry have been developed independently by Itoh and Ohtsuka [7] and Mariotti and Ridgway [8] for ground-based optical interferometers. Mariotti and Ridgway describe how the mutual coherence function can be measured by interfering light from two physically separated apertures after the beam from one aperture undergoes an optical delay relative to the other. By varying the delay and measuring the irradiance in the pupil plane where the beams are combined, the mutual coherence function for a single aperture separation, or baseline, is constructed. Analogous to image formation with ground-based interferometry, measurements can be repeated for various baselines and an image can be computed from the interferometric measurements. However, if the two apertures were allowed to overlap perfectly, this method is directly analogous to Fourier transform spectroscopy (FTS), which is a method of measuring an optical field’s spectrum, particularly in the infrared. As a consequence of this relationship, double-Fourier interferometric data can be used to synthesize the spatial-spectral distribution of an incoherent source.

A wide-field extension of double-Fourier interferometry can be obtained by adding an imaging lens after the beam combination such that a low resolution panchromatic image of the source is obtained at the detector array if either one of the beams is blocked before beam combination (see Fig. 2.1 for a simplified illustration of a wide-field spatio-spectral interferometer). In fact, the larger field-of-view (FOV) will be necessary in order to make many astrophysical observations, comprising large galaxies and collections of protostars [9]. A measurement model for an ideal interferometer of this type based on scalar wave theory has been developed by Lyon et al. [8, 9], allowing them to create an algorithm to directly construct a “dirty” high-resolution hyperspectral image of the source from an interferometric dataset. A
more detailed derivation of the measurement model and image synthesis algorithm are presented in Chapter 2 of this thesis. Another measurement model that includes polarization and other system-dependent effects was described by Elias et al. [12]. These wide-field interferometers usually consist of two radially moveable apertures that remain equidistant about the axis of the interferometer. The interferometer, or equivalently the object, can rotate in order to obtain sufficient baseline sampling azimuthally. This two-telescope design will be important for future space observatory missions when the targeted resolution prevents a single monolithic telescope from being considered due to cost, weight, and size; accordingly, NASA has identified interferometry as necessary to understand star and planet formation, as well as to verify and reduce degeneracies in simulations of galaxy formation and evolution [13]. NASA envisions that far-infrared wide-field double-Fourier interferometry will be a necessary technology in fifteen to thirty years.

1.3 Wide-Field Imaging Interferometry Testbed

For over a decade, NASA has been advancing and probing the efficacy of the wide-field spatio-spectral interferometric technique through the Wide-field Imaging Interferometry Testbed (WIIT) [7, 12–21] in preparation for future space-based observatories. This state-of-the-art testbed resides in the well-controlled environment of the Advanced Interferometry and Metrology laboratory at Goddard Space Flight Center. WIIT was designed to be a scale model of SPIRIT, as such the baseline lengths, collecting apertures, delay line, and wavelengths are all scaled down. This allowed WIIT to be developed at optical wavelengths, allowing high quality components of the system to be obtained affordably. The interferometer in WIIT is comprised of two input apertures that are on stages that can change their separation in one dimension, resulting in a minimum and maximum baseline length of 36 mm and 230 mm, respectively. The beam
from each input aperture is 25 mm in diameter. The f-number of the imaging lens after the beamsplitter is about 80 and the CCD camera’s pixel pitch is 16 microns, resulting in images that are Nyquist sampled at a wavelength of 400 nm. WIIT’s CCD camera was chosen for its large 16-bit dynamic range. The maximum scan range of the delay line is 60 mm, which is much larger than any scan range that WIIT would use in practice. A typical scan range for a WIIT dataset is about 300 microns, resulting in a spectral resolving power \( R = \frac{\lambda}{\Delta \lambda} \) ranging from about 385 to 714 over the spectral range of 420 nm to 780 nm. A metrology system [24] exists for the delay line that allows knowledge of the relative optical path difference between the two arms of the interferometer to within about 10 nm; however, the absolute path difference between the two arms of the interferometer is not known to within the desired sub-micron accuracy. Chapter 7 will discuss a method to relate the path difference measurements between delay line scans in a WIIT dataset, which is necessary for image synthesis. A typical delay line step in WIIT measurements corresponds to changing the path length between arms of the interferometer by about 100 nm.

WIIT’s light source is the Calibrated Hyperspectral Image Projector (CHIP) [25–27]. At any moment in time, CHIP is capable of producing a scene where every pixel in the scene has the same arbitrary spectrum. This is achieved by using two digital light processing (DLP) units in series. One of the DLPs controls CHIP’s spatial engine which produces an 8-bit gray-scale spatial distribution of the scene being projected. The other DLP controls the shape of the spectrum, which is realized by dispersing a broadband source onto the DLP such that each row of the DLP unit controls the relative strength of each wavelength bin in the output spectrum. A hyperspectral image can then be simulated by cycling through multiple spatially-spectrally separable images throughout the integration time of WIIT’s imaging camera. The hyperspectral
image projector is calibrated by including a fiber-coupled spectrometer into the system, allowing the spectral output of the projector to be monitored. CHIP can create spatially and spectrally diverse scenes with spatial features a tenth the size of WIIT’s CCD camera pixels and spectral features as small as 10 nm in the spectral range from 420 nm to 780 nm. The detail of using CHIP to project complicated hyperspectral scenery for use in WIIT datasets is discussed more in Chapter 5. Because CHIP is controlled digitally, source rotation is achieved by computationally rotating the scene being projected by CHIP.

1.4 Outline

This thesis provides contributions to both wide-field double-Fourier interferometry and WIIT:

- In Chapter 2, we provide a detailed derivation of the measurement model for wide-field spatio-spectral interferometry with connections to relevant coherence theory. We discuss how to synthesize a single high spatial resolution hyperspectral image from a five-dimensional double-Fourier interferometric dataset. System performance and limitations are also examined. Chapter 2 is nearly identical to a previous journal publication in the Journal of the Optical Society of America [28].

- Chapter 3 introduces a simple experiment for determining the aberrations that contribute to WIIT measurements through modern phase retrieval techniques. We make assumptions about the underlying wavefront contributing to the aberrations based on knowledge of WIIT’s imaging lens. The majority of the content of Chapter 3 was previously published in an Optical Society of America conference paper [29].

- Chapter 4 presents an algorithm capable of rotating and translating images through the use of a chirp z-transform (CZT). With the addition of nonlinear optimization techniques, we
developed an image registration algorithm that is able to retrieve relative rotations and translations between images. The results of image registration are refined by incorporating deconvolution, which is possible due to the implementation of the CZT image rotation algorithm. Chapter 4 is largely based on a previous SPIE conference proceedings paper [30].

- Chapter 5 provides a method of preparing complicated hyperspectral images for CHIP using nonnegative matrix factorization to decompose the hyperspectral scenes into a relatively small number of spatially-spectrally separable images. We discuss how to incorporate calibration data from CHIP and reduce quantization noise caused by the DLPs. The information in Chapter 5 up to Sec. 5.5 was published in an SPIE conference proceedings paper [31].

- Chapter 6 demonstrates how to simulate WIIT measurements, including image rotation, by incorporating the CZT algorithm from Chapter 4. We also show results of using the image synthesis algorithm from Chapter 2 on a simulated dataset. We show that the results of the image synthesis algorithm can be improved to better represent the underlying source by applying deconvolution. The subject matter presented in Chapter 6 has not yet been published.

- In Chapter 7, we discuss how to preprocess WIIT measurements and why the preprocessing is necessary. We then provide a method, which we call phase referencing, for registering the optical path difference measurements for each delay line scan in a dataset by using reference sources embedded into the scenes measured by WIIT. The content in Chapter 7 has not yet been published.

- Chapter 8 summarizes the contributions of this thesis and suggests opportunities for future work.
1.5 References


Chapter 2

Wide-Field Spatio-Spectral Interferometry:
Theory and Imaging Properties

2.1 Introduction

The theory and method of wide-field spatio-spectral, or double-Fourier, interferometry is intended for spectroscopically imaging extended astronomical objects with a spatial and spectral resolution that, due to cost, weight, and size limitations, cannot be achieved by any single-aperture telescope. This system design can be thought of as a combination of a Michelson stellar interferometer and a Fourier transform imaging spectrometer. Mathematical models for wide-field double-Fourier interferometric imaging have already been developed by Lyon et al. [1, 2], and by Elias et al. [3], who create a more general model that considers polarization effects. The model herein will assume the scalar field approximation, which will resemble the analysis of Lyon et al. [1, 2]; however, it will address some issues regarding the system’s spectral optical transfer function (SOTF), a term
introduced by Thurman and Fienup for Fizeau Fourier transform imaging spectroscopy [4, 5]. This interpretation will have consequences on image reconstruction and system design. A full derivation of the ideal measurement model is presented in Sect. 2.2 with a discussion of sampling considerations and resolution limits in Sect. 2.3. The image synthesis algorithm is introduced in Sect. 2.4 prior to an examination of the interferometer’s imaging properties and limitations, for which we provide some possible solutions, in Sect. 2.4.4. Concluding remarks and future work are provided in Sect. 2.5. For completeness, connections to coherence theory are provided in Sect. 2.6 (Appendix). Throughout the following derivation, we will consider the case of Fresnel propagation, which will be most pertinent for laboratory experiments but applies to astronomical imaging as well.

2.2 Measurement Model

2.2.1 Source-to-beam-combiner

Before looking into the specifics of the interferometer, we begin by determining the electric field generated by the astronomical sources of interest and propagated to the plane of the interferometer’s entrance pupil, having coordinates $x_m = (x_m, y_m)$. We start by considering only a monochromatic point source located at $\xi = (\xi, \eta)$ with wavelength $\lambda$. Due to the extremely large distance, $z$, between the astronomical source and the interferometer, we define the source position by the paraxial approximation for the direction cosines given by the angle with respect to the axis of the interferometer (line perpendicular to the entrance pupil plane of the interferometer that bisects the interferometer apertures and can also be referred to as the line-of-sight) as

$$\alpha = (\alpha, \beta) = -\frac{(\xi, \eta)}{z} = -\frac{\xi}{z}$$

(2.1)

where the negative sign in the above definition is a matter of sign convention. Note that in a laboratory setting, an effectively large distance, $z$, is attained by placing the source to be imaged at the focus of
collimating optics. In what follows, we will adopt the spectroscopy convention for the scalar wavenumber \( \kappa = 1/\lambda = \nu/c \) such that the transverse wavevector is related to the wavenumber and the direction cosines through \( \kappa = \kappa \alpha \).

Assuming \( E_s(\alpha; \kappa) \) is the field produced by the source at an arbitrary field angle and wavenumber, we can now write the Fresnel approximation, associated with that source field component, for the field incident on the aperture plane of the interferometer at time \( t \) as

\[
E_m(x_m, \alpha; \kappa) = \gamma_1 e^{i2\pi(\zeta - \alpha)} \exp \left( i\pi \frac{\kappa}{z} x_m^2 \right) E_s(\alpha; \kappa) \exp \left( i\pi \kappa \boldsymbol{z} \alpha^2 \right) \exp (i2\pi \kappa x_m \cdot \alpha),
\]

where

\[
\gamma_1 = \frac{iz}{\kappa},
\]

and \( \alpha^2 = |\alpha|^2 \). The time dependence for a monochromatic source is given by a simple time-harmonic relationship, so it is not explicitly included in the list of arguments above. Without loss of generality, we will reserve the integration over all source angles and all wavenumbers until they are needed for computing intensities. We will eventually be concerned with relative intensity values, so the exact value of \( \gamma_1 \) is not very important, but, for completeness, we will continue to redefine \( \gamma_n \) as new constants appear.

Most astronomical objects are incoherent sources, meaning the fields of all the radiators in the object are uncorrelated; this will be important for simplification later on in the derivation. The presence of spatial or spectral correlation (partial coherence) in the source could induce significant effects, such as, but not limited to, spectral shifting or broadening of the field measured by an interferometer [6,
Sect. 5.8]. The additional considerations of partially coherent and coherent sources would require significant analysis, occur rarely in nature, and will not be discussed any further.

The field just after the entrance pupil plane, having transmittance \( A(x_{in}; \kappa) \), is given by

\[
E_{ap}(x_{in}, \alpha; \kappa) = A(x_{in}; \kappa) E_{in}(x_{in}, \alpha; \kappa),
\]  \((2.4)\)

where the phase of \( A(x_{in}; \kappa) \) would be caused by aberrations or other path delays. We assume that the interferometer will have two apertures that are equally displaced about the axis of the interferometer, located in the same plane, and pointed in the same direction. The vector distance between the centers of the apertures is the baseline \( B = (B_x, B_y) \). A simplified diagram of the interferometer is shown in Fig. 2.1 for a fixed baseline. The two fields just after the equally displaced apertures in the entrance plane of the interferometer are given by

\[
E_{1,ap}(x_{in}, \alpha; \kappa) = A_1(x_{in} - \frac{B}{2}; \kappa) E_{in}(x_{in}, \alpha; \kappa),
\]  \((2.5a)\)

\[
E_{2,ap}(x_{in}, \alpha; \kappa) = A_2(x_{in} + \frac{B}{2}; \kappa) E_{in}(x_{in}, \alpha; \kappa),
\]  \((2.5b)\)

where, \( E_{1,ap} \) and \( E_{2,ap} \) are implicitly functions of \( B \) as well as \( t \). In practice, the two light-collecting apertures \( A_1 \) and \( A_2 \), which could be afocal telescopes, are likely to have some differences that would amount to different pupil aberrations, which may be wavenumber dependent, and have possible amplitude variations across each. Such an aperture function can be defined, for example, as

\[
A_n(x_{in}; \kappa) = \begin{cases} a_n(x_{n,ap}; \kappa) e^{i \Phi_n(x_{n,ap}; \kappa)} & : |x_{in}| < D/2 \\ 0 & : |x_{in}| > D/2 \end{cases},
\]  \((2.6)\)

where \( x_{n,ap} \) is the coordinate system centered on the nth aperture, \( a_n \) and \( \Phi_n \) are the nonnegative amplitude and phase across the nth aperture, respectively, and \( \| \) denotes the Euclidean norm of a
Figure 2.1. A simplified diagram of a wide-field spatio-spectral interferometer for a fixed baseline, showing coordinate relationships and basic system configuration.

vector. For the purposes of Eq. (2.6), we assumed circular apertures of diameter $D$. This definition for the aperture function transmits the correct portion of the electric field for each baseline, while preventing $a_n$ and $\Phi_n$ from having baseline dependence. Note that the support of $a_n$ need not be circular as assumed in Eq. (2.6).

From the apertures, the field in one arm of the interferometer is time (path) delayed relative to the field in the other arm before the fields are combined at a beamsplitter such that they are co-aligned and now share the same coordinate system. We denote the time delays induced by the two arms of the interferometer as $\tau_1$ and $\tau_2$. The equivalent path delays are related through $L_n = c\tau_n$. Combining the fields at the interferometer is equivalent to shifting the fields in Eq. (2.5) such that $A_1$ and $A_2$ become collocated within the established coordinate system. For simplicity, we will assume that the coordinate system where the fields are combined is given by $x_{pup} = x_{in}$. We have chosen to relabel the coordinate system at which the fields are combined because it is possible to design a system where $x_{pup}$ is shifted relative to $x_m$, which only affects the following analysis by changing the nominal values of $\tau_1$ and $\tau_2$. 

$$S_s(\alpha, \kappa) = \left| E_s(\alpha; \kappa) \right|^2$$
We combine this information with Eq. (2.5) in order express the fields from arm 1 and arm 2 of the interferometer as

\[
E_{1,\text{pup}}(x_{\text{pup}}, \alpha; \kappa) = E_{1,\text{ap}}(x_{\text{pup}} + \frac{B}{2}, \alpha; \kappa)
\]

\[
= A_1(x_{\text{pup}}; \kappa)E_{\text{in}}(x_{\text{pup}} + \frac{B}{2}, \alpha; \kappa)
\]

\[
= \gamma_1 e^{i\varphi_1(\kappa)} e^{i2\pi z (z-\kappa)} \exp\left[ i\pi \kappa \left(x_{\text{pup}} + \frac{1}{2}B\right)^2 \right] E_s(\alpha; \kappa)
\]

\[
\times A_1(x_{\text{pup}}; \kappa)\exp\left[ i\pi \kappa z^2 \right] \exp\left[ i2\pi \kappa \alpha \left(x_{\text{pup}} + \frac{1}{2}B\right) \right],
\]

\[
E_{2,\text{pup}}(x_{\text{pup}}, \alpha; \kappa) = E_{2,\text{ap}}(x_{\text{pup}} - \frac{B}{2}, \alpha; \kappa)
\]

\[
= \gamma_1 e^{i\varphi_2(\kappa)} e^{i2\pi z (z-\kappa)} \exp\left[ i\pi \kappa \left(x_{\text{pup}} - \frac{1}{2}B\right)^2 \right] E_s(\alpha; \kappa)
\]

\[
\times A_2(x_{\text{pup}}; \kappa)\exp\left[ i\pi \kappa z^2 \right] \exp\left[ i2\pi \kappa \alpha \left(x_{\text{pup}} - \frac{1}{2}B\right) \right],
\]

where \(E_{1,\text{pup}}\) and \(E_{2,\text{pup}}\) are implicitly functions of \(\tau_1\) and \(\tau_2\), and \(\varphi_1(\kappa)\) and \(\varphi_2(\kappa)\) are the additional wavenumber-dependent phases acquired during non-common path propagation from beamsplitters and reflections off mirrors in the system. Assuming that \(\varphi_1(\kappa)\) and \(\varphi_2(\kappa)\) are independent of wavenumber and that mirror reflections in the arms of the interferometer are the only contributors to \(\varphi_1\) and \(\varphi_2\), then \(\Delta \varphi = \varphi_2 - \varphi_1 = m\pi\) for any integer \(m\), depending on the number of reflections. Note that for a non-ideal system, one must pay more attention to the transmission and reflection coefficients of all optical elements in the interferometer, especially for all elements before the beamsplitter.

The above fields are then combined at the beamsplitter, often considered the exit pupil of the interferometer, generating new fields, \(E_3\) and \(E_4\), at the two output ports of the splitter:

\[
E_3 = r_{bs} E_1 + t_{bs} E_2,
\]

\[
E_4 = t_{bs} E_1 + r_{bs} E_2,
\]

(2.8a) (2.8b)
where $r_{bs}$ and $t_{bs}$ are the beamsplitter’s amplitude reflection coefficient and amplitude transmission coefficient, respectively. The above equations are valid in both the pupil and image planes, so we will be more explicit about input variables when $E_3$ and $E_4$ are computed later. For a lossless, symmetric beamsplitter, the phase difference between $r_{bs}$ and $t_{bs}$ is $\pi/2$ [7]. Also, assume a 50/50 beamsplitter such that $|r_{bs}| = |t_{bs}| = 1/\sqrt{2}$. The intensity reflection and transmission coefficients are then $R = |r_{bs}|^2 = 1/2$ and $T = |t_{bs}|^2 = 1/2$. The wide-field extension of double-Fourier interferometry can be obtained by adding an imaging lens after the beam combiner such that if either one of the beams is blocked before beam combination, a panchromatic image of the source is obtained at the detector array. The Appendix discusses connections to coherence theory, which are best formulated just after beam combination. For the time being, the most important result from the Appendix is that the source is assumed to be spatially and spectrally incoherent, resulting in the following relationships [Eqs. (2.55) and (2.56)]:

$$
\langle E_s^* (\alpha; \kappa) E_s (\alpha'; \kappa') \rangle = W_s (\alpha, \alpha'; \kappa) \delta (\kappa - \kappa') \propto \sigma S_s (\alpha; \kappa) \delta (\alpha - \alpha', \kappa - \kappa'),
$$

(2.9)

$$
\left\langle |E_s (\alpha; \kappa)|^2 \right\rangle \propto \sigma S_s (\alpha; \kappa)
$$

(2.10)

where $W_s$ and $S_s$ are the cross-spectral density and spectral density, respectively, of the source; $\sigma = \lambda^2/\pi = (\pi \kappa^2)^{-1}$ is the proportionality constant when the source is incoherent (Lambertian) [8 Eq. 5.5-19]; and $\langle \cdot \rangle$ is the ensemble average, equivalent to time average from start time $t_a$ to end time $t_b$ defined by

$$
\langle \ldots \rangle = \frac{1}{t_b - t_a} \int_{t_a}^{t_b} \ldots dt.
$$

(2.11)
Although there are proportionalities in Eqs. (2.9) and (2.10) that overlook inconsistencies with units, we will take them to be equalities since the scaling factors affect the overall scaling of the spectral density rather than its shape. The delta function in Eq. (2.9) helps to reduce the dimensionality of the integrals related to the interference of the two beams in the following section.

### 2.2.2 Wide-field Image Formation

Instead of propagating the fields $E_3$ and $E_4$ [Eq. (2.8)] just after the beamsplitter to the image plane, it may be easier to consider propagating $E_1$ and $E_2$ [Eq. (2.7)] to the image plane individually, and then generate fields $E_3$ and $E_4$ at the image plane using Eq. (2.8). The image plane coordinates, $x_{img}$, have the same transverse origin (optical axis) as the pupil coordinates, $x_{pup}$, and are defined as

$$\theta = (\theta_x, \theta_y) = \left( \frac{x_{img}, y_{img}}{f} \right) = \frac{x_{img}}{f},$$

(2.12)

where $f$ is the effective focal length of the imaging lens subsequent to beam combination. Any phases acquired after beam combination are common path, will cancel when the intensity of the combined field is taken in the measurement plane, and will not contribute to the interferogram. For this reason we will ignore the quadratic phase term that depends on image plane location and constant-phase terms in the following propagations. From Eq. (2.7),
\[
E_{1,\text{im}}(\theta, \alpha; \kappa) = \frac{\kappa}{if} \int E_{1,\text{pup}}(x_{\text{pup}}, \alpha; \kappa) e^{-i2\pi\kappa \theta \cdot x_{\text{pup}}} d^2x_{\text{pup}} \\
= \gamma_2 e^{i\phi_2(\kappa)} \exp \left( i\pi \frac{K}{4z} B^2 \right) e^{i2\pi\kappa \left[ z - c(t + \tau_1) \right]} E_s(\alpha; \kappa) e^{i\pi\kappa B} \\
\times \int A_1(x_{\text{pup}}; \kappa) \exp \left( i\pi \frac{K}{z} x_{\text{pup}}^2 \right) \exp \left[ -i2\pi\kappa \left( \theta - \alpha - \frac{B}{2z} \right) \cdot x_{\text{pup}} \right] d^2x_{\text{pup}}
\]

\[
E_{2,\text{im}}(\theta, \alpha; \kappa) = \frac{\kappa}{if} \int E_{2,\text{pup}}(x_{\text{pup}}, \alpha; \kappa) e^{-i2\pi\kappa \theta \cdot x_{\text{pup}}} d^2x_{\text{pup}} \\
= \gamma_2 e^{i\phi_2(\kappa)} \exp \left( i\pi \frac{K}{4z} B^2 \right) e^{i2\pi\kappa \left[ z - c(t + \tau_2) \right]} \exp \left( i\pi\kappa z^2 \right) \\
\times h_2 \left[ \left( \theta - \frac{B}{2z} \right) - \alpha; \kappa \right] E_s(\alpha; \kappa) e^{-i\pi\kappa B}
\]

where

\[
h_n(\theta; \kappa) = \int A_n(x_{\text{pup}}; \kappa) \exp \left( i\pi \frac{K}{z} x_{\text{pup}}^2 \right) \exp \left( -i2\pi\kappa \theta \cdot x_{\text{pup}} \right) d^2x_{\text{pup}}
\]
is the coherent impulse response, or amplitude spread function, for the \( n \)th aperture, and

\[
\gamma_2 = \gamma_1 \frac{\kappa}{if} = \frac{iz \cdot \kappa}{if} = \frac{z}{f}
\]

Note that \( \int \ldots d^2x_{\text{pup}} \) is understood to be a 2-D integral. We reserve integration over source angle and wavenumber for when we compute the image intensity, and for simplicity we also assume unit magnification between \( \alpha \) and \( \theta \). Notice that if the baseline length is not very small compared to the source distance, the image location will be a function of the baseline, where the fields from each arm shift in opposing directions. We can ignore this feature for astronomical imaging, but this effect must be considered for laboratory experiments if the source to be measured by the interferometer is not aligned properly to the front focal plane of the collimating optic, which is needed to simulate a
distant source. It is also worth noting that we can ignore the quadratic phase term in Eq. (2.14) if we carefully position the detector to be at the best focus for any given image measurement, but the quadratic phase is negligible for astronomical sources and for laboratory sources that are properly collimated.

The total intensity at the image plane is found by taking the squared magnitude of $E_3$ or $E_4$ and then taking the time average using Eq. (2.11). The total field at the image plane depends on contributions from all wavelengths and field angles (although each location in the image plane is associated with a narrow range of angles). In the rest of the derivation we will use the assumption that the sources are spatially and spectrally incoherent, summarized mathematically by Eq. (2.9) and (2.10). If we block either arm of the interferometer, we are left with the image intensity contributions from each arm independently. Using Eqs. (2.9) – (2.11) and (2.13),

$$I_1(\theta) = \left\langle \int_0^\infty \int_{-1}^1 E_{1,\text{im}}(\theta, \alpha; \kappa) d^2 \alpha d\kappa \right\rangle$$

$$= |\gamma_2|^2 \int_0^\infty \int_{-1}^1 \int_{-1}^1 \int_0^1 h_i^* \left[ \left( \theta - \frac{B}{2z} \right) - \alpha; \kappa \right] h_i \left[ \left( \theta - \frac{B}{2z} \right) - \alpha'; \kappa' \right]$$

$$\times \left\langle E_s^* (\alpha; \kappa) E_s (\alpha'; \kappa') \right\rangle d^2 \alpha' d^2 \kappa' d\kappa$$

$$= |\gamma_3|^2 \int_0^\infty \int_{-1}^1 P_{1,1} \left[ \left( \theta - \frac{B}{2z} \right) - \alpha; \kappa \right] S_s (\alpha; \kappa) d^2 \alpha d\kappa,$$

$$I_2(\theta) = \left\langle \int_0^\infty \int_{-1}^1 E_{2,\text{im}}(\theta, \alpha; \kappa) d^2 \alpha d\kappa \right\rangle$$

$$= |\gamma_3|^2 \int_0^\infty \int_{-1}^1 P_{2,2} \left[ \left( \theta + \frac{B}{2z} \right) - \alpha; \kappa \right] S_s (\alpha; \kappa) d^2 \alpha d\kappa,$$  

$$\gamma_3 = \sigma^{1/2} \gamma_2 \frac{\kappa}{i\if} = \frac{1}{\pi^{1/2} \kappa} \frac{z \kappa}{i\if} = \frac{z}{i\pi^{1/2} \if^2},$$  

where the point spread function (PSF) for the $n^{th}$ arm of the interferometer is given by
Using Eqs. (2.8), (2.9), (2.11), (2.13), (2.16), (2.17), the image intensities for the combined fields for a fixed delay difference $\Delta \tau = \tau_2 - \tau_1$ are then

\[
I_3(\theta) = \left( \int_0^\infty \int_{-1}^1 E_{3,im}(\theta, \alpha; \kappa) d^2 \alpha d\kappa \right)^2
\]

\[
= \left( \int_0^\infty \int_{-1}^1 \left[ n_{bs} E_{1,im}(\theta, \alpha; \kappa) + i_{bs} E_{2,im}(\theta, \alpha; \kappa) \right] d^2 \alpha d\kappa \right)^2
\]

\[
= RI_1(\theta) + TI_2(\theta)
\]

\[
+ 2 \text{Re} \left[ i_{bs}^* i_{bs} \int_0^\infty \int_{-1}^1 \left( E_{1,im}^*(\theta, \alpha; \kappa) E_{2,im}(\theta, \alpha'; \kappa') \right) d^2 \alpha' d^2 \kappa' d\kappa d\kappa' \right]
\]

\[
= \frac{1}{2} \left[ I_1(\theta) + I_2(\theta) \right] + \text{Re} \left[ \frac{i \pi}{2} \int_0^\infty \int_{-1}^1 W_{12}^{im}(\theta; \kappa') \delta(\kappa - \kappa') d\kappa' d\kappa \right]
\]

\[
= \frac{1}{2} \left[ I_1(\theta) + I_2(\theta) \right] - \text{Im} \left[ \int_0^\infty W_{12}^{im}(\theta; \kappa) d\kappa \right],
\]

\[
I_4(\theta) = \left( \int_0^\infty \int_{-1}^1 E_{4,im}(\theta, \alpha; \kappa) d^2 \alpha d\kappa \right)^2
\]

\[
= \frac{1}{2} \left[ I_1(\theta) + I_2(\theta) \right] + \text{Im} \left[ \int_0^\infty W_{12}^{im}(\theta; \kappa) d\kappa \right],
\]

where

\[
W_{12}^{im}(\theta; \kappa) = \int_{-1}^1 \int_{-1}^1 \left( E_{1,im}^*(\theta, \alpha; \kappa) E_{2,im}(\theta, \alpha'; \kappa) \right) d^2 \alpha' d^2 \alpha
\]

\[
= |\gamma_2|^2 \int_{-1}^1 \int_{-1}^1 e^{i \Delta \phi(\kappa)} \exp \left( -i \pi \frac{\kappa}{4z} B^2 \right) e^{-i \pi \kappa z \alpha^2} e^{-i 2 \pi \kappa [z - c(t + \tau)]} \times e^{-i \pi \kappa \alpha B} \exp \left( i \pi \frac{\kappa}{4z} B^2 \right) e^{i \pi \kappa z \alpha^2} e^{i 2 \pi \kappa [z - c(t + \tau)]} e^{-i \pi \kappa \alpha' B}
\]

\[
\times h_1^* \left[ \left( \theta - \frac{B}{2z} + \alpha; \kappa \right) \right] h_2^* \left[ \left( \theta + \frac{B}{2z} - \alpha; \kappa \right) \right]
\]

\[
\times \left( E_s^*(\alpha; \kappa) E_s(\alpha'; \kappa) \right) d^2 \alpha' d^2 \alpha
\]

\[
= |\gamma_3|^2 e^{i \Delta \phi(\kappa)} e^{-i 2 \pi \kappa c \Delta \tau} \int_{-1}^1 h_1^* \left[ \left( \theta - \frac{B}{2z} - \alpha; \kappa \right) \right] h_2 \left[ \left( \theta + \frac{B}{2z} - \alpha; \kappa \right) \right]
\]

\[
\times e^{-i 2 \pi \kappa \alpha B} S_s(\alpha; \kappa) d^2 \alpha.
\]

\[
p_{n,n}(\theta, \kappa) = |h_n(\theta, \kappa)|^2.
\]
is the cross-spectral density between the fields from the interferometer’s entrance apertures after propagation to the interferometer’s image plane. In the above equations we applied the assumptions that \( R = T = 1/2 \) and that the phase difference between \( \eta_{bs} \) and \( \tau_{bs} \) is \( \pi/2 \). We also assume that the field is stationary in the wide sense so that the interference terms, \( \langle E_{1,im}^* E_{2,im} \rangle \) and \( W_{12}^{im} \), do not vary with start time \( t_a \) in Eq. (2.11), but is implicitly a function of \( \Delta \tau \), and that \( S_s(\alpha;\kappa) \) does not vary over the course of data collection. This technique is not well suited for imaging quickly changing sources. Eqs. (2.19a) and (2.19b) can be combined into a single equation for the image intensity:

\[
I_{3,4}(\theta) = \frac{1}{2} \left[ I_1(\theta) + I_2(\theta) \right] + \text{Im} \left[ \int_0^\infty W_{12}^{im}(\theta;\kappa) d\kappa \right].
\]

(2.21)

The coherent impulse responses due to the apertures are now addressed. Eq. (2.13) shows that, in general, the locations of the amplitude spread functions depend on the ratio of the baseline separation to the source distance. For astronomical sources, the source distance is much greater than the baseline length, allowing the baseline dependence to be dropped if the image shift is negligible compared to the interferometer’s finest resolution [see Eq. (2.32)]

\[
\frac{|B|_{\text{max}}}{2\pi} \left( \kappa_{\text{max}} \frac{1}{|B|_{\text{max}}} \right)^{-1} \ll 1,
\]

\[
z \gg \frac{1}{2} \kappa_{\text{max}} |B|_{\text{max}}^2,
\]

(2.22)

where \( \kappa_{\text{max}} \) is the largest measured wavenumber and \( |B|_{\text{max}} \) is the longest baseline length. For experimental setups that must simulate very distant sources using an image at the focus of a collimating mirror, a steering mirror can be used to make sure that the coherent impulse responses overlap for all baseline lengths. For these reasons, the baseline dependence on image location will be ignored for the rest of this paper, and Eqs. (2.16) and (2.20) become
\[ I_1(\theta) = |\gamma_3|^2 \int_0^\infty \int_{-1}^1 p_{1,1}(\theta - \alpha; \kappa)S_s(\alpha; \kappa) d^2\alpha d\kappa, \]  

(2.23a)

\[ I_2(\theta) = |\gamma_3|^2 \int_0^\infty \int_{-1}^1 p_{2,2}(\theta - \alpha; \kappa)S_s(\alpha; \kappa) d^2\alpha d\kappa, \]  

(2.23b)

\[ W_{12}^{im}(\theta; \kappa) = |\gamma_3|^2 e^{i\Delta\phi(\kappa)} e^{-i2\pi\kappa \Delta L} \int_{-1}^1 p_{1,2}(\theta - \alpha; \kappa) e^{-i2\pi\kappa B_s(\alpha; \kappa)} d^2\alpha, \]  

(2.24)

where the optical delay has been converted to units of length using \( \Delta L = c\Delta \tau \) and where the spectral point spread functions (SPSFs) \([4, 5]\) of the apertures are defined as

\[ p_{m,n}(\theta; \kappa) = h_m^*(\theta; \kappa) h_n(\theta; \kappa) \]  

(2.25)

where \( m \) and \( n \) are the indices of the apertures in the interferometer. Inserting Eq. (2.24), Eq. (2.21) becomes

\[ I_{3,4}(\theta) = \frac{1}{2} \left[ I_1(\theta) + I_2(\theta) \right] + |\gamma_3|^2 \int_0^\infty \int_{-1}^1 S_s(\alpha; \kappa) \text{Im} \left[ e^{i\Delta\phi(\kappa)} p_{1,2}(\theta - \alpha; \kappa) e^{-i2\pi\kappa (\alpha B + \Delta L)} \right] d^2\alpha d\kappa. \]  

(2.26)

Analogous to how the van Cittert-Zernike theorem is applicable to source distances that are shorter than allowed by the Fraunhofer approximation, Eq. (2.26) was derived using the more general Fresnel propagations, yet most of the quadratic phase terms have vanished. Only the quadratic phase term of Eq. (2.14) remains, but its impact is negligible for astronomical sources, especially when Eq. (2.22) holds, and can be mitigated in the lab by placing the detector at the best focus of the imaging system.

Eq. (2.26) represents a complicated Fourier relationship. Since the order of integration over source angle and wavenumber can be interchanged for an incoherent source, we will consider integration over source angle first; with this assumption, (i) \( \alpha \) and \( \kappa B \), and (ii) \( \kappa \) and \( \Delta L \) (or \( \nu \) and \( \Delta \tau \)) are revealed to be Fourier conjugate variables. In fact, the name double-Fourier interferometry \([9]\) comes from the idea that we have two Fourier transform relationships, one spatial and one spectral, contributing to each measurement.
To simplify Eq. (2.26) even further, we temporarily assume that both of the interferometer’s collecting apertures are perfectly symmetric and identical, in which case

\[ p_{1,1}(\theta; \kappa) = p_{2,2}(\theta; \kappa) = p_{1,2}(\theta; \kappa), \tag{2.27} \]

and

\[ I_1(\theta) = I_2(\theta), \tag{2.28} \]

and

\[
I_{3,4}(\theta) = I_1(\theta) \mp \gamma_3 \int_0^\infty \int_{-1}^1 p_{1,1}(\theta - \alpha; \kappa) S_\alpha(\kappa) \text{Im} \left[ e^{i \Delta \varphi(\kappa)} e^{-i2\pi\kappa(\alpha B + \Delta L)} \right] d^2\alpha d\kappa
\]

\[ = I_1(\theta) \pm \gamma_3 \int_0^\infty \int_{-1}^1 p_{1,1}(\theta - \alpha; \kappa) S_\alpha(\kappa) \times \sin \left[ 2\pi\kappa(\alpha \cdot B + \Delta L) - \Delta \varphi(\kappa) \right] d^2\alpha d\kappa. \tag{2.29} \]

Due to the assumption of perfect symmetry, the phase of \( p_{1,2} \) above has vanished. In practice, instrumental phases including the phase of \( p_{1,2} \) and \( \Delta \varphi(\kappa) \) can be calibrated if they are known. This simplified model of the measured intensity shows that if we take image measurements for many different delay values at a single baseline, a fringe packet as a function of \( \Delta L \) exists for every image location \( \theta \). The position of the fringe is dependent on both the source location and baseline, while the width and shape of the fringe packet depend on both the source spectral density at the corresponding source location and \( \Delta \varphi(\kappa) \). A simplified measurement model similar to the form shown in Eq. (2.29) has been used to demonstrate how a high resolution hyperspectral image can be reconstructed from a set of these interferometric measurements [1, Eq. (12); 2, Eq. (4)]; however, we will start our derivation of the “dirty” (i.e. direct Fourier inversion) high resolution hyperspectral image directly from the more general Eq. (2.26).
2.3 Measurement Set and Sampling

Before getting into the details of reconstructing a high resolution hyperspectral image, it is instructive to discuss the set of interferometric measurements that are used to generate the high resolution image. Given a fixed baseline and delay length, the interferometer captures a two-dimensional panchromatic image of the astronomical source having the resolution of the individual apertures (assumed to be of the same size). We assume the sampling of the detector is matched to the resolution of a single aperture so that the Nyquist sampling criterion is satisfied. It might also be possible to sub-Nyquist sample the images and obtain super-resolved images relative to the pixel pitch through a post-processing dealiasing technique developed for Fizeau Fourier transform imaging spectroscopy (FTIS) [10]; however, we will continue to assume that all measured intensities are Nyquist sampled according to the diameters of the input apertures. For the sake of simplicity, all measurements will be assumed to be completely discrete in the sense that all parts of the interferometer are stationary for each measurement with fixed baseline and delay values. Keeping the baseline fixed, a set of measurements are taken for various delay lengths resulting in one measurement cube where two dimensions are the spatial dimensions of the array detector and the third dimension is delay length, $\Delta L$, which can be monitored using laser metrology [11]. Due to the Fourier relationship between $\Delta L$ and wavenumber, $\kappa$, sampling theory can be used to determine the desired scan range and sampling of the delay line to match the desired spectral resolution and range of the final reconstructed image:

\[
\text{resolution: } \Delta \kappa_{\text{min}} \approx \frac{1}{\Delta L_{\text{max}} - \Delta L_{\text{min}}} = \frac{1}{2\Delta L_{\text{max}}}, \tag{2.30}
\]

\[
\text{range: } \kappa_{\text{max}} \approx \frac{N_L}{2} \Delta \kappa_{\text{min}} = \frac{N_L}{4\Delta L_{\text{max}}}, \tag{2.31}
\]

where $N_L$ is the number of Nyquist-sampled delay-line samples from $-\Delta L_{\text{max}}$ to $\Delta L_{\text{max}}$. The position of the fringe packet is linearly related to image location, so, in order to measure the fringe for
all field points, the delay line length is assumed to extend equally in both directions about the zero
delay position, meaning $\Delta L_{\text{min}} = -\Delta L_{\text{max}}$. We will return to this idea shortly in order to discuss the
interferometer’s field-of-view (FOV). Conversely, the range of recovered wavenumbers is $\kappa_{\text{max}}$
because only positive values of $\kappa$ are physical. In fact, due to system transmissivity and detector
limitations, the interferometer’s bandpass is less than $\kappa_{\text{max}}$, suggesting $0 < \kappa_{\text{min}} < \kappa_{\text{max}}$, where $\kappa_{\text{min}}$
is the smallest measured wavenumber; knowledge of the bandpass can be exploited, provided the
measurements have an adequate signal-to-noise ratio, allowing the delay line sampling to be relaxed
as for FTIS [12].

The angular resolution of a sparse-aperture interferometer is related to the inverse of the largest
spatial frequency probed by the imaging interferometer, $\kappa_{\text{max}} |B|_{\text{max}}$. Using this idea, the resolution
of an imaging interferometer is

$$\Delta \alpha_{\text{min}} = \left( \kappa_{\text{max}} |B|_{\text{max}} \right)^{-1} = \frac{\lambda_{\text{min}}}{|B|_{\text{max}}} \quad (2.32)$$

Although this equation defines the best possible angular (spatial) resolution of the interferometer, the
overall quality of the reconstructed image at this resolution is also related to baseline sampling as well
as the signal-to-noise ratio of each measurement. We will briefly return to the idea of baseline
sampling in Sect. 2.4.4. It should be noted that images captured at different baseline orientations are
rotated according to the baseline orientation because we assume that, by design, the entire
interferometer, including all optical elements from the input apertures to the detectors, is rotating about
its line-of-sight while the source orientation stays fixed (or equivalently that the source is rotating
while the interferometer orientation stays fixed), and this rotation must be accounted for.

The full FOV of a double-Fourier interferometer could be limited by various aspects of the
system, depending on overall system design, including the full FOV of each individual aperture, the
size of steering mirrors in both arms of interferometer, the number of samples across the array detector, and the length of the delay line. The dependency on $\Delta L_{\text{max}}$ is due to the fact that, in order to accurately reconstruct the spectra of off-axis sources, the center of the fringe, whose position depends on source location and baseline length, must be measured. Specifically, the argument of the sinusoid in Eq. (2.29) includes the term $(\alpha \cdot B + \Delta L)$, from which the zero path difference (ZPD), defined as the center of the fringe packet, is determined to be

$$\Delta L_{\text{ZPD}} = -\alpha \cdot B.$$  \hspace{1cm} (2.33)

If we assume $\Delta \varphi(\kappa)$ is independent of $\kappa$ in Eq. (2.29), then the envelope of the fringe packet is symmetric about $\Delta L_{\text{ZPD}}$, where the envelope is maximal. If we keep the assumption that the fringe packets are symmetric, and assume that $\Delta L_{\text{max}}$ and $|B|_{\text{max}}$ are fixed, then the FOV is limited by

$$\text{FOV} = |\alpha|_{\text{max}} - |\alpha|_{\text{min}} \approx -\frac{\Delta L_{\text{min}}}{|B|_{\text{max}}} + \frac{\Delta L_{\text{max}}}{|B|_{\text{max}}} = \frac{2\Delta L_{\text{max}}}{|B|_{\text{max}}}.$$  \hspace{1cm} (2.34)

If $\Delta \varphi(\kappa)$ is arbitrary, the fringe envelope is not necessarily symmetric, so it is important that $\Delta L_{\text{max}}$ is large enough to measure out to $\pm |\Delta L_{\text{ZPD}}|_{\text{max}} = \pm |\alpha \cdot B|_{\text{max}}$ at the least. In this sense, Eq. (2.34) is a slightly overstated estimate for the full FOV of an arbitrary wide-field double-Fourier interferometer when $\Delta L_{\text{max}}$ and $|B|_{\text{max}}$ are fixed.

### 2.4 Direct Hyperspectral Image Synthesis

#### 2.4.1 Preprocessing of Data

Now that the interferometric measurement set and spatial/spectral resolution limits have been discussed, the algorithm to reconstruct a high-resolution hyperspectral image can be introduced. Let us assume that we have already preregistered all images in the measurement set using a method akin
to phase referencing in optical stellar interferometry [13, 2.2.2], which has been introduced for double-Fourier interferometry by Mariotti and Ridgway [9]. The image reconstruction approach provided herein is based on processing the entire FOV at once, as opposed to existing algorithms that have been demonstrated on a pixel-by-pixel basis [14]. The first few steps involved in reconstructing the “dirty” image cube are very similar to reconstructing spectra in Fourier transform spectroscopy measurements. Reconsider the result from Eq. (2.26) to be a five-dimensional measurement set where two dimensions correspond to image location \( \theta \), two dimensions correspond to the baseline vector \( B \), and one dimension to the delay \( \Delta L \) between arms of the interferometer. Including \( B \) and \( \Delta L \) explicitly in the list of arguments, we have

\[
I_{3,4}(\theta, B; \Delta L) = I_{\text{bias}}(\theta) \mp \sqrt{3} \int_{-1}^{1} S_\alpha(\alpha; \kappa) \times \text{Im} \left[ e^{i\Delta \phi(\kappa)} p_{1,2} (\theta - \alpha; \kappa) e^{-i2\pi \kappa (\alpha B + \Delta L)} \right] d^2 \alpha d\kappa,
\]

(2.35)

where we define

\[
I_{\text{bias}}(\theta) = \frac{1}{2} \left[ I_1(\theta) + I_2(\theta) \right]
\]

(2.36)

which is independent of \( \Delta L \) and \( B \). The limit in Eq. (2.36) applied to the interference term in in Eq. (2.35) approaches a value of zero because the interference term is essentially the sum of many sinusoids of varying amplitude and frequency, and the integral of each sinusoid approaches zero when \( \Delta L_{\text{max}} \to \infty \). The validity of the approximation in Eq. (2.36), which allows the estimation of \( I_{\text{bias}}(\theta) \) directly from the datacube for each baseline, depends on the delay line range and sampling because we want the fluctuations in the interference term to approach a value of zero when averaging over all delay line positions for a single baseline.
Assuming that measurements at all baselines have been rotated and registered to one orientation, a bias-subtracted interferogram can be generated by subtracting off the fringe bias at each pixel:

$$I_{bs}^r(\theta, B; \Delta L) = I_{3,4}(\theta, B; \Delta L) - I_{bias}(\theta)$$

$$= \mp |\gamma_3|^2 \int_0^{t_1} \int_{-1}^1 S_s(\alpha; \kappa) \text{Im} \left[ e^{i \Delta \phi(\kappa)} P_{1,2} (\theta - \alpha; \kappa) e^{-i 2 \pi \kappa (\alpha \cdot B + \Delta L)} \right] d^2 \alpha d\kappa. \tag{2.37}$$

The superscript is meant to convey that the above array consists of strictly real values.

### 2.4.2 Spectral Processing

Although $I_{bs}^r$ is real valued, it is derived from an underlying analytic signal describing the spectral distribution of the source, where only positive wavenumbers are physical. This suggests that the imaginary values associated with the analytic signal can be determined from the real values through a Hilbert transform over the delay line variable, $\Delta L$:

$$I_{bs}^i(\theta, B; \Delta L) = H \left[ I_{bs}(\theta, B; \Delta L) \right]$$

$$= \pm |\gamma_3|^2 \int_0^{t_1} \int_{-1}^1 S_s(\alpha; \kappa) \text{Re} \left[ e^{i \Delta \phi(\kappa)} P_{1,2} (\theta - \alpha; \kappa) e^{-i 2 \pi \kappa (\alpha \cdot B + \Delta L)} \right] d^2 \alpha d\kappa, \tag{2.38}$$

and we can define the complex analytic signal by

$$I_{bs}(\theta, B; \Delta L) = I_{bs}^r(\theta, B; \Delta L) + i I_{bs}^i(\theta, B; \Delta L)$$

$$= \pm i |\gamma_3|^2 \int_0^{t_1} \int_{-1}^1 P_{1,2}(\theta - \alpha; \kappa) S_s(\alpha; \kappa) \exp \left[ i \Delta \phi(\kappa) - i 2 \pi \kappa (\alpha \cdot B + \Delta L) \right] d^2 \alpha d\kappa. \tag{2.39}$$

Notice that when the analytic signal is generated from the measured interferogram, the result looks like the same Fourier transform relationship as in Eq. (2.26). Consequently, the spectrum of each spatial sample in the image of the source for a given baseline can be recovered by taking the inverse Fourier transform of $I_{bs}$ over $\Delta L$:
\[ S_i(\theta, B; \kappa) = \int_{-\Delta L_{\text{max}}}^{\Delta L_{\text{max}}} I_{\text{bs}}(\theta, B; \Delta L) e^{i2\pi \kappa \Delta L} d\Delta L \]

\[ = \pm i \gamma_3 \left[ \int_0^{\Delta L_{\text{max}}} e^{-i2\pi \kappa \Delta L} e^{i2\pi \kappa \Delta L} d\Delta L \right] \int_{-1}^{1} p_{1,2}(\theta - \alpha; \kappa') \]

\[ \times S_s(\alpha; \kappa') \exp \left[ i\Delta \phi(\kappa) - i2\pi \kappa(\alpha \cdot B + \Delta L) \right] d^2 \alpha d\kappa \]

\[ = \pm i \gamma_3 \left[ \int_0^{\Delta L_{\text{max}} \sin \left[ 2\Delta L_{\text{max}}(\kappa - \kappa') \right] \int_{-1}^{1} p_{1,2}(\theta - \alpha; \kappa') \right) \]

\[ \times S_s(\alpha; \kappa') e^{i\Delta \phi(\kappa')} \exp \left[ -i2\pi \kappa' \alpha \cdot B \right] d^2 \alpha d\kappa' \]

\[ = \pm i 2\Delta L_{\text{max}} |\gamma_3|^2 \sin \left( 2\Delta L_{\text{max}} \kappa \right) \times \left[ \int_{-1}^{1} p_{1,2}(\theta - \alpha; \kappa) \right] S_s(\alpha; \kappa) e^{i\Delta \phi(\kappa)} \exp \left[ -i2\pi \kappa \alpha \cdot B \right] d^2 \alpha \]

\[ = \pm i 2\Delta L_{\text{max}} |\gamma_3|^2 \sin \left( 2\Delta L_{\text{max}} \kappa \right) \times \left[ S_s(\theta; \kappa) \exp \left( -i2\pi \kappa \theta \cdot B \right) \right] \theta \int_{-1}^{1} p_{1,2}(\theta; \kappa) e^{i\Delta \phi(\kappa)} \right], \]

where the in-line asterisk \(*\) denotes a convolution in the \( \kappa \) dimension and \( \theta \) represents a 2-D convolution over the dimensions of \( \theta \). The above result is now an array of low-spatial-resolution, spatially-spectrally-filtered hyperspectral images, where the resolution in the spectral domain is limited by the sinc function of width \( (2\Delta L_{\text{max}})^{-1} \) and the spatial resolution is limited by the width of \( p_{1,2} \), which is approximately \( (\kappa D)^{-1} \) for individual pupils contained within a circle of diameter \( D \).

Although at this point in the image processing Eq. (2.40) is a low-spatial-resolution hyperspectral image with spatial sampling related to \( (\kappa_{\text{max}} D)^{-1} \), the following computations assume that we are first able to upsample the spatial components of Eq. (2.40) to a finer spatial sampling related to \( \Delta \alpha_{\text{min}} \) in Eq. (2.32), corresponding to an upsampling factor of \( |B|_{\text{max}} D^{-1} \). The image sampling has now been increased but the spatial resolution of the images remains unchanged.

The linear phase term associated with the spectral density in Eq. (2.40) is what will allow the collection of low-spatial-resolution measurements to eventually become a single high-spatial-
resolution hyperspectral image because it relates the baseline to the spatial frequencies of the source. We can see this relationship between the interferometer baseline and the spatial frequencies of the source spectral density by taking the inverse Fourier transform from $\theta$ to $f_\theta$ of Eq. (2.40):

$$S_i(f_\theta, B; \kappa) = \int_{-1}^{1} S_i(\theta, B; \kappa) \exp(2\pi \theta \cdot f_\theta) d^2\theta$$

$$= \pm i 2\Delta L_{\text{max}} |\gamma_3| \int_{-1}^{1} \int_0^{\infty} \text{sinc} \left[ 2\Delta L_{\text{max}} (\kappa - \kappa') \right]$$

$$\times \left\{ S_s(\theta; \kappa') \exp \left( i 2\pi \kappa' \cdot B \right) \right\}^\theta \left[ p_{1,2}(\theta; \kappa') e^{i \Delta \phi(\kappa')} \right]$$

$$\times \exp \left( i 2\pi \theta \cdot f_\theta \right) d\kappa' d^2\theta$$

$$= \pm i 2\Delta L_{\text{max}} |\gamma_3| \int_{-1}^{1} \int_0^{\infty} \text{sinc} \left[ 2\Delta L_{\text{max}} (\kappa - \kappa') \right]$$

$$\times \left[ \tilde{S}_s(f_\theta - \kappa B; \kappa') \tilde{p}_{1,2}(f_\theta; \kappa') e^{i \Delta \phi(\kappa')} \right] d\kappa'$$

$$= \pm i 2\Delta L_{\text{max}} |\gamma_3| \text{sinc} \left( 2\Delta L_{\text{max}} \kappa \right)$$

$$\times \left[ \tilde{S}_s(f_\theta - \kappa B; \kappa) \tilde{p}_{1,2}(f_\theta; \kappa) e^{i \Delta \phi(\kappa')} \right],$$

where

$$\tilde{p}_{1,2}(f_\theta; \kappa) = \int_{-\infty}^{\infty} p_{1,2}(\theta; \kappa) \exp(2\pi \theta \cdot f_\theta) d^2\theta$$

is the non-normalized SOTF of the system, and a tilde over a variable indicates a continuous Fourier transform from $\theta$ to $f_\theta$. Eq. (2.41) shows that the measurement cube for each baseline probes a different range of spatial frequencies, centered about $f_\theta = \kappa B$, of the source spectral density. The information content of a measurement cube for each baseline can be visualized as truncated oblique cones in a volume defined by $f_\theta$ and $\kappa$, as seen in Fig 2.2.

### 2.4.3 Image Processing and Effective Transfer Function

From Eq. (2.40) it is evident that there is another Fourier transform relationship between $\alpha$ (or $\theta$) and $\kappa B$ that will provide a means of recovering a high-spatial-resolution hyperspectral image where the spatial resolution for the largest wavenumber is given by Eq. (2.32). This relationship,
Figure 2.2. An illustration of the information content probed by the datacube associated with each baseline vector. Each datacube maps out a truncated oblique cone in the spectral-spatial frequency domain whose location depends on the particular value of the baseline vector.

as well as the imaging properties of the system, is best understood through the effective transfer function of the interferometer system. Taking an inverse discrete Fourier transform (DFT) over the \( N_b \) baselines and exploiting the Fourier transform relationship between \( \theta \) and \( \kappa B \), we obtain the “dirty” high-resolution hyperspectral image:

\[
S_1^{hr}(\theta; \kappa) = \sum_{n=1}^{N_b} S_i(\theta, B_n; \kappa) e^{i2\pi \theta B_n} = \pm i2\Delta L_{\text{max}} \left[ \gamma_3 \sum_{n=1}^{N_b} \left( \frac{\kappa - \kappa'}{2\Delta L_{\text{max}}} \right) \right]
\]

\[
\times \int_{-1}^{1} S_s(\alpha; \kappa') p_{1,2}(\theta - \alpha; \kappa') e^{i\Delta \phi(\kappa')} \exp\left[ i2\pi \left( \kappa \theta - \kappa' \theta \right) \cdot B_n \right] d^2 \alpha d\kappa'.
\] (2.43)

Substituting \( \kappa \theta - \kappa' \alpha = \kappa' (\theta - \alpha) + (\kappa - \kappa') \theta \), we can rewrite Eq. (2.43) in the form of two convolutions:
The linear phase term associated with the sinc wavenumber convolution kernel suggests that the delay line sampling for each baseline and point in the FOV should be centered about its associated $\Delta L_{ZPD}$. In practice this means that $\Delta L_{max}$ must be large enough to capture $\pm |\Delta L_{ZPD}|_{max}$ as mentioned near the end of Sect. 2.3; fortunately, the $\Delta L_{max}$ needed to obtain the desired wavenumber resolution is typically large enough to satisfy this condition when compared to the largest baseline length and FOV that would be considered for infrared astronomical imaging interferometry. The other linear phase term associated with the SPSF $p_{1,2}(\theta; \kappa')$ is instead related to both the spatial frequency content associated with the baseline and the effective transfer function of the interferometer.

In order to obtain the effective transfer function, we compute the 2-D spatial (angular) inverse Fourier transform of Eq. (2.44) using $\theta$ and $f_\phi$ as conjugate variables and employ the convolution theorem:
\( \tilde{S}_i^{hr}(f_{\theta};\kappa) = \int_1^{l_1} S_i^{hr}(\theta;\kappa) \exp(i2\pi\theta \cdot f_{\theta}) d^2\theta \)

\[ = \pm i2\Delta L_{\text{max}} \gamma_3^2 \int_1^{l_1} \sum_{n=1}^{N_{b}} \int_0^{\infty} \text{sinc} \left[ 2\Delta L_{\text{max}}(\kappa - \kappa') \right] \exp \left[ i2\pi (\kappa - \kappa') \theta \cdot B_n \right] \]

\[ \times \left\{ S_s(\theta;\kappa) \ast \left[ p_{1,2}(\theta;\kappa) e^{i\Delta \varphi(\kappa)} \exp(i2\pi\kappa\theta \cdot B_n) \right] \right\} \exp(i2\pi\theta \cdot f_{\theta}) d\kappa' d^2\theta \] \tag{2.45}

\[ = \pm i2\Delta L_{\text{max}} \gamma_3^2 \int_0^{\infty} \text{sinc} \left[ 2\Delta L_{\text{max}}(\kappa - \kappa') \right] \]

\[ \times \sum_{n=1}^{N_{b}} \left\{ \tilde{S}_s \left[ f_{\theta} + (\kappa - \kappa') B_n; \kappa' \right] \tilde{p}_{1,2} \left( f_{\theta} + \kappa B_n; \kappa' \right) \right\} d\kappa'. \]

The effective OTF of a double-Fourier interferometer is the summation of shifted SOTFs, where the shift, \( \kappa B_n \), is equal to the product of the baseline and wavenumber. This is most easily observed if we assume that the sinc convolution term in Eq. (2.45) is narrow enough to be approximated by a delta function, resulting in the following effective transfer function:

\[ \text{OTF}_{\text{eff}}(f_{\theta};\kappa) \approx \sum_{n=1}^{N_{b}} \tilde{p}_{1,2} \left( f_{\theta} + \kappa B_n; \kappa \right). \] \tag{2.46}

We will return to the consequences of the above equation in Sect. 2.4.4 to discuss imaging properties of the interferometer.

We can now relate Eq. (2.41) to Eq. (2.45) through

\[ \tilde{S}_i^{hr}(f_{\theta};\kappa) = \sum_{n=1}^{N_{b}} \tilde{S}_i \left( f_{\theta} + \kappa B_n, B_n; \kappa \right), \] \tag{2.47}

which allows us to combine Eqs. (2.41) and (2.45) – (2.47) to create a new procedure for computing Eq. (2.44) from Eq. (2.40). We start by computing \( \tilde{S}_i \left( f_{\theta}, B_n; \kappa \right) \), Eq. (2.41), for all baselines \( B_n \). We then apply Eq. (2.47) by shifting all \( \tilde{S}_i \left( f_{\theta}, B_n; \kappa \right) \) by \( \kappa B_n \) before summing the results. Finally, we obtain the ultimate “dirty” high resolution spectral image \( S_i^{hr}(\theta;\kappa) \) by taking the 2-D Fourier transform of \( \tilde{S}_i^{hr}(f_{\theta};\kappa) \) from \( f_{\theta} \) to \( \theta \). This procedure could possibly provide computational speed
advantages if we consider that we can combine spatial upsampling with the computation of Eq. (2.41) instead of first upsampling Eq. (2.40) before the multiplications and summation of Eq. (2.43).

Note that the spectral density of the object is real valued, so its Fourier transform exhibits Hermitian symmetry, \( \hat{S}_{i}^{hr}(-\mathbf{f}_{\theta}; \kappa) = \hat{S}_{i}^{hr}(\mathbf{f}_{\theta}; \kappa) \), halving the number of spatial frequencies that must be measured. This just means that baselines \( \mathbf{B}_{n} \) and \( -\mathbf{B}_{n} \) provide redundant information.

If we replace the “dirty” high-resolution hyperspectral image of Eq. (2.43) with a guess for the hyperspectral object being measured, the procedure for obtaining the “dirty” hyperspectral image can be reversed in order to simulate double-Fourier interferometric measurements in either the form of Eq. (2.26) or the bias-subtracted form of Eq. (2.37). Those simulations can be incorporated into a nonlinear optimization algorithm to recover the high-resolution hyperspectral image from a measurement set, allowing for the inclusion of regularization metrics to enforce prior knowledge of the object being measured. An inverse approach to image reconstruction has advantages that would mitigate issues associated with instrumental effects and inadequate sampling, which will be discussed further in Sect. 2.4.4. Simulations, such as those generated by the Far-Infrared Interferometer Instrument Simulator [14], can be made to include many different instrumental effects, including, but not limited to, telescope pointing errors, background or detector noise, and thermal effects. Generating simulated measurements with instrumental effects, such as telescope pointing errors, provides a means for understanding the impact of instrumental errors on the quality of the hyperspectral image obtained after reconstruction [14].
2.4.4 Imaging Properties

The effective transfer function of the interferometer not only provides insight into the image reconstruction procedure but also into the system’s imaging properties, including the final image quality at each sampled wavenumber. Notice that the OTF in Eq. (2.46) is dependent on baseline sampling. This dependency indicates that different baseline separations probe different spatial frequencies in the source, and the effective OTF is then the sum of shifted versions of the SOTFs. Fig. 2.2 illustrates this idea by showing that the information content of a measurement at a single baseline maps out a volume in the spectral-spatial frequency domain that is a truncated oblique cone having a vertex at the (0,0;0) coordinate, and that each baseline vector corresponds to different portions of the object’s spectral-spatial frequencies. For this reason, baseline sampling is chosen, time permitting, such that the volume describing the object’s spectral-spatial frequencies is populated without gaps. Furthermore, instead of simply summing components of the OTF, one would choose to weight them to arrive at a uniformly weighted effective transfer function. However, there are some spatial frequencies to which we do not have access without additional hardware.

Because it is physically impossible for the two apertures in a double-Fourier interferometer to coincide ($B_n \neq 0$), the effective OTF must necessarily vanish at and around the DC spatial frequency, meaning that the lowest spatial frequencies are not measured, depicted in Fig. 2.2 by the empty volume traced in red. Without the low spatial frequency content, the recovered high-resolution images at each wavenumber will be zero-mean. However, conventional Fourier transform spectroscopy (FTS) is a means of recovering some of the missing information [9]. This could be achieved with an additional beamsplitter, indicated by a thin line, and a mechanism to alter the interferometer configuration such that wide-field FTS is performed with a single aperture, as shown in Fig. 2.3, in addition to the double-Fourier measurements. If such increased system complexity is not an option, then we must find a way
Figure 2.3. A simplified diagram of a wide-field spatio-spectral interferometer with an additional beam-splitter and mirror to demonstrate that the same system can be used for conventional Fourier transform imaging spectroscopy.

to estimate the low spatial frequencies that are not measured. A simple solution to this problem is to use the fringe bias from Eq. (2.36) to replace the low spatial frequencies. This is only an estimate because the fringe bias is a panchromatic image of the source, and this is equivalent to approximating the spectral density to be independent of wavenumber: $S_s(\alpha, \kappa) \approx \psi g(\alpha)$. A more appropriate assumption would be a gray-world approximation for the spectral density, $S_s(\alpha, \kappa) \approx \psi(\kappa)g(\alpha)$, which can be used to recover a more accurate estimate for the low spatial frequencies of the spectral image through the use of reconstruction algorithms [5]. The result of the algorithm developed for Fizeau Fourier transform imaging spectroscopy not only yields a gray-world estimate for low spatial frequencies, but also provides a method for Wiener filtering the measurements using the gray-world estimate [5]. This technique could also be applied to double-Fourier measurements, and the result could even be refined by incorporating Eqs. (2.44) or (2.45) into a nonlinear optimization algorithm, where the Wiener filtered result would provide a starting guess to the algorithm. This approach would allow for additional regularization in the presence of noise and missing, or redundant, spatial frequencies.
Aside from missing spatial frequencies at and around the DC spatial frequency, there could be other missing spatial frequencies if the angular or radial collection of baselines is insufficient. Such sparsity in the spatial frequency domain will cause artifacts in the image domain that can be mitigated with regularization parameters, such as nonnegativity and L1-minimization, within the image reconstruction procedure. As mentioned above, the quality of the “dirty” image can be improved by appropriately weighting the spatial frequency content of $\tilde{S}^{hr}_1(f_\theta; \kappa)$ provided that there are no gaps in spatial frequency coverage. Another option, which is particularly important in the presence of missing spatial frequencies, is to use deconvolution techniques that have been developed for imaging interferometry such as CLEAN [15] or maximum entropy methods [16]. Some newer deconvolution methods are discussed by Thiébaut and Young [17] and have been tested in SPIE’s interferometric imaging beauty contest [18].

Also, the system’s effective OTF has a complicated geometry, related to that for Fizeau Fourier transform imaging spectroscopy [4, 5], where the measured spatial frequencies scale with both baseline and wavelength, meaning that the spatial frequency content of each baseline measurement is slightly different for all wavelengths in the hyperspectral image. The effect of this Fourier geometry on image quality requires further investigation, but it is likely that similar image reconstruction and regularization techniques will be useful to mitigate artifacts in the resulting image.

### 2.5 Conclusion

A complete derivation of the measurement model for wide-field spatio-spectral interferometry based on Fresnel propagations and a two-aperture interferometer has been presented. This led to a generalization of the van Cittert-Zernike theorem that relates the spectral density of an incoherent source to the irradiance measured by the interferometer. We discussed delay line sampling in the
context of desired spectral resolution, as well as the impact of the delay line scan range on the spatial
FOV of the interferometer. We also provided an estimate of the spatial (angular) resolution of an
imaging interferometer based on the largest measured baseline length and wavenumber, but actual
image quality and resolution are also dependent on baseline sampling and noise. A method for
recovering a high resolution hyperspectral image from a noiseless measurement set was derived. We
made connections between baseline sampling and spectral OTF (SOTF) coverage, and how they are
related to imaging properties of the system through the interferometer’s effective transfer function.
Possible methods for overcoming the issue of missing low spatial frequencies, as well mitigating
artifacts due to irregular baseline sampling, were proposed and based on existing algorithms for
imaging interferometry.

Chapter 6 demonstrates the image reconstruction algorithm presented herein on simulated data.
Chapter 6 also briefly examines how the lack of low spatial frequencies affects image synthesis and
describes an attempt to recover low spatial frequencies without conventional wide-field FTS
measurements. Future work will be necessary to investigate the use of a regularized nonlinear
optimization algorithm to recover an improved high-resolution hyperspectral image.

2.6 Appendix

2.6.1 Relation to Coherence Theory

We begin by determining the intensities for each beam that would be measured if a large array
doctor were placed just before the beamsplitter. The intensities of the individual beams before the
beamsplitter are found by taking the squared magnitude of Eq. (2.7) and applying Eq. (2.11),
accounting for contributions from all field angles and wavelengths:
\[ I_1(x_{\text{pup}}) = \left| \int_0^\infty \int_{-1}^1 E_{1,\text{pup}}(x_{\text{pup}}, \alpha; \kappa) \, d^2 \alpha \, d\kappa \right|^2 \]  
\tag{2.48a}

\[ = |\gamma_1|^2 \int_0^\infty \int_{-1}^1 |A_1(x_{\text{pup}}; \kappa)|^2 \int_{-1}^1 \left| E_s(\alpha; \kappa) \right|^2 \, d^2 \alpha \, d\kappa, \]

\[ I_2(x_{\text{pup}}) = |\gamma_1|^2 \int_0^\infty \int_{-1}^1 |A_2(x_{\text{pup}}; \kappa)|^2 \int_{-1}^1 \left| E_s(\alpha; \kappa) \right|^2 \, d^2 \alpha \, d\kappa. \]  
\tag{2.48b}

Without considering all field angles and wavelengths simultaneously, we would be implying assumptions about the coherence properties of the source field. We can, however, change the order of integration over time, angles, and wavenumber without loss of generality. Using Eqs. (2.7), (2.8), (2.11) and (2.48), the measured intensities after the beamsplitter for fixed delay \( \Delta \tau \) are

\[ I_3(x_{\text{pup}}) = \left| \int_0^\infty \int_{-1}^1 E_{3,\text{pup}}(x_{\text{pup}}, \alpha; \kappa) \, d^2 \alpha \, d\kappa \right|^2 \]  

\[ = \left\langle \int_0^\infty \int_{-1}^1 \left[ \eta_{bs} E_{1,\text{pup}}(x_{\text{pup}}, \alpha; \kappa) + \eta_{bs} E_{2,\text{pup}}(x_{\text{pup}}, \alpha; \kappa) \right] \, d^2 \alpha \, d\kappa \right|^2 \]  

\[ = RI_1(x_{\text{pup}}) + TI_2(x_{\text{pup}}) + 2 \Re \left[ \eta_{bs}^* \eta_{bs} \int_0^\infty \int_{-1}^1 \int_{-1}^1 \left( E_{1,\text{pup}}(x_{\text{pup}}, \alpha; \kappa) E_{2,\text{pup}}(x_{\text{pup}}, \alpha'; \kappa') \right) \, d^2 \alpha' \, d^2 \alpha \, d\kappa \, d\kappa' \right] \]  

\[ = \frac{1}{2} \left[ I_1(x_{\text{pup}}) + I_2(x_{\text{pup}}) \right] \]  

\[ - \Im \left[ \int_0^\infty \int_{-1}^1 \int_{-1}^1 \left( E_{1,\text{pup}}^*(x_{\text{pup}}, \alpha; \kappa) E_{2,\text{pup}}^*(x_{\text{pup}}, \alpha'; \kappa') \right) \, d^2 \alpha' \, d^2 \alpha \, d\kappa \, d\kappa' \right], \]  
\tag{2.49a}

and similarly,

\[ I_4(x_{\text{pup}}) = \frac{1}{2} \left[ I_1(x_{\text{pup}}) + I_2(x_{\text{pup}}) \right] \]  

\[ + \Im \left[ \int_0^\infty \int_{-1}^1 \int_{-1}^1 \left( E_{1,\text{pup}}^*(x_{\text{pup}}, \alpha; \kappa) E_{2,\text{pup}}^*(x_{\text{pup}}, \alpha'; \kappa') \right) \, d^2 \alpha' \, d^2 \alpha \, d\kappa \, d\kappa' \right], \]  
\tag{2.49b}

where the intensities from the output arms of the beamsplitter, \( I_{3,4} \), are implicitly functions of \( \tau_1, \tau_2 \) and \( B \). In the above equations we applied the assumptions that \( R = T = 1/2 \) and that the phase difference between \( \eta_{bs} \) and \( \tau_{bs} \) is \( \pi/2 \). We also assume that the field is stationary in the wide sense,
so that the interference term \( \langle E_{1,\text{pup}}^* E_{2,\text{pup}} \rangle \) does not vary with integration start time \( t_a \). Eqs. (2.49a) and (2.49b) can be combined into a single equation for the pupil intensity:

\[
I_{3,4}(x_{\text{pup}}) = \frac{1}{2} \left[ I_1(x_{\text{pup}}) + I_2(x_{\text{pup}}) \right] + \text{Im} \left[ \int_{-\infty}^{\infty} \int_{0}^{\infty} \int_{-1}^{1} \langle E_{1,\text{pup}}^* (x_{\text{pup}}, \alpha; \kappa) E_{2,\text{pup}}(x_{\text{pup}}, \alpha' \kappa') \rangle d^2 \alpha d^2 \alpha' d\kappa d\kappa' \right].
\] (2.50)

In order to simplify the above equation any further, special attention must be paid to the integration of the cross term, which gives rise to an interference pattern that varies with induced time delay difference \( \Delta \tau = \tau_2 - \tau_1 \). Assume that \( \Delta \tau \) and \( B \) are fixed during a single measurement; then

\[
\langle E_{1,\text{pup}}^* E_{2,\text{pup}} \rangle \text{ is a measure of the cross-spectral density, } W_{12}(\kappa), \text{ requiring that } [6, \text{Sect. 4.3.2]}
\]

\[
\langle E_{1,\text{pup}}^* (\kappa) E_{2,\text{pup}}(\kappa') \rangle = W_{12}(\kappa) \delta(\kappa - \kappa').
\] (2.51)

From the sifting property of the delta function, Eq. (2.50) simplifies to

\[
I_{3,4}(x_{\text{pup}}) = \frac{1}{2} \left[ I_1(x_{\text{pup}}) + I_2(x_{\text{pup}}) \right] + \text{Im} \left[ \int_{0}^{\infty} W_{12}^\text{pup}(x_{\text{pup}}, \kappa) d\kappa \right],
\] (2.52)

where

\[
W_{12}^\text{pup}(x_{\text{pup}}, \kappa) = \int_{-1}^{1} \int_{-1}^{1} \langle E_{1,\text{pup}}^*(x_{\text{pup}}, \alpha; \kappa) E_{2,\text{pup}}(x_{\text{pup}}, \alpha' \kappa') \rangle d^2 \alpha d^2 \alpha'
\] (2.53)

is a measure of cross-spectral density in the interferometer’s exit pupil. Combining Eqs. (2.7) and (2.53), we have
\[ W_{12}^{\text{pup}}(x_{\text{pup}}, \kappa) = |\gamma_1|^2 \int_{-1}^{1} \int_{-1}^{1} e^{i \Delta \phi(\kappa)} e^{-i 2 \pi \kappa \left[ z-c(t+t_1) \right]} \exp \left[ -i \pi \frac{\kappa}{z} \left( x_{\text{pup}} + \frac{1}{2} B \right)^2 \right] \]
\[ \times E_s^* (\alpha; \kappa) A_1^* (x_{\text{pup}}; \kappa) e^{-i \pi \kappa \alpha^2} \exp \left[ -i 2 \pi \kappa \alpha \cdot \left( x_{\text{pup}} + \frac{1}{2} B \right) \right] \]
\[ \times e^{i 2 \pi \kappa \left[ z-c(t+t_2) \right]} \exp \left[ i \pi \frac{\kappa}{z} \left( x_{\text{pup}} - \frac{1}{2} B \right)^2 \right] E_s (\alpha'; \kappa) \]
\[ \times A_2 (x_{\text{pup}}; \kappa) e^{i \pi \kappa \alpha'^2} \exp \left[ i 2 \pi \kappa \alpha' \cdot \left( x_{\text{pup}} - \frac{1}{2} B \right) \right] \]
\[ = |\gamma_1|^2 \int_{-1}^{1} \int_{-1}^{1} e^{i \Delta \phi(\kappa)} e^{-i 2 \pi \kappa \Delta \tau} \exp \left( -i 2 \pi \frac{\kappa}{z} x_{\text{pup}} \cdot B \right) \]
\[ \times \left( E_s^* (\alpha; \kappa) E_s (\alpha'; \kappa') \right) A_1^* (x_{\text{pup}}; \kappa) A_2 (x_{\text{pup}}; \kappa) \exp \left[ i \pi \kappa \left( \alpha'^2 - \alpha^2 \right) \right] \]
\[ \times \exp \left[ -i \pi \kappa \left( (\alpha + \alpha') \cdot B - 2 (\alpha' - \alpha) \cdot x_{\text{pup}} \right) \right] d^2 \alpha' d^2 \alpha. \]  

We can simplify further by recalling, again, that the astronomical source is assumed to be spatially incoherent, which is mathematically expressed as

\[ \left\langle E_s^* (\alpha; \kappa) E_s (\alpha'; \kappa') \right\rangle = W_s (\alpha, \alpha'; \kappa) \delta (\kappa - \kappa') \]
\[ \propto \sigma S_s (\alpha; \kappa) \delta (\alpha - \alpha', \kappa - \kappa'). \]  

Similarly, the spectral density for an incoherent source obeys

\[ \left\langle |E_s (\alpha; \kappa)|^2 \right\rangle \propto \sigma S_s (\alpha; \kappa) \]  

Using Eqs. (2.55) and (2.56), we can simplify Eqs. (2.48) and (2.54) further, integrating over \( \alpha' \) when needed:

\[ I_1 (x_{\text{pup}}) = |\gamma_4|^2 \int_0^{\infty} \left| A_1 (x_{\text{pup}}; \kappa) \right|^2 \int_{-1}^{1} S_s (\alpha; \kappa) d^2 \alpha d\kappa, \]  

\[ I_2 (x_{\text{pup}}) = |\gamma_4|^2 \int_0^{\infty} \left| A_2 (x_{\text{pup}}; \kappa) \right|^2 \int_{-1}^{1} S_s (\alpha; \kappa) d^2 \alpha d\kappa, \]  

and
\[ W_{12}^{\text{pup}}(x_{\text{pup}}, \kappa) = |\gamma_4|^2 \int_{-1}^{1} e^{i \Delta \varphi(\kappa)} e^{-i 2 \pi \kappa \Delta \tau} A_1^*(x_{\text{pup}}; \kappa) A_2(x_{\text{pup}}; \kappa) \]
\[ \times \exp \left( -i 2 \pi \frac{\kappa}{z} x_{\text{pup}} \cdot B \right) W_{12}^{\text{in}}(\alpha; \kappa) d^2 \alpha. \]  

(2.58)

where

\[ W_{12}^{\text{in}}(\alpha; \kappa) = S_s(\alpha; \kappa) e^{-i 2 \pi \alpha B}, \]  

(2.59)

and

\[ \gamma_4 = \gamma_1 \sigma^{1/2} \]
\[ = \frac{i z}{\pi^{1/2} \kappa^2}. \]  

(2.60)

\[ W_{12}^{\text{in}}(\alpha; \kappa) \] is the cross-spectral density of the field in the entrance pupil plane of the interferometer due to a point object at \( \alpha \). Eq. (2.59) has a special functional form arising from the far-field propagation of an incoherent source that shows the cross-spectral purity of the field in the pupil plane. Namely, the fields at each aperture have identical spectral densities and the combined field from both apertures has the same spectral density as the individual fields [6, Sect. 4.5.1; 8, Sect. 5.3.2; 19].

Eq. (2.53) can only be related to the mutual coherence function,

\[ \Gamma(-B/2, B/2, \Delta \tau) = \Gamma(B, \Delta \tau) = \Gamma_{12} (\Delta \tau), \] in the pupil plane if \( \Delta \varphi \), \( A_1 \) and \( A_2 \) are all independent of wavenumber, including phase aberrations, which is an idealized assumption that depends on characteristics of the interferometer [6, Sect. 4.3.2; 8, Sect. 5.2.5]. In general, the mutual coherence function and the cross-spectral density are related through the Wiener-Khintchine theorem [6, Sect. 4.3.2; 8, Sect. 3.4.2], which becomes apparent after integration over all \( \kappa \):

\[ \Gamma_{12} (\tau) = \int_0^\infty W_{12} (\nu) e^{-i 2 \pi \nu \tau} d\nu \]
\[ = \int_0^\infty W_{12} (\kappa) e^{-i 2 \pi \kappa \tau} d\kappa. \]  

(2.61)

We will now incorporate the preceding assumptions for the time being for the sole purpose of making a clear connection between the intensity in the exit pupil of the interferometer and the mutual coherence function through application of the Wiener-Khintchine theorem:
\[ I_{3,4}(x_{\text{pup}}) = \frac{1}{2} I_1(x_{\text{pup}}) + I_2(x_{\text{pup}}) + c^{-1/4} \left| \gamma_4 \right|^2 \text{Im} \left\{ e^{i\Delta \phi} A_1^*(x_{\text{pup}}) A_2(x_{\text{pup}}) \right\} \]
\[ \times \int_{-1}^{1} c \int_0^\infty \exp \left\{ -i 2 \pi c \kappa \left( \Delta \tau + \frac{x_{\text{pup}} B}{c z} \right) \right\} W_{12}^{\text{in}}(\alpha, \kappa) d\kappa d^2 \alpha \]

(2.62)

where \( \Gamma_{12}^{\text{in}}(\alpha; \Delta \tau) \) is the mutual coherence in the entrance pupil of the interferometer for each source angle and position in the pupil. The above equation gives insight into the physical quantities being probed by double-Fourier interferometry. In the most general sense, this technique makes measurements of both spatial and temporal coherence in the entrance pupil through the mutual coherence function. This is ultimately a generalization of the van Cittert-Zernike theorem that exploits the propagation of mutual intensity from an incoherent source; instead, double-Fourier interferometry exploits the propagation of the more general mutual coherence function. Although we had assumed that \( \Delta \phi, A_1 \) and \( A_2 \) were independent of wavelength, we could have relaxed this assumption and the intensity in the exit pupil of the interferometer would still be related to the mutual coherence function in the interferometer’s entrance pupil with additional convolution kernels that depend on the path difference \( \Delta \tau \). The convolutions imparted by the spectral variations in the complex-valued aperture functions can be calibrated out of the measurements during post processing if they are measured or known \textit{a priori}. Analogous to the van Cittert-Zernike theorem, the intensity in the interferometer’s exit pupil can also be related directly to the source spectral density. For this purpose, we return to the more general case where spectral variations exist and combine Eqs. (2.52), (2.58), and (2.59):
\[ I_{3,4}(x_{\text{pup}}) = \frac{1}{2} \left[ I_1(x_{\text{pup}}) + I_2(x_{\text{pup}}) \right] + |\gamma_4|^2 \text{Im} \left\{ \int_{-1}^{1} \int_{0}^{\infty} e^{i\Delta \varphi(\kappa)} A_1^*(x_{\text{pup}}, \kappa) A_2(x_{\text{pup}}, \kappa) \right. \\
\left. \times \exp \left[ -i2\pi c\kappa \left( \Delta \tau + \frac{B}{cz} + \frac{\alpha \cdot B}{c} \right) \right] S_s(\alpha; \kappa) d\kappa d^2\alpha \right\}. \tag{2.63} \]

The linear phase term proportional to \( x_{\text{pup}} \cdot B \) can be dropped for astronomical distances, as well as for meticulously designed lab experiments, as long as Eq. (2.22) is satisfied. The remaining phase terms proportional to \( \kappa \Delta \tau \) and \( \kappa \alpha \cdot B \) are the necessary terms required for conventional FTS and Michelson stellar interferometry, respectively. Conventional spatio-spectral interferometry, as discussed by Mariotti and Ridgway [9, Eq. (6)] for example, would stop here and place a large bucket detector after the beamsplitter. One would typically simplify Eq. (2.63) even more by making further idealizations (identical apertures and aberrations, etc.) about the system in order to get better insight into how different source spectral densities will be manifested in the measurements. Such a measurement regime provides a limited FOV and has demanding baseline sampling requirements in order to form a hyperspectral image from the measurements. Instead, a wide FOV extension to spatio-spectral interferometry has been developed as described in [1–3] Sect. 2.2.2.
2.7 References


Chapter 3

Phase Retrieval for the Wide-Field Imaging Interferometry Testbed

3.1 Introduction

The purpose of the work in this chapter is to leverage knowledge about the WIIT system in order to improve the quality of WIIT simulations and WIIT dataset processing. The whole WIIT system, including the light source CHIP, were introduced in Chapter 1. In order to synthesize or simulate WIIT datasets, it is crucial to have knowledge of the optical system’s aberrations and related point spread function (PSF) as a function of wavelength. We used CHIP and WIIT to perform an experiment to recover aberrations from the system’s PSF.

Because WIIT’s optical components that are non-common path are well-corrected flat mirrors with known surface profiles, we suspect that most of the system’s aberrations are caused by an imaging lens after beam combination or by optical components before the entrance aperture of the
interferometer. Knowing that the imaging lens is comprised of three achromatic doublets, it is likely that the system has axial chromatic aberrations that vary quadratically with wavelength. The isoplanatic patch size of the system is assumed to be at least as large as the full field-of-view of the image, which is supported in part by the fact that the imaging camera has a large f-number of about 80. For simplicity, we assume that the beams from both apertures overlap perfectly at the beam splitter and that the combined aperture is uniformly illuminated. Because the non-common path aberrations are small compared to the rest of the system’s aberrations, we assume that the aberrations contributing to WIIT images derive from one underlying wavefront, modeled as having wavelength independent aberrations in addition to axial chromatic aberration that varies quadratically with wavelength. We describe the measurements used to retrieve the wavefront through modern phase retrieval techniques [1, 2] in Sect. 3.2 and provide the results in Sect. 3.3.

### 3.2 Phase Retrieval Measurements and Wavefront Model

The nonzero portion of CHIP’s spectral response ranges from about 420 nm to 780 nm, peaking around 555 nm. CHIP’s spatial engine is able to produce images at ten times the image resolution of WIIT’s 16 micron camera pixels, so it is able to produce scenes that contain what are effectively point sources to WIIT’s imaging camera. For phase retrieval experiments, we used a spatial scene comprised of 5 well-separated, point-like sources, with one central source surrounded by 4 sources closer to the corners of the image plane. We took three sets of measurements with about 27 nm bandwidths centered on 475 nm, 575 nm, and 675 nm, where the detector was placed near the focus of the system as it would be during the system’s normal mode of operation. These three sets of measurements were taken at WIIT’s shortest baseline length of 36 mm. A fourth set of measurements was taken with a 27 nm bandwidth centered on 575 nm, where the baseline length was increased to 230 mm in order to include the possibility that the projection mirror, which makes the scene appear to
be in the far field of the interferometer’s input aperture, has a significant impact on the system’s aberrations. Fig. 3.1(a) shows the spectra used to collect the phase retrieval measurements as compared to the spectrum produced by the bulb within CHIP.

Before performing phase retrieval, we preprocessed the data by first averaging 720 images of each scene (to increase signal-to-noise ratio) and performing background subtraction, which is required because CHIP generates a spatially variant background pattern on top of which our scene is displayed. We then crop out all five PSFs in each image so that the resultant dataset has twenty images of 28x28 pixels each. Phase retrieval was then performed jointly on all twenty images. Fig. 3.1(b) shows an example of the scenes collected by WIIT after preprocessing and averaging the measured images for the case involving a baseline of 36 mm and a spectrum centered at 575 nm.

We incorporated the assumptions mentioned in Sect. 3.1 and made additional simplifications to the phase retrieval problem. Our computational model always allowed the tip/tilt terms of the wavefront to vary individually for each of the 20 PSFs. Initially, defocus was also allowed to vary

Figure 3.1. (a) The spectra used for the phase retrieval experiment in comparison to the bulb’s spectrum in CHIP and (b) an example image taken with WIIT using a 36 mm baseline length with a spectrum centered at 575 nm after removing CHIP’s background signatures and averaging over 720 individual measurements.
individually for each PSF while all other higher-order aberrations were common to all of them. The unified wavefront for the higher-order aberrations was described by Zernike polynomials through the 8th radial order (41 terms after removing piston, tip, tilt and defocus). Each individually modeled PSF had a 27 nm spectrum, where the spectra shown in Fig. 3.1(a) were sampled in 1 nm increments. The defocus in microns, along with all other wavefront terms, was assumed to be constant over the entire 27 nm bandwidth.

Using the model described above, where tip/tilt/defocus are independent variables for all 20 PSFs, we performed phase retrieval via nonlinear optimization of a bias-invariant error metric [1, 2], the square root of which is the normalized root mean-squared error (NRMSE) between the modeled PSFs and the measurements. We fit a quadratic to the retrieved defocus values as a function of wavelength and used that curve to compute new values for defocus at each of the 3 central wavelengths, effectively incorporating our assumption that the axial chromatic aberration has a quadratic shape with wavelength. We then re-optimized the Zernike coefficients with the fixed defocus values.

### 3.3 Phase Retrieval Results

We found that, after the first round of optimization where defocus was freely varied for all 20 PSFs, the NRMSE was 0.0426. After fixing the defocus values to lie on the fitted curve and re-optimizing, the final NRMSE was 0.0435. Fig. 3.2(a) shows a plot of recovered peak-to-valley (PV) defocus values in microns as a function of wavelength and the curve that was fit to the data. The final wavefront without tip/tilt/defocus is shown in Fig. 3.2(b) with units of microns. Fig. 3.3 contains examples of modeled (top row) and measured (bottom row) PSFs for the on-axis sources, where the intensity values have been stretched to the 0.6 power after setting negative values to zero. There is particularly good agreement near the centers of the PSFs, and noise begins to dominate prior to reaching the inner edge.
of the red circles in Fig. 3.3. The absolute residuals between the modeled PSFs and the measured PSFs are shown in Fig. 3.4 stretched to the 0.6 power in addition to the individual NRMSE for each PSF. Each row shows a different wavelength and baseline combination, where the first two rows are for a central wavelength of 575 nm and the last two rows correspond to central wavelengths of 675 nm and

![Defocus vs Wavelength](image1)

![Wavefront](image2)

Figure 3.2. Phase retrieval results. (a) Retrieved defocus in microns as a function of wavelength, and (b) all higher order aberration through 8th radial order Zernikes shown in microns with tip/tilt/defocus removed.

![Modeled PSFs](image3)

![Measured PSFs](image4)

Figure 3.3. (a–d) Modeled PSFs shown stretched to the 0.6 power after gain and bias adjustment, and (e–h) measured on-axis PSFs. For (a, e) 575 nm source at 36 mm baseline, (b, f) 575 nm source at 230 mm baseline, (c, g) 675 nm source at 36 mm baseline, and (d, h) 475 nm source at 36 mm baseline. Note the colorbar shows stretched intensity values. The red circles indicate boundaries where noise is assumed to dominate the PSF measurements, so only pixels inside the red circles contributed to the optimizations.
475 nm, respectively. All rows correspond to 36 mm baselines except for the second row, which is for measurements with a 230 mm baseline. Again, noise dominates far from the center of the PSFs. However, there are small features near the centers of the PSFs that are likely due to a model mismatch. It is possible that a model including more chromatic effects, such as allowing defocus to vary across the 27 nm bandwidths and incorporating transverse chromatic aberration, would provide better agreement with the measurements. Despite the mismatch, the retrieved PSFs in Fig. 3.3 agree quite well with the measurements, and our simplified wavefront model will allow for more accurate simulations of WIIT, as well as improved data processing, than assuming a diffraction-limited wavefront.

Figure 3.4. Absolute residuals between modeled and measured PSFs stretched to the 0.6 power for central wavelengths of (a)-(j) 575 nm, (k)-(o) 675 nm, and (p)-(t) 475 nm. All measurements were taken at a baseline of 36 mm except the second row (f)-(j) taken at a baseline of 230 mm.
3.4 Conclusion

We have devised an experiment to recover aberrations that are representative of the WIIT system. Using a simplified model for the system’s aberrations, we demonstrated that phase retrieval techniques can recover aberrations that agree with the measured data to within better than 4.4% on average. These results will allow for more accurate WIIT simulations, as well as provide the information needed to simultaneously synthesize and deconvolve WIIT datasets.
3.5 References


Chapter 4

Chirp Z-transform Image Registration

4.1 Introduction

Image registration techniques are required in many imaging applications from tracking changes in medical images to super-sampling of under-sampled images. For the latter, accurate registration is essential for overall performance. This is also the case for wide-field spatio-spectral interferometric imaging. Datasets for such an interferometer include measurements collected with many different baseline lengths and orientations. For a dual aperture interferometer, the orientation of the baseline is typically varied by rotating the entire interferometer, causing a rotation of collected images. For experimental interferometers, it is often easier to rotate the input scene while keeping the interferometer fixed, as is the case for WIIT. In order to reduce the dataset to a single high-spatial-resolution hyperspectral image using the synthesis algorithm from Chapter 2, all images in the dataset must be registered to a single orientation.
Although spatio-spectral image synthesis algorithms have been demonstrated on simulated data [1, 2], experimental effects create considerable challenges. The experimental realization that we are most concerned with is WIIT, which was discussed in greater detail in Chapter 1. Despite residing in the well-controlled environment of the Advanced Interferometry and Metrology laboratory at Goddard Space Flight Center (GSFC), there is heat generated by the imaging camera and CHIP that causes mild image motion between measurements. In an absolutely ideal environment there would not be any image shifts and the rotation angles would be known exactly for WIIT measurements, so registration would not necessarily be needed; however, we would still need to orient all measurements to a single rotation angle so that all images are co-aligned.

This chapter discusses a very general approach to image registration for WIIT measurements. We will demonstrate the accuracy of the image registration process on simulated WIIT measurements for several baseline lengths and orientations. The center of rotation and image translations are degenerate parameters, so we will take the fast Fourier transform (FFT) convention for the DC pixel as the center of rotation. This is particularly useful because we implement image resampling with a chirp z-transform (CZT) algorithm, which itself is computed using multiple FFTs. The image registration algorithm as a whole, including the CZT algorithm and its incorporation into an image registration procedure, is included in Sec. 4.2. Sec. 4.3 describes spatio-spectral interferometric measurements, including how to preprocess the data prior to image registration. The results of image registration on simulated data are shown in Sec. 4.4, and some concluding remarks and future work are given in Sec. 4.5.
4.2 Image Registration Via Nonlinear Optimization

We first describe the CZT algorithm that we use to perform image resampling, including rotations and translations, as well as possible up-sampling or down-sampling. We will then discuss nonlinear optimization and how it facilitates image registration through the CZT algorithm.

4.2.1 Chirp Z-transform Resampling

CZT resampling has previously been applied to a variety of problems, including blind deconvolution, phase retrieval, and super-sampling \[3,4\]. Not all implementations of the CZT algorithm are equivalent; for example, we have chosen to employ the 2D CZT rotation algorithm formulated by Myagotin and Vlasov \[5\] instead of successive 1D CZT operations \[3,4\]. We will derive the 2D CZT rotation algorithm here in a style similar to that of Myagotin and Vlasov \[5\]; although, the factorization that facilitates this 2D method was first published by Tong and Cox \[6\].

Let \(G_{mn}\) be the discrete Fourier transform (DFT) of the image \(g_{xy}\) we want to rotate. Now consider the inverse DFT of \(G_{mn}\):

\[
g_{xy} = \sum_{m,n=-N/2}^{N-1} G_{mn} \exp \left[ i \frac{2\pi}{N} (mx + ny) \right]
\]

\[
= \sum_{m,n=0}^{N-1} G_{mN/2,nN/2} \exp \left[ i \frac{2\pi}{N} \left( m - \frac{N}{2} \right)x + \left( n - \frac{N}{2} \right)y \right], \tag{4.1}
\]

where we have assumed \(G_{mn}\) is a square array. \(G_{m-N/2,n-N/2}\) is equivalent to performing an fftshift on \(G_{mn}\). Explicitly discretize the x- and y-dimensions: \((x, y) = (r, s) \Delta_{xy}\), where \(r, s = 0, ..., R - 1\), and \(\Delta_{xy}\) is the pixel spacing and is assumed to be equal in both dimensions. If we rotate the discretized x- and y-coordinates about the point \((x_0, y_0)\) by an angle \(\theta\), the rotated coordinates become
\[ x' = (\Delta_{xy} r - x_0) \cos \theta - (\Delta_{xy} s - y_0) \sin \theta + x_0 , \]  
(4.2)

\[ y' = (\Delta_{xy} r - x_0) \sin \theta + (\Delta_{xy} s - y_0) \cos \theta + y_0 . \]  
(4.3)

Replacing \((x, y)\) with \((x', y')\) in Eq. (4.1), we obtain

\[ g'_{rs} = g_{rs'} = \sum_{m,n=0}^{N-1} G \left( \frac{m-N}{2}, \frac{n-N}{2} \right) \exp \left\{ i \frac{2\pi}{N} \left[ (m-N/2)x + (n-N/2)y \right] \right\}. \]  
(4.4)

Using Eqs. (4.2) and (4.3), we can expand Eq. (4.4) to take the following form:

\[ g_{rs} = \exp \left\{ -i\pi \left[ (\Delta_{xy} r - x_0)(\cos \theta + \sin \theta) + (\Delta_{xy} s - y_0)(\cos \theta - \sin \theta) \right] \right\} \]
\[ \times \sum_{m,n=0}^{N-1} G \left( \frac{m-N}{2}, \frac{n-N}{2} \right) \exp \left\{ i \frac{2\pi}{N} \left[ (m-N/2)x + (n-N/2)y \right] \right\} \]
\[ \times \exp \left\{ -i \frac{2\pi}{N} \left[ (x_0 \cos \theta - y_0 \sin \theta)m + (x_0 \sin \theta + y_0 \cos \theta)n \right] \right\} \]
\[ \times \exp \left\{ i \frac{2\pi \Delta_{xy}}{N} \left[ \cos \theta (mr + ns) + \sin \theta (nr - ms) \right] \right\}. \]  
(4.5)

In order to take advantage of the CZT algorithm, we will need to write Eq. (4.5) in the form of a convolution; to do this, we adopt the expansion first published by Tong and Cox [6]:

\[ (mr + ns) = -(s - m)(r - n) + mn + rs \]
\[ 2(nr - ms) = (s - m)^2 - (r - n)^2 + (n^2 - m^2) + (r^2 - s^2). \]  
(4.6)

Eq. (4.6) allows us to rewrite Eq. (4.5) in the form of a 2D CZT. After rearranging and grouping terms, Eq. (4.5) becomes

\[ g_{rs} = A_{rs} \sum_{m,n=0}^{N-1} B_{mn} G \left( \frac{m-N}{2}, \frac{n-N}{2} \right) H_{(s-m)(r-n)}, \]  
(4.7)

where

\[ A_{rs} = \exp \left\{ -i 2\pi \alpha rs - i \pi \beta \left( r^2 - s^2 \right) \right\} \]
\[ \times \exp \left\{ -i \pi \left[ (\Delta_{xy} r - x_0)(\cos \theta + \sin \theta) + (\Delta_{xy} s - y_0)(\cos \theta - \sin \theta) \right] \right\}, \]  
(4.8)

\[ B_{mn} = \exp \left\{ -i 2\pi \alpha mn - i \pi \beta \left( n^2 - m^2 \right) \right\} \exp \left\{ i \frac{2\pi}{N} \left[ (m-N/2)x + (n-N/2)y \right] \right\} \]
\[ \times \exp \left\{ -i \frac{2\pi}{N} \left[ (x_0 \cos \theta - y_0 \sin \theta)m + (x_0 \sin \theta + y_0 \cos \theta)n \right] \right\} \]
\[ \times \exp \left\{ i \frac{2\pi \Delta_{xy}}{N} \left[ \cos \theta (mr + ns) + \sin \theta (nr - ms) \right] \right\}. \]  
(4.9)
\[ H_{mn} = \exp\left[i2\pi\alpha mn + i\pi\beta\left(n^2 - m^2\right)\right], \]  
(4.10)

and

\[ (\alpha, \beta) = -\frac{\Delta_{xy}}{N}(\cos \theta, \sin \theta). \]  
(4.11)

Although Eq. (4.7) appears as a regular convolution, it is important to point out that, the way the equation is written, the convolution kernel \( H_{mn} \), or equivalently the input array \( G_{mn} \) and \( B_{mn} \), must have its axes swapped prior to performing the convolution. Fortunately, this can be solved using a simple transpose operation (as opposed to transpose-conjugate).

A sub-pixel image shift prior to the rotation can be performed by adding another linear phase term to \( B_{mn} \), or directly to the input array \( G_{mn} \), which does not change the computational complexity of the CZT rotation algorithm. In addition to rotation and translation, this same algorithm is capable of changing the sampling rate in the output plane with the variable \( \Delta_{xy} \), which is possible because the desired sample spacing in the output plane is arbitrary. This means that with the addition of padding and cropping operations, this CZT algorithm can also simulate changes in the magnification of an optical system, where \( \Delta_{xy} < 1 \) increases the number of samples in the output array compared to the input array by a factor of \( \Delta_{xy} \). It should be noted that the arguments of the 2D CZT algorithm in Eq. (4.7) are subject to the same padding requirements as the 1D CZT algorithm as described by Rabiner et al. [7] and Bailey and Swarztrauber [8]. For two dimensions, the padding considerations are described by Jurling and Fienup [9], who also describe how the 2D CZT can be performed with three FFTs and that a prudent choice of padding size can improve the performance of the algorithm. We also note that it is possible to incorporate various system transfer functions in the spatial frequency domain as part of the CZT algorithm, allowing simulations of system aberrations, pixel transfer
functions, or atmospheric turbulence. Incorporation of a transfer function will be discussed further in Sect. 4.4.

Rotation via the CZT algorithm is equivalent to using 2D sinc interpolation to perform the shift, rotation and resampling but with lower computational complexity [3,7,8]. Due to the frequency-based nature of this method, it works best for bandlimited images in order to prevent the algorithm from inducing unwanted aliasing; however, this problem can be overcome by paying special attention to sampling and zero-padding throughout the computation. In total, the CZT image rotation algorithm requires four FFTs, typically of a larger size than the original image to be rotated. Accordingly, the computation time for this rotation algorithm is much greater than for bilinear resampling, but CZT resampling provides more accurate rotations than the bilinear resampling method. We now turn our attention toward incorporating the CZT algorithm described in this section into an image registration algorithm through nonlinear optimization of the registration parameters.

4.2.2 Nonlinear Optimization

In this section we use nonlinear optimization to perform image registration, employing the rotation and translation algorithm from Sec. 4.2.1. Assume that we have a set of $K$ measured images $\tilde{g}_k(r,s)$ to be registered to a single reference image $f(r,s)$. Starting with $K$ estimates for the image registration parameters, particularly rotation and translation, we apply the CZT resampling algorithm from Sec 4.2.1 $K$ times to the reference image $f(r,s)$ so that we now have $K$ rotated and shifted images $\hat{f}_k(r,s;\theta_k,x_{0,k},y_{0,k})$, where $\theta_k$ is the rotation angle, $(x_{0,k},y_{0,k})$ are the translation values for the x- and y-dimensions, and the hat denotes that the variable is an estimate for the measured image. We can then compare the estimated images $\hat{f}_k$ to the measurements $\tilde{g}_k$. 
Now that we have a forward model, optimization algorithms require that we have a scalar error metric to determine how well the simulated images predicted with the forward model compare to the measured images. We have chosen a gain and bias invariant normalized mean square error metric (NMSE) between the predicted images \( \hat{f}_k(r, s; \theta_k, x_{0,k}, y_{0,k}) \) and measured images \( \tilde{g}_k(r, s) \) to promote data consistency [10]:

\[
E = \frac{1}{K} \sum_{k=1}^{K} \sum_{r,s} w_k(r,s) \left[ \sigma_k \hat{f}_k(r, s; \theta_k, x_{0,k}, y_{0,k}) + \epsilon_k - \tilde{g}_k(r, s) \right]^2
\]

\[\text{(4.12)}\]

where \( \sigma_k \) is the signal gain, \( \epsilon_k \) is the signal bias, and \( w_k(r,s) \) is a weighting function that can be used to emphasize particular pixels or ignore dead pixels. We used the weighting functions to mask out the areas in \( \hat{f}_k(r, s; \theta_k, x_{0,k}, y_{0,k}) \) where \( f(r, s) \) did not contribute any information, often in the corners of the transformed images. Given a particular measurement and model image, the optimal gain and bias can be computed independently of other parameters [10], and they are helpful for mitigating detector-induced errors for experimental datasets. If pixel values are optimized directly, say for the case of using this algorithm to perform a deconvolution, rather than for the sole purpose of recovering image registration parameters, it is possible to incorporate regularization metrics; however, we did not use regularization for any of our results in this chapter. It is worth noting that it is possible to register the images jointly, as suggested by Eq. (4.12), or individually for each \( \tilde{g}_k(r, s) \). We chose to register the images jointly in order to avoid recalculating intermediate variables required for the CZT resampling algorithm from Sec. 4.2.1.

Nonlinear optimization algorithms require one to supply a gradient of the error metric with respect to all optimization variables; in this case, we are optimizing over rotation angles and image...
translations. Instead of deriving analytic expressions for the gradients, we employed algorithmic differentiation, which is a formulaic method for computing analytic gradients in a step-by-step fashion where every computation in the forward model has a corresponding step in the gradient model. By breaking up the gradient computation into small pieces, it becomes much less daunting to compute analytic gradients for complicated forward models. The idea of algorithmic differentiation has recently been made more accessible for image reconstruction and phase retrieval by Jurling and Fienup [11]. We have followed their formulation of algorithmic differentiation in order to compute the required gradients. The nonlinear optimization algorithm we utilized in our simulations was the limited-memory Broyden-Fletcher-Goldfarb-Shanno algorithm with bounds (L-BFGS-B) from the SciPy package for the Python computing language, although we did not impose any bounds on the optimization variables for image registration. Various stopping criteria are allowed and are ultimately problem dependent. We chose to set a tolerance of $10^{-8}$ on the decrease in the value of the error metric between iterations, corresponding to about a 0.01% decrease in the error metric for the majority of cases presented in this chapter.

4.3 Wide-field Spatio-spectral Interferometric Simulations

This section describes the simulated dataset and how the data are preprocessed before applying the image registration algorithm. The intricacies of implementing the simulations are beyond the scope of this chapter. For a single vector baseline of the interferometer, many image measurements are taken while scanning the path delay between the arms of the interferometer, resulting in a single measurement cube. If we assume that the interferometer’s apertures are identical so that they have the same aberrations, a measurement cube can be represented by Eq. (2.29)
\[ I_{3,d}(\theta, B, \Delta L) = I_1(\theta) \pm |\gamma_3| \int_0^1 \int_{-1}^1 p_{1,1}(\theta - \alpha; \kappa) S_s(\alpha; \kappa) \times \sin\left[2\pi\kappa(\alpha \cdot B + \Delta L) - \Delta \varphi(\kappa)\right] d^2\alpha d\kappa, \tag{4.13} \]

where, as a reminder,
\[ I_1(\theta) = |\gamma_3| \int_0^1 \int_{-1}^1 p_{1,1}(\theta - \alpha; \kappa) S_s(\alpha; \kappa) d^2\alpha d\kappa, \tag{4.14} \]

\( p_{1,1}(\theta; \kappa) \) is the PSF of either aperture function independently, \( S_s(\alpha; \kappa) \) is the spectral density of the source, \( B \) is the vector baseline, \( \Delta L \) is the optical path difference between the arms of the interferometer, \( \kappa \) is the spectroscopy convention for wavenumber (the inverse of the wavelength), \( \alpha \) is the direction cosine of the source position relative to the line-of-sight of the interferometer, and \( \theta \) is the angular position of the image pixels relative to the optical axis of the imaging system within the interferometer. An entire dataset, comprised of many measurement cubes, is generated by repeating the measurement process at various vector baselines.

We preprocessed the measurement cubes by taking the average over the delay dimension, which is a method of approximating Eq. (4.14) from Eq. (4.13), and performed image registration using the result. Due to delay-line sampling, noise, and other detector effects, Eq. (4.14) cannot be recovered exactly but can be well approximated, resulting in averaged images that are very weakly dependent on \( B \). In this sense, the images to be registered are not exactly the same but are derived from the same underlying high-spatial-resolution object. In fact, if the PSF is not radially symmetric, the images to be registered can be significantly different because rotation occurs prior to application of the PSF. The PSF in our simulations is similar to that observed in WIIT and does not have perfect rotational symmetry. The images in Eq. (4.13) and (4.14) are bandlimited by the diameters of the input apertures, and WIIT is designed to be at least Nyquist sampled according to the diameter of a single aperture. Details of WIIT’s optical parameters and source capabilities were discussed in Chapter 1 and can be found in other publications. A typical WIIT datacube contains between two and three thousand
images, one for each of the sampled delay positions $\Delta L$, so the act of preprocessing reduces the standard deviation of the noise in the average of Eq. (4.13) by the square root of the number of averaged images, which is about a factor of 50 in this case.

The underlying hyperspectral image that we used for simulations was created by summer intern David Blankenship using far-infrared sky simulation code developed by Dr. Alexander Kashlinsky at GSFC and is similar to a Hubble deep-field image but was based on a model of the far infrared sky. After passing through the interferometer’s optical system, the image of the scene becomes blurred, due to the modest aperture size, so that it is difficult to discern the many galaxies from the diffuse foreground emission and light from nearby galaxies. We also injected point-like reference sources outside of the core of the deep-field scene that will eventually be useful for calibration. This scene was developed to probe the efficacy of spatio-spectral interferometry with WIIT, and was used to take measurements with WIIT [12]. The simulated dataset we used consisted of one datacube that served as a reference image whose rotation and translations were defined to be identically zero, while the rest of the dataset consisted of 10 datacubes whose translations were unknown and drawn from a uniform distribution that allowed a maximum image shift of 2.5 pixels in either dimension. The rotations of the 10 datacubes were assumed known to the nearest half a degree, where the noise added to the rotation angles was also drawn from a uniform distribution. Fig. 4.1(b) is the result of averaging Eq. (4.13) for the reference image in our simulations.

We investigated how various noise models in the datacube measurements, Eq. (4.13), affect registration accuracy. The first noise model only takes into account the effect of detector quantization, which also limits the overall dynamic range. WIIT has a high-quality 16-bit CCD camera with unit gain. We assumed that the peak value of the entire dataset was 40% of the 16-bit maximum, or 26,214 counts, for all results shown in this paper. This is also the value we use when referring to peak signal-
to-noise (pSNR) values, which, for Poisson noise, is then $\sqrt{26,214} \approx 162$. In conjunction with Poisson noise and quantization noise, we added different levels of read noise, where the pSNRs for read noise were 100, 10, and 1. In all, we tried 5 different noise combinations: quantization noise only, quantization noise and Poisson noise, and the combination of quantization noise, Poisson noise and read noise for the 3 different read noise levels. Fig. 4.1(a) shows the fringes of Eq. (4.13) as a function of $\Delta L$ at a particular pixel for various noise levels, excluding the read noise case with a pSNR of 1 because it would make it more difficult to identify the fringe near the center of the plot. Five images from the dataset are displayed in Fig. 4.2 for the various noise levels. To show more dim background detail, all subsequent images have been stretched to the 0.3 power after setting negative values to zero.

Figure 4.1. Example of simulated WIIT data. (a) The fringes at a particular pixel of the datacube for four of the five noise realizations used in this paper, and (b) the average along the delay dimension, where the highlighted pixel in the lower left corner is the location of the fringes in (a).

4.4 Image Registration Results

The initial values of the registration parameters are important because nonlinear optimization routines can be prone to local minima issues that can prevent the algorithm from finding the global minimum. The rotations, as mentioned in Sec. 4.3, were known a priori to the nearest half a degree. The initial translation values were assumed to be zero. Another source of error comes from noise in
the reference image, which is assumed to come from the overall dataset, even though it is considered separate from the dataset for the purposes of this demonstration. In order to reduce the impact of noise in the reference image, we obtained a new estimate of the reference image after performing one run of the registration process by transforming the entire dataset to the same orientation as the original reference image and taking the average of the transformed images. This provided a new, less noisy estimate of the reference image with which we performed a second round of registration. The original reference image is shown in Fig. 4.3 for all of the various noise levels, and the updated reference images are displayed in Fig. 4.4.

Figure 4.2. Five images from the dataset with different rotations and translations, stretched to the 0.3 power after setting negative values to zero. Images shown with increasing noise levels of (a) quantization only, (b) quantization and Poisson noise, and (c) through (e) have quantization noise, Poisson noise, and Gaussian read noise with pSNR values of 100, 10, and 1, respectively.

For quality assessment we computed both the mean absolute deviation (MAD) from the truth and the standard deviation (StD) from the truth:

\[
MAD = \frac{1}{K} \sum_{k} |\hat{\phi}_k - \phi|, 
\]  

(4.15)
\[ \text{StD} = \sqrt{\frac{1}{K} \sum_{k} (\hat{\phi}_k - \phi_i)^2} \]  

where \( \hat{\phi}_k \) is the estimate variable \( \phi \) in the \( k \)th image, and \( \phi_i \) is the truth value of the variable. Tables 2.1 and 2.2 show the results for the two applications of the registration algorithm, where Table 2.2 values were obtained after computing a new estimate of the reference image by averaging over the dataset. The translation error is for both x and y translations together.

Tables 4.1 and 4.2 reveal some interesting information about the registration technique. First, the translation registration results appear to be virtually unaffected by the noise models, and there is only a slight improvement after updating the reference image, resulting in registrations accurate to within a tenth of a pixel. The apparent drop in both the MAD and StD for translation at the higher noise levels is insignificant because the standard deviations of the MAD calculations are about half the MAD values themselves. On the other hand, the rotation registration accuracy begins to degrade at the

![Figure 4.3. Reference images, stretched to the 0.3 power after setting negative values to zero, for increasing noise levels of (a) quantization only, (b) quantization and Poisson noise, and (c) through (e) have quantization noise, Poisson noise, and Gaussian read noise with pSNR values of 100, 10, and 1, respectively.](image-url)
highest noise level, and there is typically a significant improvement in the registration accuracy after updating the reference image. Even at the highest noise level, we still managed to recover the rotation angles to within 3 arcminutes. We also notice that rotation is recovered more accurately than translation. We believe that this is due to the asymmetry of the aberrations used in the simulations. Imagine the case of image aberrations being dominated by strong coma. The rotation angle is able to be recovered more accurately than translation because the rotation angle depends on the global features of the image. Once the images are resampled to the same orientation, the blur from the coma will occur in different directions, causing uncertainty in the translation. This problem led us to make further modifications to our image registration procedure.

In addition to simulating the WIIT optical system, we simulated a dataset where the optical system was assumed to be free of all aberrations except defocus. Although we will not be able to make measurements from an interferometer with such an ideal optical system, we can still learn from investigating an optical system with a radially symmetric PSF. For this idealized dataset, the
Table 4.1. Results of image registration using the initial reference image for all of the noise levels, where the final error metric value is provided along with the mean absolute deviations and standard deviations for the translation and rotation parameters.

<table>
<thead>
<tr>
<th></th>
<th>Final NMSE</th>
<th>Translation Errors (pixels)</th>
<th>Rotation Errors (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(E)</td>
<td>(MAD)</td>
</tr>
<tr>
<td><strong>Quantization Only</strong></td>
<td>1.64e–03</td>
<td>8.60e–02</td>
<td>1.04e–01</td>
</tr>
<tr>
<td><strong>Quantization + Poisson</strong></td>
<td>1.64e–03</td>
<td>8.60e–02</td>
<td>1.04e–01</td>
</tr>
<tr>
<td><strong>Quantization + Poisson + Read Noise (pSNR=100)</strong></td>
<td>1.65e–03</td>
<td>8.60e–02</td>
<td>1.05e–01</td>
</tr>
<tr>
<td><strong>Quantization + Poisson + Read Noise (pSNR=10)</strong></td>
<td>1.94e–03</td>
<td>8.54e–02</td>
<td>1.04e–01</td>
</tr>
<tr>
<td><strong>Quantization + Poisson + Read Noise (pSNR=1)</strong></td>
<td>3.28e–02</td>
<td>8.29e–02</td>
<td>9.73e–02</td>
</tr>
</tbody>
</table>

Table 4.2. Results of image registration using the updated reference image for all of the noise levels, where the final error metric value is provided along with the mean absolute deviations and standard deviations for the translation and rotation parameters.

<table>
<thead>
<tr>
<th></th>
<th>Final NMSE</th>
<th>Translation Errors (pixels)</th>
<th>Rotation Errors (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(E)</td>
<td>(MAD)</td>
</tr>
<tr>
<td><strong>Quantization Only</strong></td>
<td>5.36e–04</td>
<td>8.34e–02</td>
<td>1.01e–01</td>
</tr>
<tr>
<td><strong>Quantization + Poisson</strong></td>
<td>5.36e–04</td>
<td>8.34e–02</td>
<td>1.01e–01</td>
</tr>
<tr>
<td><strong>Quantization + Poisson + Read Noise (pSNR=100)</strong></td>
<td>5.38e–04</td>
<td>8.35e–02</td>
<td>1.02e–01</td>
</tr>
<tr>
<td><strong>Quantization + Poisson + Read Noise (pSNR=10)</strong></td>
<td>6.82e–04</td>
<td>8.29e–02</td>
<td>1.01e–01</td>
</tr>
<tr>
<td><strong>Quantization + Poisson + Read Noise (pSNR=1)</strong></td>
<td>1.50e–02</td>
<td>8.00e–02</td>
<td>9.37e–02</td>
</tr>
</tbody>
</table>
registration accuracy for translation improved by as much as a factor of 50, while the accuracy of the rotation angle improved by a factor of 5. This suggests that the algorithm is impacted more by model mismatch, meaning that the images are not identical after rotating to the same orientation, than by noise. One can get an understanding of the model mismatch by looking at the NMSE values for the quantization-only case, which we found to be about two orders of magnitude smaller for the case of radially symmetric aberrations. Because of the difference in performance in the presence of asymmetric aberrations, we have developed a modified image registration procedure, involving deconvolution, that incorporates knowledge of the system’s aberrations, when available. Fortunately, the addition of deconvolution does not cause much of an increase in computational complexity because it is easy to include transfer functions in the CZT algorithm described in Sec. 4.2.1.

The inclusion of deconvolution into the image registration procedure would be most valuable in narrowband image measurements; however, WIIT measurements are broadband in nature and all images are panchromatic. An ideal deconvolution of the broadband imagery would involve obtaining the deconvolved image for each spectral component of a hyperspectral image before summing over all spectral components to obtain a panchromatic image. Unfortunately, we do not have access to the spectral components of the panchromatic image unless other techniques are used to measure the same scene, such as conventional Fourier transform imaging spectroscopy. As a result, the panchromatic PSF of the system, regardless of asymmetric aberrations, must be approximated as

\[ p_{pc}(\theta) = \sum_{\kappa} p_{1,1}(\theta; \kappa) \] (4.17)

for the purposes of deconvolution. The quality of a deconvolution using this approximation depends on the spectral and spatial content of source being imaged. The approximation of Eq. (4.17) means that the deconvolution is finding an effective monochromatic source given by
Combining Eqs. (4.17) and (4.18), we can approximate measured images given by Eq. (4.14) as

\[ I_i(\theta) \approx \left[ \sum \frac{s_r}{k} \right] \int_{-1}^{1} p_{pc}(\theta - \alpha) S_{\text{eff}}(\alpha) d^2 \alpha. \]  

(4.19)

By incorporating the transfer function associated with the panchromatic PSF of Eq. (4.17) into our rotation registration algorithm and including analytic gradients with respect to the image pixels, we can recover a deconvolved image that approximates the actual source according to Eq. (4.18).

Using the deconvolution idea, we modified our original approach to image registration. Previously, after performing an initial round of the registration procedure, we updated the reference images by averaging over all the reoriented images in the dataset. This time we did not perform the average of the images after the first round of registration. Instead, we used our estimate for the panchromatic PSF to perform a deconvolution of the original reference images. We then performed a second round of image registration using the deconvolved image as the reference image, where only the registration variables could vary. A final registration was performed where the deconvolved image was optimized in addition to all registration variables. The final deconvolved reference images are shown in Fig. 4.5. The noise-free ideal deconvolution for our simulation is also shown in Fig. 4.5 for comparison, where each spectral component of the image was deconvolved independently.

Table 4.3 displays the results of the new registration procedure. Like the previous procedure, translation registration results appear to be virtually unaffected by the noise models. The new procedure improved registration accuracy by more than a factor of two compared to the previous procedure, resulting in translation accuracy to better than a twenty-fifth of a pixel. There was less of an improvement in rotation registration, except at the most severe noise level, with ultimate angle registration accuracies ranging from 0.12 to 1.2 arcminutes. Although this procedure that incorporates
Figure 4.5. Reference images after deconvolution, stretched to the 0.3 power after setting negative values to zero, for increasing noise levels of (a) quantization only, (b) quantization and Poisson noise, and (c) through (e) have quantization noise, Poisson noise, and Gaussian read noise with pSNR values of 100, 10, and 1, respectively. The ideal deconvolution (f) is included for comparison.

Table 4.3. Final results of image registration that incorporates deconvolution for all noise levels, where the final error metric value is provided along with the mean absolute deviations and standard deviations for the translation and rotation parameters.

<table>
<thead>
<tr>
<th></th>
<th>Final NMSE</th>
<th>Translation Errors (pixels)</th>
<th>Rotation Errors (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>E</td>
<td>MAD</td>
<td>StD</td>
</tr>
<tr>
<td>Quantization Only</td>
<td>1.53e-05</td>
<td>2.85e-02</td>
<td>3.49e-02</td>
</tr>
<tr>
<td>Quantization + Poisson</td>
<td>1.54e-05</td>
<td>2.85e-02</td>
<td>3.50e-02</td>
</tr>
<tr>
<td>Quantization + Poisson + Read Noise (pSNR=100)</td>
<td>1.71e-05</td>
<td>2.86e-02</td>
<td>3.50e-02</td>
</tr>
<tr>
<td>Quantization + Poisson + Read Noise (pSNR=10)</td>
<td>2.00e-04</td>
<td>2.90e-02</td>
<td>3.54e-02</td>
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<tr>
<td>Quantization + Poisson + Read Noise (pSNR=1)</td>
<td>1.83e-02</td>
<td>2.86e-02</td>
<td>3.38e-02</td>
</tr>
</tbody>
</table>
deconvolution does result in comparable results to measurements with radially symmetric aberration, it does provide some improvement in registration accuracy in the presence of asymmetric aberrations.

4.5 Conclusion

We have demonstrated the performance of an image registration algorithm for both rotation and translation using a CZT algorithm to perform resampling and nonlinear optimization to solve for the unknown registration parameters. This technique has been shown to have the added benefit of being able to perform deconvolution by incorporating knowledge of the optical system’s aberrations, which can also reduce registration errors caused by asymmetric aberrations. Although this technique was developed for registering spatio-spectral interferometric images as a precursor to image synthesis, this technique is very general and could be a viable approach for many other imaging applications where rotation and translation between similar images is unknown. With careful consideration to image sampling, this procedure could also be applied to images that are under-sampled with respect to the Nyquist criterion for imaging, and in that case, can even be used to perform simultaneous super-resolution and image registration.
4.6 References


Chapter 5

Nonnegative Matrix Factorization for a Calibrated Hyperspectral Image Projector

5.1 Introduction

Hyperspectral imaging is an important modality for the identification and classification of objects and materials within a scene. In particular, the astrophysics community relies on hyperspectral measurements to make inferences about, and develop models of, distant astronomical objects. Although ground-based interferometric measurements are in widespread use among astrophysicists, single-aperture telescopes remain the main source of space-based hyperspectral measurements. There will come a time, however, when a monolithic aperture will be unable to meet the demands for high spatial resolution astronomical imagery due to excessive cost, weight, and size, especially at infrared wavelengths. This is when space-based observatories will likely employ wide-field spatio-spectral interferometry, which was described in detail in Chapter 2.
Although the theoretical groundwork for double-Fourier interferometry has already been established, the space-based spatio-spectral interferometric imaging technique needs further characterization before an interferometric observatory, such as the NASA proposed Space Infrared Interferometric Telescope (SPIRIT) [1], ever becomes a reality. This prompted NASA to build the Wide-field Imaging Interferometry Testbed (WIIT), which was described in more detail in Chapter 1. For the purpose of this chapter, we are primarily concerned with the Calibrated Hyperspectral Image Projector (CHIP) [2], which is the light source for the interferometer that is integrated into WIIT.

Hyperspectral image projectors (HIPs) were developed by the Optical Technologies Division at the National Institute of Standards and Technology to solve the problem of testing and characterizing hyperspectral imaging techniques with known hyperspectral scenes [3–5]. These projectors allow for a scene to be displayed such that every object in the scene has the same arbitrary spectrum, resulting in a spatially-spectrally separable image at any given time. This is achieved using two digital light processing (DLP) units. One of the DLP units determines the gray-scale spatial distribution of the image being projected. The other unit controls the shape of the arbitrary spectrum, which is achieved by spectrally dispersing a broadband source onto the DLP such that each row of the DLP chip controls the relative strength of each wavelength bin of the output spectrum. The number of wavelength bins and the spectral range together determine the spectral resolution of the HIP’s spectral output. A hyperspectral image can then be simulated with a HIP by cycling through multiple spatially-spectrally separable images that can be added together throughout the integration time of the camera to simulate the measurement of a spatially-spectrally complicated hyperspectral image. A calibrated HIP, such as NASA’s CHIP, is constructed by including a fiber-coupled spectrometer into the design such that the spectral output of the projector can be monitored [2]. Combined, WIIT and CHIP provide
a controlled means of probing the intricacies of spatio-spectral interferometry in a shot-noise-limited regime.

In this chapter, we describe how CHIP will be used to obtain interferometric measurements of realistic astronomical test scenes from WIIT in a time-efficient manner. In Section 5.2 we will motivate how CHIP can expedite the measurement of spatially-spectrally complex images in combination with the data decomposition technique known as nonnegative matrix factorization (NMF). A brief introduction to NMF and how it applies to hyperspectral image decomposition is included in Section 5.3. The result of decomposing an astronomically realistic test scene as a function of the number of images through which the CHIP must cycle to represent the scene is presented in Section 5.4. Other considerations for the preparation of scenes for CHIP, including limitations intrinsic to CHIP and limitations imposed by the interferometric technique, are provided in Section 5.5, followed by concluding remarks in Section 5.6.

5.2 Time-efficient WIIT Data Collection Using CHIP

The WIIT is invaluable for demonstrating the effectiveness of double-Fourier interferometric imaging, but the data collection process for test scenes of moderate spatial and spectral complexity can be quite lengthy. To start, there are aspects to the data collection process that are intrinsic to the technique that cannot be altered. For example, the sample spacing of the OPD dimension limits the range of spectral frequencies in the reconstructed hyperspectral image, and the range of the delay line limits spectral resolution of the recovered image. This means that the number of delay line positions for a single baseline is predetermined. In the same regard, the number of baseline measurements required to fully measure the spatial frequency domain, or u-v space, is dependent on both the maximum baseline length and the size of the individual apertures of the interferometer. One could use
sparse sampling to reduce total measurement time, but we are assuming a general object that may not be conducive to sparse sampling. On the other hand, an aspect of the system over which we do have control is the integration time of the camera for each delay line position. Because datacubes are collected for many baselines, and many delay line positions are required for each datacube, decreasing the integration time for each delay line position will result in a reduction in total data collection time for WIIT. When experimentally simulating the measurement of hyperspectral images, however, the integration time is intimately related to implementing hyperspectral scene generation with CHIP.

Recall that CHIP generates a hyperspectral image by cycling through multiple spatially-spectrally separable images that can be added together throughout the integration time of the camera to simulate the measurement of a spatially-spectrally complicated hyperspectral image. Imagine a simple scene with two astronomical objects having different spectra on a blank background. For this simple scene, the CHIP would have to display the image of one object with its spectrum, followed by the image of the second object with its different spectrum. As the number of objects with different spectra gets larger, the number of images through which CHIP has to cycle grows proportionally until the number of spectrally diverse objects is the same as the number of spectral bins of CHIP. At that point, one could just cycle through all of CHIP’s spectral bins independently. There is another option, however, that provides a trade-off between the number of images through which the CHIP cycles and how accurately the projected image matches the original hyperspectral test scene, in a manner similar to principal component analysis (PCA). Unfortunately, PCA on an arbitrary hyperspectral image will result in basis spectra and basis images, sometimes referred to as abundance maps, with negative values, which we cannot represent using CHIP. Nonnegative matrix factorization (NMF) is similar to PCA except all values in the basis spectra and basis images are restricted to be nonnegative, so it can be used to decompose hyperspectral test scenes for CHIP. The idea for time-efficient projection using
the CHIP is to decompose the test scene into as few basis images and basis spectra as possible while maintaining a specified accuracy between the projected image and the original test scene.

5.3 Hyperspectral Image Decomposition Using NMF

NMF is the process in linear algebra of approximating an arbitrary nonnegative $n \times m$ matrix $A$ by decomposing it into the matrix product of two smaller nonnegative matrices $W$ and $H$, with sizes $n \times p$ and $p \times m$, respectively, expressed as

$$A \approx WH. \quad (5.1)$$

The goal is to keep the size of $p$ as small as possible while maintaining an accurate representation of the original matrix $A$, which is analogous to PCA with the exception that all three matrices are restricted to have nonnegative values. Most algorithms for NMF, including both the multiplicative update algorithm [6,7] and the alternating least squares algorithm [7,8], are based on iteratively reducing some cost function, usually the square of the Frobenius norm of the residual matrix:

$$\|A - WH\|_F^2 = \sum_{m,n} (A_{mn} - WH_{mn})^2. \quad (5.2)$$

The results shown later in this chapter employ the alternating nonnegative least squares algorithm using projected gradient methods [9] to minimize this cost function because of its speed and consistency. Due to the imperfect data compression associated with approximating the matrix $A$ with fewer than $nm$ elements, all NMF optimization routines suffer from many local minima. This means that the starting values of $W$ and $H$ are important for finding a good solution. Instead of trying many random starting points and choosing the best result, we have found that applying nonnegative double singular value decomposition [10] to $A$ provides an initial estimate that consistently outperforms the random guess method. The stopping criteria are related to the norm of the projected gradient as well.
as a maximum number of iterations. Figure 5.1 is an attempt to visualize how Eq. (5.1) applies to a hyperspectral image. The left side of Fig. 5.1 shows the original hyperspectral image and the right side shows how it can be approximated by summing over a sequence of spatially-spectrally separable images, represented as basis spectra (top) and basis images (bottom). We can now discuss how to apply NMF to a hyperspectral image to obtain the plots and images on the right side of Fig. 5.1.

Adapting NMF for decomposition of a hyperspectral image into basis spectra and basis images relies on array manipulations, which are simple tasks for computing languages such as Numpy/Python and Matlab. A hyperspectral image is often thought of as a three-dimensional array where two dimensions correspond to spatial coordinates and the other to the spectrum, but we can reshape the three dimensional array into a two-dimensional array such that the two spatial coordinates are collapsed down to a single dimension. This means that a hyperspectral image of size \( j \times k \times l \) is reshaped to size \( j \times kl \), where \( j \) indexes the spectral dimension and \( k \) and \( l \) are pixel indices. The result is an array that describes the spectrum for each image pixel and can now be assigned to the matrix \( A \) in Eqs. (5.1) and (5.2) such that \( m = k \) and \( n = kl \). After applying NMF to the hyperspectral matrix \( A \), the \( p \) columns of \( W \) will be the computed basis spectra and the \( p \) rows of \( H \) will contain the

---

**Figure 5.1.** An illustration of Eq. (5.1) showing how a hyperspectral image (left) can be approximated by the sum of various spatially-spectrally separable images (right), with the basis spectra on top and the associated basis images on the bottom.
corresponding basis images. In order to visualize the basis images, we must reshape the matrix \( H \) from size \( p \times kl \) to \( p \times k \times l \). The basis spectra now correspond to the spectra for each of the spatially-spectrally separable images through which CHIP will cycle, while the basis images become the 8-bit gray scale images applied synchronously to the other DLP. Because spatially-spectrally complicated hyperspectral images can be decomposed into a handful of basis spectra and basis images suitable as inputs for CHIP, NMF is a natural solution to the problem of decomposing complicated hyperspectral images so that time-efficient data collection can be performed with WIIT.

### 5.4 Results for an Astronomically Realistic Test Scene

The hyperspectral test scene used for our simulations was one of a handful created by NASA to demonstrate the viability of a space-based double-Fourier interferometer for future far-infrared (FIR) missions, such as SPIRIT [1], because high resolution images of the FIR sky do not yet exist. The chosen test scene, a panchromatic image of which is shown in Fig. 5.2, is a deep field image comprised of many spectrally varying sources and possesses the most spatial-spectral complexity of any of the test scenes generated by NASA. The galaxy population in redshift space was chosen using a Monte Carlo procedure, so the galaxies’ spectral lines can appear at any spectral bin. This is optimal for demonstrating NMF decomposition because it will require the most basis spectra (the largest value of \( p \)) to accurately represent the complexity of the test scene. The dimensions of the test scene are \( j \times k \times l = 376 \times 375 \times 375 \).

In order to perform time-efficient data collection as discussed in Section 5.2, we want to decompose the deep field image into as few basis spectra as possible while maintaining adequate spatial/spectral complexity. We compare the results of the NMF estimated image with the input image
using the normalized mean squared error (NMSE), which is a normalized version the cost function used by the NMF algorithm:

\[
\text{NMSE} = \frac{\sum_{m,n} (A - WH)_{mn}^2}{\sum_{m,n} A_{mn}^2}.
\]

(5.3)

We performed NMF as described in Section 5.3 for integer values of \( p \) ranging from 4 to 15, and more sparsely sampled for values of \( p \) out to 100. Figure 5.3 is a plot of the NMSE on a log scale as a function of \( p \), showing that as few as 8 basis spectra represents the original test scene with an NMSE < 0.01, about 27 basis spectra represents the scene with an NMSE < 0.001, and about 58 basis spectra produce an NMSE < 0.0001. The spectral average of the estimated hyperspectral image for \( p = 8 \) is shown in Fig. 5.4; it is visually similar to Fig. 5.2.

As expected, the NMSE between the original and decomposed image monotonically decreases

![Spectral Average of Hyperspectral Image](image)

Figure 5.2. Spectral average of the simulated far-IR hyperspectral image to be decomposed (stretched to 0.25 power to show more faint spatial detail).
Figure 5.3. Plot of normalized mean squared error versus the number, $p$, of basis spectra fitted by the NMF algorithm.

Figure 5.4. Spectral average of the decomposed far-IR hyperspectral image produced by NMF algorithm for $p = 8$ (stretched to 0.25 power).
as the number $p$ of recovered basis spectra increases. We can expect that the NMSE will continue to decrease, approaching zero as either the value of $p$ reaches the number of spectral bins in the test image, 376 in this case, or as the value of $p$ reaches the total number sources with unique spectra. Figure 5.5 shows the reconstructed basis spectra and basis images corresponding to the approximated panchromatic image in Fig. 5.4. The first basis spectrum and basis image recovered by the NMF algorithm in Fig. 5.5 are nearly identical for all values of $p$ because together they describe the foreground infrared radiation from dust in the solar system and interstellar dust in the Milky Way galaxy, which contribute energy to every pixel in the scene. The shape of the remaining basis spectra and basis images are dependent on the value of $p$.

![Figure 5.5. Basis spectra and basis images (stretched to 0.5 power) produced by the NMF algorithm for $p = 8$. The plotted spectra include all 376 spectral bins of the simulated deep field scene.](image-url)
Figure 5.6 shows the spectral average of the residual images, $A - WH$, for various values of $p$ between 6 and 50 along with their associated NMSE values. Bright and dark features in the residuals

Figure 5.6. Spectral average of the residual hyperspectral image as the number of estimated basis spectra varies from $p = 6, 8, 14, 20, 30, 50$. Note the difference in colorbar values.
correspond to spectral sources that are under-estimated and over-estimated, respectively. Notice that as the value of $p$ increases, the NMF algorithm tends to improve the brightest and darkest spatial features, which contribute the most energy to the residual images. This is what we should expect because the algorithm is minimizing the Frobenius norm of the residual images, as discussed in Section 5.3. The remaining spatial features in the residual images likely have sharp spectral lines not shared by the majority of astronomical sources within the scene.

### 5.5 Preparing Scenes for CHIP

So far, we have assumed that CHIP is capable of producing arbitrarily complicated spectra and images. However, just as a spectrometer’s spectral resolution is limited by design, the same is true for CHIP’s spectral engine. We can incorporate knowledge of CHIP’s spectral resolution in order to improve our ability to project a desired scene by using calibration information measured by a spectrometer. The spectral resolution of CHIP is about 10 nm, as measured by the spectrometer, so, when there are very narrow spectral features, CHIP is unable to produce the scene with arbitrary accuracy regardless of the number of frames used. In addition, the illumination source for CHIP has a spectral range from about 420 nm to 780 nm, but the usable portion for hyperspectral decomposition is about 260 nm wide about the peak of the spectrum (see Fig. 5.7). Also, because CHIP uses two DLPs for the spectral and spatial engines of CHIP, there is a limit to the dynamic range of both the spectra and images that can be produced at any given time. In Section 5.5.3, we discuss how to reduce the impact of quantization error imparted by the gray levels produced by the DLPs in the scene preparation procedure.

There are other considerations related to WIIT as a whole that limit how CHIP is used. For example, as the number of basis spectra and images projected by CHIP increases, the integration time of WIIT’s camera must increase and the speed of the delay line translation stage must decrease. The
delay line translation stage in WIIT is prone to sticking at very low speeds, so in practice, we found that it is beneficial to limit $p$ to a maximum value of 16. Of those 16 basis frames, we reserve at least four frames for phase reference sources, which are point sources embedded outside of the science scene and are intended to help with registering delay line measurements prior to image reconstruction.

5.5.1 Hyperspectral Mapping

Before we can decompose an arbitrary hyperspectral image for use with CHIP, we must consider how to map the spectral range of the hyperspectral image to the usable portion of CHIP’s spectrum. This problem is related to the number of samples in the spectral dimension of the hyperspectral image to be decomposed, as well as its spectral features. It is useful to crop the spectrum to reduce the number of spectral samples in the hyperspectral image before decomposition if the image has spectral features that are narrower than can be displayed by CHIP or if the spectral range of the image exceeds the spectral range of CHIP. Similarly, the spatial size of the hyperspectral image must be cropped so that the number of samples is less than the number of samples that can be displayed by CHIP’s spatial engine.

The far IR test scene shown in previous sections was designed to be representative of a deepfield scene with 376 samples over the spectral range of 25 to 400 microns. A simple choice for mapping the far infrared spectrum to CHIP’s visible spectrum would be arbitrarily deciding to map each 1 micron increment of the far IR scene to 1 nm in the visible spectrum. Unfortunately, the usable spectral range of CHIP is only about 260 nm, so the spectral range of the deepfield scene must be cropped to accommodate CHIP. Also, the scene contains plenty of emission line features that are about 1 micron wide, which cannot be adequately projected given CHIP’s spectral resolution if we were to map 1 micron to 1 nm. If we crop the deepfield’s spectral range enough and resample over CHIP’s usable spectral range, the emission line features will broaden, making it easier for CHIP to represent
the features even if they are still not 10 nm wide. Convolution could also be used to broaden the features but we have not chosen to take that route.

The hyperspectral image from the previous sections contained 375x375 spatial samples, but we have chosen to use only 280x280 samples from the upper left corner of the original image in order to reduce the number of unique spectra in the scene. We have also chosen to split the original spectral range of 376 samples into quarters, resulting in 94 spectral samples per quarter, which broadens spectral line features to about 2.8 nm given CHIP’s usable spectral range of 260 nm. Although the emission line features remain narrower than CHIP’s 10 nm resolution, they are broader than they were previously.

5.5.2 Incorporating Calibrated Spectra

Before we can discuss how the calibration data are used, we must first describe how the data are obtained. The spectral engine of CHIP incorporates a DLP where each row of the DLP corresponds to a wavelength bin in the output spectrum. The number of columns used in each row determines the relative strength of each wavelength bin. The number of columns for CHIP in WIIT is 768, which will be important later when discussing how to reduce quantization error. Calibration data are obtained by using a spectrometer to measure the spectra produced by using all columns of the DLP, one row at a time. Fig. 5.7 shows CHIP’s spectrum as well as three of the spectra for individual rows of the DLP of the spectral engine. This information is used to construct a calibration matrix, $B$, where one dimension contains the spectral bins corresponding to the rows of the DLP and the other dimension corresponds to the spectra measured by the spectrometer. Incorporating the calibration data is necessary to know how to control the spectral engine’s DLP in order obtain a desired output spectrum.

The spectral resolution of CHIP is about 10 nm, as measured by the spectrometer. The far IR test scene shown in previous sections has emission line features that are 2.8 nm wide after
hyperspectral mapping, so CHIP is unable to produce the scene with arbitrary accuracy regardless of
the number of frames used. This is where the calibration matrix becomes important. We can model
the basis spectra as another matrix multiplication between the calibration matrix and a coefficient
matrix, $C$, with dimensions corresponding to the number of rows, which in our case is 1024 elements,
of the DLP and the number of desired basis spectra, $p$. If $W$ in Eq. (5.1) is assumed to contain the basis
spectra and $H$ is assumed to contain the basis images, then we can rewrite Eq. (5.1) as

$$A \approx BCH.$$ 

where the values of $B$ are known through spectrometer measurements, and the values of $C$ and $H$ must
be determined through optimization.

![CHIP Spectrum](image)

Figure 5.7. CHIP’s spectrum and a few of the spectra that comprise the calibration data.
The process of using NMF for determining the basis images remains the same as long as the spatial extent of the desired hyperspectral image fits on the 1024x768 DLP. The basis spectra, however, must now be determined by solving for the coefficient matrix $C$. A starting guess for $C$ can be obtained by using the pseudoinverse of the calibration matrix $B$ and multiplying it by the basis spectra matrix $W$ that is found prior to considering the calibration information. Then $C$ and $H$ can be refined by performing nonlinear optimization on the data consistency metric given by

$$\text{NMSE} = \frac{\sum_{m,n} (A - BCH)^2}{\sum_{m,n} A_{mn}^2} \text{ s.t. } C \geq 0, H \geq 0. \quad (5.5)$$

It can be useful to alternate between optimizing $C$ and $H$ prior to optimizing both jointly.

Figure 5.8 shows spectra at two spatial positions within the hyperspectral image indicated by the blue and orange boxes within the spectral averaged images of Fig. 5.8(b) and Fig. 5.8(c). The top and bottom rows show the spectra and spectrally averaged image for the hyperspectral image before and after incorporating the calibration data, respectively, for the case of 12 basis spectra and images. Notice that the calibration data causes the emission peaks to broaden and weaken in magnitude. This is a direct result of the input hyperspectral image having spectral features narrower than can be reproduced by CHIP. If the input hyperspectral image were designed with CHIP’s spectral resolution in mind, the resulting spectra after incorporating calibration data would look much more like the input spectra. Another consequence of CHIP’s spectral resolution is that the smooth continuum portion of the original spectra look noisy after decomposition. This is likely due to the algorithm trying to fit spectral features at other spatial locations, which is in part due to the limited number of basis spectra and images. The spatial images in Fig. 5.8 are stretched to the half power to help distinguish the difference between the input image and the image after decomposition. They are in quite good agreement aside from stronger hazy features surrounding the galaxies in the scene. This is in part due
to the inability of the algorithm to accurately match the spectra in the scene given CHIP’s spectral resolution, as well as quantization of the basis images.

Figure 5.8. Spectra at two spatial locations (left) and spectrally averaged hyperspectral images stretched to the half power (right) before (top) and after (bottom) incorporating CHIP’s calibration measurements for the case of \( p = 10 \).

### 5.5.3 Minimizing Quantization Error

After optimization, the coefficient matrix and basis images are supplied to the spectral and spatial engines of CHIP. The spectral engine of CHIP has 769 gray levels from 0 to 768, which is limited by the number of elements along each column of the DLP. Each element of the DLP is limited to binary “on” and “off” positions, resulting in the aforementioned gray levels in the spectral engine.
The spatial engine has a DLP that produces 8-bit images with gray levels ranging from 0 to 255. As a result, the coefficient matrix and basis images must be constrained to have values that are consistent with the quantization produced by their respective gray levels. The values of \( C \) and \( H \) are discretized according to

\[
C_q = \frac{1}{768} \text{round} \left[ \frac{768}{\max(C)} C \right], \quad (5.6)
\]

\[
H_q = \frac{1}{255} \text{round} \left[ \frac{255}{\max(H)} H \right], \quad (5.7)
\]

where \( C_q \) and \( H_q \) are the quantized approximations of \( C \) and \( H \).

The process of discretizing continuous values induces quantization error. The quantization error can be mitigated by a simple iterative optimization procedure in which either \( C \) or \( H \) is discretized and held fixed while the other matrix is optimized. Due to the spectral engine having more gray levels than the spatial engine, the quantization error induced by discretizing \( C \) is less than the error caused by discretizing \( H \). This also suggests that \( C \) can be used to partially compensate for the error in the discretization of \( H \). In practice, this means that it is often useful to start by quantizing the values in \( H \) before optimizing the values in \( C \), as seen in Fig 5.9. Then \( C \) is quantized and held fixed while the values in \( H \) are optimized, after which the values of \( H \) are quantized again. Then we compute the error \( E_q \) between \( CH \) and \( C_qH_q \), given by

\[
E_q = \frac{\sum_{m,n} (BCH - BC_q H_q)_{mn}^2}{\sum_{m,n} (BCH)_{mn}^2}, \quad (5.8)
\]

and compare to a threshold value before deciding to end or repeat the optimization process.
Incorporating this quantization error reduction procedure tends to reduce the overall normalized root mean-squared error (NRMSE), given by the square root of Eq. (5.8), when compared to simply quantizing the values of $C$ and $H$ that result after optimizing for Eq. (5.5). This is shown in Fig. 5.10 for values of $p$ ranging from 6 to 12. In general, we expect the NRMSE to decrease as $p$ increases, but we have found that is not always the case due to features in the datacube having spectral features finer than CHIP’s spectral resolution. Those spectral features cannot be fit regardless of the value of $p$. The quantization reduction process does not result in a NRMSE that monotonically decreases as $p$ increases, which is likely due to the fact that when $p$ changes, the dynamic range of $C$ can change quite drastically, especially when strong emission lines are being fit near either end of CHIP’s usable spectral range. The quantization error, especially for $C$, can be reduced by using a smaller portion of CHIP’s spectral range about the peak of its emission or by using a hyperspectral image better suited for projection by CHIP. The quantization error for $C$ can also be reduced by inspecting the dynamic range of the coefficients; sometimes there are very few outlying large coefficients that can be truncated to a smaller value before quantization without sacrificing much of a loss in the overall NRMSE.
Figure 5.10. Plot showing the square root of Eq. (5.8) after quantizing the values in $C$ and $H$ both with (green) and without (orange) application of the quantization error reduction procedure in this section. The plot also shows the NRMSE before quantization (blue) for comparison.

5.6 Conclusion

Nonnegative matrix factorization has been applied to an astronomically realistic test scene that will be used by NASA’s CHIP and WIIT for further characterization of double-Fourier interferometric imaging. We demonstrated that the NMSE between the original and decomposed hyperspectral image decreases monotonically as a function of the value of $p$, dropping below 0.01, 0.001, and 0.0001 for as few as 8, 27, and 58 basis spectra and basis images, respectively. We then described how to prepare the decomposed scenes for CHIP by accounting for spectral limitation of the projector and incorporating calibration data. We also presented a simple method to reduce quantization error
introduced by CHIP’s DLPs. Although this chapter describes how to accurately represent a complicated test with CHIP with a limited number of basis spectra and images, it is worth noting that, for the purposes of experimentation with WIIT, the test scene actually projected by CHIP only needs to be a reasonably close approximation of the simulated far-infrared sky as long as the test scene actually projected by CHIP is known. The decomposed hyperspectral image presented in this chapter were used to make the WIIT measurements shown in Chapter 7.
5.7 References


Chapter 6

WIIT Simulations and Image Synthesis

6.1 Introduction

Accurate simulations are important for understanding and developing new imaging modalities and image processing techniques. In the case of spatio-spectral interferometry, the Far-Infrared Interferometer Instrument Simulator (FIInS) [1] is capable of combining instrument features into simulated data: a few different baseline geometries, thermal effects, pointing errors, detector and background noise, and velocity errors due to delay line scanning. FIInS was developed without the wide-field aspect of an interferometer in mind, so, in order to simulate a wide-field interferometer, each pixel of the image plane would be computed independently. For WIIT in particular, the Spatio-spectral Interferometer Computational Optical Model (SsICOM) [2] was developed in the optical engineering software FRED to simulate the WIIT system both with and without metrology measurements of various system components, allowing ideal and as-built models of the system to be compared. Although both of these simulators are
flexible and able to include various instrument parameters, the nature of their computations make them very time intensive and do not lend themselves well to simulating source or instrument rotation during image measurement.

We have developed a simulator based on physical propagation models, allowing for faster simulations. This model is capable of incorporating wavefront errors and image rotations with arbitrary baseline configurations. It is also possible to include wavelength dependent transmissivity of the system and pointing errors with the addition of some extra computations, but we did not include this in any of the simulations shown in this chapter. The process of simulating a datacube for any particular baseline length and rotation is provided in Sect. 6.2. In Sect. 6.3, a simulated WIIT dataset is described and used to demonstrate the image synthesis algorithm from Chapter 2.4.

6.2 Simulating Wide-field Spatio-spectral Measurements

In this section we describe how to simulate wide-field double-Fourier measurements described by Eq. (2.29) for simplicity, but this procedure also applies for the more general Eq. (2.26), which would require one to generate more SPSFs, given by Eq. (2.25), than for Eq. (2.29). First, we assume that SPSFs have already been generated for the system, using, say, transform techniques that allow for arbitrary sampling [3, Chap. 5.3; 4, Chap. 3.3] such that detector sampling is the same for every wavelength in the simulation. For our purposes, what we really need are the SOTFs of the system, given by Eq. (2.42), which are related to the SPSFs by a 2D inverse spatial Fourier transform for every spectral component. We will also need the CZT algorithm described by Eqs. (4.7)–(4.11), which will allow us to simulate measurements in the presence of source or interferometer rotations, as well as allow for faster computation of interferometric measurements by avoiding evaluation of unnecessary values.
Eq. (2.29) can be split into two independent terms that we must compute. Eq. (2.29) is repeated here for the purposes of description, where the scalar multiple has been subsumed into the SPSFs:

\[ I_{3,4}(\theta) = \int_0^\infty \int_{-1}^1 p_{1,1}(\theta - \alpha; \kappa) S_s(\alpha; \kappa) d^2\alpha d\kappa \]

\[ \pm \int_0^\infty \int_{-1}^1 p_{1,1}(\theta - \alpha; \kappa) S_s(\alpha; \kappa) \sin\left[ 2\pi\kappa(\alpha \cdot B + \Delta L) - \Delta \varphi(\kappa) \right] d^2\alpha d\kappa. \]  

(6.1)

If we assume that the interferometer rotates as a whole, from baseline mirrors to detector, it is equivalent to keeping the orientation of the interferometer fixed while the source rotates. As a result, all instances of \( \alpha \) in Eq. (6.1) can be replaced by \( \alpha' = (\alpha \cos \omega, \beta \sin \omega) \), where \( \omega \) is the angle of rotation.

First we discuss how the CZT algorithm is used and how it is beneficial for faster simulations. The CZT algorithm allows us to compute the rotated 2D Fourier transform, or its inverse, for each wavelength (or wavenumber) in the source spectral density \( S_s \), represented by a 3D hyperspectral array. The input hyperspectral array has much finer sampling than the low-spatial-resolution images measured by the interferometer. The ratio of spatial samples in the input images and output images depends on system parameters. Assuming that the measured images are at least Nyquist sampled for all wavelengths being measured, the input/output sampling ratio is related to the ratio between the maximum baseline length and the diameter of the input apertures to the interferometer. For the case of WIIT, which is additionally limited by the spatial sampling of CHIP, the input hyperspectral image is sampled 10 times more finely in either spatial dimension than the measured images. As a result of the Nyquist sampling assumption and multiplication by the SOTFs in the spatial frequency domain, many fewer spatial frequencies contribute to each measurement than would be computed with a fast Fourier transform (FFT). The CZT algorithm allows us to compute only the spatial frequencies in the
central portion of an FFT window that contribute to the measured images, allowing for faster computation of the measurements. The ability to rotate and avoid computing unnecessary samples can also be accomplished through a matrix triple product [3, Chapter 5.2.4], but the CZT algorithm is faster for the sizes of the arrays used for our simulations.

We start by describing the first term in Eq. (6.1), which does not vary with delay line position, because it will help us in describing the interferometric term later. We first compute the inverse Fourier transform using the CZT algorithm, resulting in a rotated 2D inverse Fourier transform of the input image over the window of interest for each wavelength. We multiply the result by the SOTF of the system, \( \tilde{p}_{1,1}(f_{\theta}; \kappa) \), which has values that fall to zero before the edge of the array of the computed spatial frequencies. Using a 2D FFT, we Fourier transform the resulting arrays, which are now much smaller than the input arrays. After taking the sum over all wavelengths, we are left with the first term of Eq. (6.1).

The first few operations for computing the interferometric term are similar to what is described above and are performed for each spectral component independently. Again, we used the CZT algorithm to compute the rotated inverse Fourier transform of the input images over the region of interest around the DC pixel of the array. This time, however, we used the CZT algorithm to perform a shift in addition to the rotation, where the shift occurs after the rotation but is computed in one step. The rotation and shift cause the spatial frequency given by \( \kappa B_n \) (see Eq. (2.45)) to be located at the DC pixel of the array. We then apply the SOTF to incorporate the imaging properties of the system and use a 2D FFT to return to the spatial domain, as described for the first term of Eq. (6.1). This time we do not take the sum over all wavelengths; instead, we apply a 1D DFT to every pixel in the scene, which relates the spectral content at each pixel to the
interferogram caused by scanning the delay line. Now we have both components of Eq. (6.1) and can simulate wide-field double-Fourier measurements.

6.3 Simulated WIIT Dataset and Image Synthesis

We used the procedure described in the previous section to generate a dataset that is similar to data that can be collected with WIIT. The scene used for simulations is the scene described in Chapter 5.4 and 5.5, with the assumption that CHIP could generate arbitrarily complicated hyperspectral imagery for WIIT. In this case we map the 1 micron increments of the 25–400 micron far infrared scene to 1 nm increments from 420–795 nm. We assumed that the aberrations from both arms of the interferometer are identical, and we used the phase retrieval results from Chapter 3 to generate the system SOTF. The scan range was set to create path length differences ranging from -140 to 140 microns with 0.1 micron spacing. The baseline lengths ranged from 36 to 226 mm in 10 mm increments. The rotation angles were chosen to also mimic those that WIIT would measure. If $|B_n|_{\text{mm}}$ is the baseline length in mm and $\lfloor \cdot \rfloor$ denotes rounding down to the nearest integer, the number of angles for each baseline length in the range from $[0^\circ, 180^\circ]$ is given by

$$\left\lfloor \frac{\pi |B_n|_{\text{mm}}}{20 \text{ mm}} \right\rfloor + 1.$$ (6.2)

After simulating the dataset, we added noise representing a realistic model that is shot-noise limited. First we assumed that the integration time of the camera is constant for the entire data collection and that the peak signal for the dataset filled up 80 percent of the 16-bit well-depth of the camera. The camera gain was assumed to be 1, which is true for WIIT’s CCD camera, so 52,428 photoelectrons were assumed measured for the strongest signal in the dataset, corresponding to a peak signal-to-noise-ratio (pSNR) of 229. Gaussian read noise with a standard
deviation of 10 photoelectrons was also added to the dataset. An example image and fringes from the dataset are shown in Fig. 6.1. The image on the right is the average over the delay line dimension, which is an approximation of the first term in Eq. (6.1) (see Eq. (2.36)). The image on the left shows the fringes at two pixels corresponding to strong sources in the scene.

The CZT algorithm was used to resample all of the images from the dataset to a single orientation prior to applying the image synthesis algorithm from Chapter 2.4 to process the described dataset into a single high-spatial-resolution hyperspectral image. The top row of Fig. 6.2 shows the result of the synthesis algorithm, and, for comparison, the bottom row shows the ideal result of filtering each spectral component of the input hyperspectral image by the effective OTF of the interferometric system, which is equivalent to convolving each spectral component by the “dirty” beam of the interferometer. Aside from the noise, there is excellent agreement between the synthesized image and the ideal result.

It is worth noting that the synthesized and ideal spectra are greatly impacted by the effective OTF of the interferometer, where both baseline sampling and characteristics of the interferometer’s imaging camera play a role. The quality of the images and spectra can be

![Figure 6.1](image)

Figure 6.1. An example from the simulated dataset after adding noise where (a) shows an example of two fringes from a single datacube and (b) shows the average along the OPD dimension for the same datacube.
Figure 6.2. The top row shows the result of applying image synthesis to a simulated interferometric dataset, and the bottom row shows the result of passing the original scene through the effective OTF of the system for each spectral sample. (a) and (c) are the spectra from 4 spatial pixels and (b) and (d) are the spectral averages of the hyperspectral images.

improved by applying deconvolution techniques, such as the one described by Thurman and Fienup [5] for Fizeau Fourier transform spectroscopy, which is also capable of reconstructing information about low spatial frequencies that were not measured through the use of a gray-world approximation. In this technique, a Wiener filter is used to deconvolve spatial frequencies covered by the effective OTF of the system, while the first term of Eq. (6.1) is combined with a gray-world approximation to estimate low-spatial frequencies that are not measured interferometrically. We applied this technique to our synthesized data and found that the
accuracy of the spectra was greatly improved. This time, we compare our results to a perfect deconvolution, which is equivalent to filtering each spectral component of the input image with the support of the interferometer’s effective OTF. We included the low spatial frequencies in the support for the effective OTF because we applied the gray-world approximation in our deconvolution in an attempt to recover some of the missing low spatial frequencies.

Fig. 6.3 shows the comparison between the (top) deconvolution technique that we applied and the (bottom) ideal deconvolution. The noise was amplified much more at shorter wavelengths because there was much less signal and spatial structure at those wavelengths. Aside from the noise, there is quite good agreement between the two scenes. The overall shape of the spectral continua match that for the ideal deconvolution, and most of the strong emission lines are recovered, although the strength of some of the emission lines are over- or underestimated, possibly due in part to noise. All emission lines that are comparable in strength to the noise are indiscernible. The impact of missing low spatial frequencies is most noticeable at the shorter wavelengths where the scene is primarily a uniform value for all pixels due to zodiacal emission in the modeled scene. This is observed in Fig. 6.3-(d) as the diffuse background between the galaxies in the scene. The diffuse zodiacal emission cannot be captured well without conventional Fourier transform imaging spectroscopy measurements. The effect of missing low spatial frequencies was not as detrimental at longer wavelengths where the scene is dominated by strong galaxies that are spatially sparse. We expect the impact of missing low spatial frequencies to be more dramatic for scenes having more low-order spatial content, such as for the image of a single galaxy and for extended infrared cirrus emission from interstellar dust particles.
Figure 6.3. The top row shows the result of applying Wiener filtering and gray-world approximations to the synthesized hyperspectral image, and the bottom row shows the result of an ideal deconvolution for each spectral sample, including the low spatial frequencies that were not captured in the simulated dataset. (a) and (c) are the spectra from 4 spatial pixels and (b) and (d) are the spectral averages of the hyperspectral images.

It is possible to filter the data to make the continuum spectra smoother through convolution, particularly with some prior knowledge about the spectra. If we know that the spectral features are composed of emission lines on top of the continuum spectra, a median filter can be used to remove the emission lines so that we can apply a convolution or basis decomposition to the continuum to make it smoother. Fig. 6.4 shows the result of applying a median filter to remove emission lines before convolving the spectral continua by a Gaussian
with a full-width at half-max of 10 spectral bins. The emission lines are added back to the spectra after smoothing the continua. Fig. 6.4 (c) and (d) are again the result of the ideal deconvolution, are identical to Fig. 6.3 (c) and (d), and are included for comparison. It might also be possible to improve the effects of noise by using a nonlinear deconvolution algorithm so that regularization metrics, such as L-1 norms in the spatial domain and total variation metrics in the spectral domain, can be incorporated.

Figure 6.4. The top row shows the result of smoothing the spectral continua of the deconvolved hyperspectral image with a Gaussian filter while leaving emission lines intact by first removing them using a median filter, and the bottom row shows the result of an ideal deconvolution for each spectral sample, including the low spatial frequencies that were not captured in the simulated dataset. (a) and (c) are the spectra from 4 spatial pixels and (b) and (d) are the spectral averages of the hyperspectral images.
6.4 Conclusion

We have described how to simulate wide-field spatio-spectral measurements using a CZT algorithm, allowing for source/interferometer rotations and the inclusion of system aberrations through SOTFs. We then simulated a dataset that is representative of a somewhat idealized version of WIIT, where we incorporated the aberrations estimated with phase retrieval in Chapter 3. The synthesis algorithm from Chapter 2.4 was used to reduce the simulated dataset into a single high-spatial-resolution hyperspectral image. We improved upon the results from the synthesis algorithm through Wiener filtered deconvolution and estimation of low spatial frequencies with a gray-world approximation. Finally, we showed that prior knowledge of the scene’s spectra can be used to help reduce the noise in the deconvolved image, and we proposed that the result could possibly be improved through nonlinear deconvolution and the incorporation of regularization metrics, which we leave for future work. These techniques need to be applied to experimental datasets from WIIT, but the experimental datasets must be preprocessed sufficiently before the techniques in this chapter can be applied.
6.5 References


Chapter 7

WIIT Data Preprocessing and Phase Referencing

7.1 Motivation

In order for WIIT measurements to be analyzed and synthesized, they require preprocessing to remove known system-induced features and errors. Although WIIT resides in the well-controlled environment of the Advanced Interferometry and Metrology laboratory at Goddard Space Flight Center (GSFC), there are aspects of the interferometer that require the data to be preprocessed prior to employing image synthesis techniques. For example, there is heat generated by WIIT’s imaging camera and CHIP that cause mild motion between image measurements for a single baseline length and orientation. Also, the lamp that provides the illumination for CHIP fluctuates in its overall intensity during each delay line scan, causing features in the collected data that appear as low-order signatures for the fringes at each pixel. Due to the micromirror devices that control how the hyperspectral projector operates, CHIP also imparts a spatially variant background on the scenes being measured. The detector of WIIT’s
imaging camera also has hot pixels, typically comprising the bottom few rows of pixels in the image frame, that must be removed in processing. Finally, baseline orientations are varied in WIIT by digitally rotating the scene being projected by WIIT, so all baseline measurements must be registered and resampled to the same orientation prior to applying the synthesis algorithm in Chapter 2.4. The preprocessing of WIIT data can be thought of in two categories: 1) preprocessing that occurs separately for each baseline length and orientation and 2) preprocessing that requires the entire dataset or subsets of the dataset. To aid in the preprocessing of the data described in Sect. 7.2, a collection of image measurements are taken to identify the background imparted by CHIP and the imaging camera before starting each delay line scan.

The final step in the preprocessing procedure is known as phase referencing. Although it is part of preprocessing, we think about it separately from the rest of the preprocessing steps because it has some unique challenges and is vital for relating each datacube within the entire WIIT dataset to the other datacubes, which is a necessity for image synthesis. Phase referencing for WIIT is needed due to limitations in the laser metrology system [1] used to monitor the optical path difference (OPD) during delay line scans. During any particular scan of the delay line, the metrology system provides relative OPD measurements to within 10 nm. There are also encoders on the delay line stage that provide coarser (~100 nm) knowledge of its absolute position. However, there is no accurate knowledge of the absolute path length in both arms of the interferometer. Additionally, there is a tip/tilt mirror in one arm of the interferometer to make sure that the images from both arms are overlapping in the image plane, which creates an additional unknown for determining the absolute OPDs during data collection. These complications require that the absolute OPDs be estimated after data collection by using fringes in the measurements generated by known reference sources embedded into the scene being
projected by CHIP. In Sect. 7.3, we will discuss how these references sources can be used in a phase referencing procedure. This procedure is important because the location of the zero path difference (ZPD), which is the center of the fringe packet (see Eq. (2.33)), for any particular pixel provides information about the sub-pixel location of the sources within that pixel.

The results shown in this chapter are from a WIIT dataset that resulted from applying the approach in Chapter 5 to prepare the deepfield scene, shown throughout this thesis in various forms, for CHIP. For phase referencing and analysis purposes, the scene has various reference sources embedded outside of the deepfield portion of the scene, which were also included in the simulated scene in Chapter 4. There are 4 point-like reference sources aligned with the corners of the deepfield scene and 4 binary reference sources aligned with the edges of the deepfield scene, all of which rotate with the scene. The binary sources consist of two point-like sources separated by distances small enough that they appear as single sources in WIIT measurements. An additional 4 point-like sources, located furthest from the deepfield scene, remain fixed throughout scene rotation. The fixed sources are masked out for the purposes of registration in the preprocessing procedure of Sect. 7.2. All of the point-like sources have a diameter of a single CHIP pixel and use the entire spectral output of CHIP’s light source. Because the scene is rotated digitally, the single CHIP pixel is typically resampled over the nearest 4 pixels.

7.2 WIIT Data Preprocessing

The procedure for preprocessing WIIT datasets starts by processing each datacube, comprising all measurements associated with the delay line scan for each baseline length and angle, independently. First, the fixed background created by the hot pixels of WIIT’s imaging camera are removed from all background and data measurements. Because the portion of the detector used for each dataset is not the same, the hot pixels are identified for each dataset prior
to processing. Before averaging the remaining background frames together, the background frames are registered using a cross-correlation approach [2] such that they no longer suffer from image shifts caused by the mild turbulence in the system. All image measurements within each datacube are also registered to the first measured frame of the delay line scan. Prior to removing the background caused by CHIP from all the data frames, we first remove the intensity fluctuations caused by CHIP’s illumination source. This is accomplished by using a high-pass filter designed with the spectral bandpass of CHIP in mind, which leaves us with the desired fringes while removing the low-order signatures caused by the lamp fluctuations. Fig. 7.1 shows (a) the high-pass filter in the wavenumber domain and (b) its Fourier transform pair that shows what the convolutional filter looks like in the OPD domain. The filter shown in Fig. 7.1-(a) might not be ideal but appears to produce reasonable results. We removed the mean value at each pixel prior to filtering and added the mean value back after filtering. The averaged background image is then registered to the average of the data frames prior to removing the background from Figure 7.1. (a) The high-pass filter in the wavenumber domain and (b) the truncated convolutional high-pass filter in the OPD domain. (a) is the Fourier transform pair of (b).
all data frames in the datacube. Fig. 7.2 shows the OPD averaged image (b) before and (d) after this preprocessing procedure for a WIIT measurement with a baseline length of 226 mm and a source rotation of zero degrees. Fig. 7.2 (a) and (c) show the interferograms for 4 of the point-like reference sources before and after preprocessing, respectively. We show the preprocessing result for the 226 mm baseline because the visibilities of the reference source fringes are smaller than for shorter baselines, demonstrating the importance of filtering out the lamp fluctuations, which can be comparable in amplitude to the desired visibilities. The filtering process could be

![Graph](image1)

**Figure 7.2.** Results from preprocessing a single datacube by removing unwanted background illumination and correcting for fluctuations in CHIP's light source, where (a) and (c) show fringes for the 4 point-like reference sources that rotate with the scene before and after preprocessing, respectively. (b) and (d) show the images resulting from averaging over the path length domain before and after preprocessing, respectively.
improved in the future by making independent measurements of the light source’s fluctuations during data collection. In an ideal system the vertical displacement of the interferograms in Fig. 7.2 (a) and (c) should be identical, but a nonuniform spatial illumination in CHIP causes the mismatch in measured irradiance of the reference sources. Future work will be needed to account for the nonuniform illumination in CHIP. We also notice that the background illumination from CHIP takes up about three thousand counts of the imaging camera’s 16-bit well-depth, accounting for about 5 percent of the usable dynamic range.

Once each datacube has been processed independently, registration of the entire dataset is possible. First, we take the average data frame from each datacube. We then use the chirp z-transform (CZT) image registration approach from Chapter 4 to register all the images to the image from the first datacube in the dataset. The rotation angle between measurements is known very well because source rotation is performed digitally prior to image projection, but there are remaining image translations that must be recovered. We use the point spread function of WIIT’s imaging camera that we estimated in Chapter 3 to aid in the CZT registration procedure. The CZT algorithm is then used to orient all data frames based on the results of the image registration algorithm. It might be possible to improve the results of the registration by taking the results of the initial registration for all datacubes and averaging them together to improve the signal-to-noise ratio and using that as the master frame for a second registration.

7.3 Phase Referencing for WIIT

Phase referencing is the name for the procedure we use to relate the OPD measurements between datacubes and recover absolute OPD values from the relative OPD values that are measured. Due to the fact that the tip/tilt mirror in the system is fixed for all orientations at a given baseline length and changed for each new baseline length, the phase referencing procedure
we have developed is performed independently for each baseline length. Our phase referencing procedure relies on the reference sources embedded within the scenes projected by CHIP. The point-like, broadband nature of the reference sources provides access to narrow fringe packets with high visibility out to the longest baseline length of WIIT. We use the inner 4 reference sources that rotate with the science scene that we intend to recover, as seen in Fig 7.2 (a) and (b). Although the sources appear arranged in a square pattern, the arrangement is slightly trapezoidal to remove symmetry. Now that we have described the reference sources, we can describe how we use them to perform phase referencing.

The phase referencing technique relies on tracking the ZPD positions of all the reference sources as the baseline length and orientation change. Accordingly, the first step in the phase referencing procedure is to estimate the ZPD locations for all reference sources in each datacube. The ZPD locations correspond to the location of maximum fringe visibility, which is also where the envelope of the fringe packet is maximal. The envelope of the fringe can be computed using a Hilbert transform of the fringe to obtain the complex analytic signal (see Eq. (2.39)). The envelope is then obtained by taking the absolute value of the analytic signal. In an ideal case, we could just find the location of the envelope’s peak and use that as an estimate for the ZPD. In experimental applications, sampling and system imperfections require additional steps to get a better estimate of the ZPD location. One complication in WIIT measurements is that the fringe packets are not quite symmetric about the ZPD as they would be in an ideal system. This, combined with sampling effects, can produce envelopes that are skewed or have multiple peaks. We combat this issue by fitting a Gaussian curve to the calculated envelopes and using the peak of the Gaussian as the location of the envelope’s peak. Fig. 7.3 shows the fringes for the 4 phase reference sources for a baseline length of 56 mm and an orientation of 22.5 degrees, providing
separation between the fringe packets and large visibilities. Fig. 7.3 (a) shows the envelope computed by the Hilbert transform technique and (b) shows the Gaussian fit to the original envelopes. The markers below the fringes in (a) and above the fringes in (b) show the estimated ZPD values recovered from using the envelopes in each plot.

Figure 7.3. Example fringes from a WIIT measurement with a 56 mm baseline and a scene oriented at 22.5 degrees showing (a) the envelopes of the fringes using a Hilbert transform method and (b) after fitting Gaussians to the envelopes from (a). The markers below the fringes in (a) and above the fringes in (b) show the estimated ZPD values recovered from using the envelopes in each plot.
envelope. In WIIT, the fringes are created by the interferometric term in Eq. (2.29), so they are the sum of many sinusoids; therefore, the value of the interferometric term is zero at the ZPD, so we find the zero-crossing closest to the envelope’s peak and use that as the ZPD. Fig. 7.3 shows the locations of estimated ZPD’s below the fringes in (a) and above the fringes in (b), allowing for better visualization of how using the Gaussian fit changes the predicted ZPD location. In this example, using the Gaussian fit changes the location of the predicted ZPDs by 0 to 635 nm. For any particular baseline length, we now have estimates for the ZPDs of each reference source at all source orientation.

We know how the ZPDs of the reference sources should change with baseline length and source orientation through Eq. (2.33), which we repeat here:

$$\Delta L_{ZPD} = -\alpha \cdot B.$$  \hspace{1cm} (7.1)

In WIIT, the baseline vector is always aligned along a single dimension, so \( B = (B_x, 0) \). The location of the source along the x-axis, \( \alpha_x \), changes as the scene is rotated, allowing us to expand Eq. (7.1) to allow for changing scene orientation through the nominal rotation angle \( \phi \):

$$\Delta L_{ZPDn} = -\alpha_x B_x$$

$$= -\left[ (\alpha_{xn} - \alpha_{xc}) \cos \phi - (\alpha_{yn} - \alpha_{yc}) \sin \phi + \alpha_{xc} \right] B_x,$$  \hspace{1cm} (7.2)

where \((\alpha_{xn}, \alpha_{yn})\) is the location of the \(n\)th phase reference source when \(\phi = 0\), and \((\alpha_{xc}, \alpha_{yc})\) is the center of rotation for the scene, which is the same for all of the reference sources. For the purposes of creating a model to which we can fit our estimated ZPD values, we must also account for the fact that the measured OPD values that we have are all relative. For this we add an extra term \(\Delta L_{ZPDo}\) to Eq. (7.2) that is constant for all rotation angles at a given baseline length:

$$\Delta L_{ZPDn} = -\left[ (\alpha_{xn} - \alpha_{xc}) \cos (\phi + \Delta \phi) - (\alpha_{yn} - \alpha_{yc}) \sin (\phi + \Delta \phi) + \alpha_{xc} \right] B_x + \Delta L_{ZPDo},$$  \hspace{1cm} (7.3)
where $\Delta \phi$ accounts for an unknown deviation from the nominal rotation angle between the image produced by CHIP and WIIT’s baseline mirrors. $\Delta \phi$ should be constant for all measurements in a WIIT dataset, and if WIIT itself is not perturbed, it should also be constant across datasets.

The unknown rotation angle $\Delta \phi$ can be isolated from all other phase referencing parameters by taking advantage of known source separations. In WIIT, we know the source separations with high accuracy because we create the scene. To exploit this information for any particular baseline length, we subtract ZPD location of the $m$th phase reference source from the $n$th:

$$\Delta L_{ZPDn} = \Delta L_{ZPDn} - \Delta L_{ZPDm} = -\left[\left(\alpha_{xn} - \alpha_{xm}\right)\cos(\phi + \Delta \phi) - \left(\alpha_{yn} - \alpha_{ym}\right)\sin(\phi + \Delta \phi)\right] B_z,$$

(7.4)

where the nominal rotation angles $\phi$ and relative source positions $\left(\alpha_{xn} - \alpha_{xm}, \alpha_{yn} - \alpha_{ym}\right)$ are known exactly, and the baseline length is known very well through stage encoder readings. Because we have 4 phase reference sources, we have estimates for source ZPD separations that trace out 3 sinusoids. We use this model, along with nonlinear optimization, to solve for $\Delta \phi$ for each baseline length independently. With the WIIT dataset shown in this chapter, we found that $\Delta \phi = -0.518^\circ \pm 0.107^\circ$. Figure 7.4-(a) shows the estimated ZPD separations and how the model fits to those estimates for a baseline length of 66 mm, where $\Delta \phi$ was set to $-0.518^\circ$. Figure 7.4-(b) shows the differences between the estimated ZPD separations and the model for the same 66 mm baseline length. The average residual between the modeled and estimated ZPD separations is 41 nm with a standard deviation of 298 nm. The OPD measurements are sampled every 100 nm for this dataset, so the error in the residuals corresponds to an error of about 3 OPD samples.
To recover the rest of the phase reference parameters, we return to the ZPD location model of Eq. (7.3). The remaining unknown parameters are \((\alpha_x, \alpha_y)\) and \(\Delta L_{ZPD0}\). The pixel locations of the phase reference sources, \((\alpha_x, \alpha_y)\), are determined by taking the centroids in the measured images after registration combined with knowledge of WIIT’s plate scale. The values for \((\alpha_x, \alpha_y)\) and \(\Delta L_{ZPD0}\) are determined for each baseline length independently through nonlinear optimization of Eq. (7.3) with respect to the estimated ZPD values for all 4 reference sources. Fig. 7.5 (a) shows the estimated ZPDs and how the model fits to those estimates for a baseline length of 66 mm, and Fig. 7.5 (b) shows the residuals between the modeled and estimated ZPDs.

Figure 7.4. Recovery of the deviation from the nominal rotation angles between the source image and the baseline mirrors where (a) is a plot showing estimated (markers) and modeled (lines) ZPD separations, and (b) shows the residuals between the modeled and estimated ZPD separations.

Figure 7.5. Recovery of phase referencing parameters from ZPD locations that were estimated from phase reference sources, where (a) is a plot showing estimated (markers) and modeled (lines) ZPDs, and (b) shows the residuals between the modeled and estimated ZPDs.
estimated ZPDs. The average residual is 0.0 nm with a standard deviation of 230 nm, corresponding to an error larger than 2 OPD samples. The similarity in the standard deviations of the residuals in Fig. 7.4 (b) suggests that determining the source positions through a centroiding process is likely not the largest contributor to errors in phase referencing. Ideally, we would like to know the ZPD locations with comparable accuracy to our knowledge of the OPD values, which we know to within ±10 nm. More studies need to be performed to understand how much of an error in estimates of absolute OPD values can be tolerated to successfully reconstruct an image. This is important because we are now going to discuss using the phase reference parameters to correct the measured OPD values, but we will show that our current results do not appear to be adequate for synthesizing a hyperspectral image.

We can determine the corrected OPD values $\Delta L^{corr}$ from the measured OPD values $\Delta L$ by using our estimates of the phase referencing parameters through

$$
\Delta L^{corr} = \Delta L - \left\{ -\left[ -\alpha_{xc} \cos (\phi + \Delta \phi) - \left( -\alpha_{xc} \right) \sin (\phi + \Delta \phi) + \alpha_{xc} \right] B_x + \Delta L_{ZPD0} \right\} \\
= \Delta L - \left\{ \alpha_{xc} \cos (\phi + \Delta \phi) - \alpha_{sc} \sin (\phi + \Delta \phi) - \alpha_{xc} \right\} B_x + \Delta L_{ZPD0} 
$$

(7.5)

The corrected OPD values are computed for each baseline length independently. If $(\alpha_{sc}, \alpha_{xc})$ are determined as values relative to the DC image pixel, the DC pixel now has an OPD value of zero for all rotations and all baseline lengths. These corrected values can now be used in the image reconstruction process. The image synthesis algorithm also requires that we know the correct rotation angle, so the corrected rotation angles are given by $\phi^{corr} = \phi + \Delta \phi$.

We performed phase referencing for the entire dataset that we have used throughout this chapter, but we did not see the desired results when we tried to perform image reconstruction. For example, if we apply image synthesis to WIIT measurements at any particular baseline
length, we should obtain a dirty image that resembles something close to the ideal dirty image, which can be computed with knowledge of the effective transfer function of the system at the desired baseline length. Figure 7.6-(b) shows the spectral average of the ideal dirty image for the 4 phase reference sources for a baseline length of 66 mm and also incorporated the aberrations of the imaging lens that were estimated in Chapter 3 through phase retrieval. Figure 7.6-(a) shows the result of applying our image synthesis algorithm to the WIIT measurements taken at the same baseline length and taking the spectral average. It is obvious that Fig. 7.6-(a) looks very different from Fig. 7.6-(b). We are currently uncertain if the discrepancy in Fig. 7.6 is the result of inadequacies of the phase referencing approach or additional systematic errors induced by WIIT itself. It is possible that we are unable to achieve adequate image synthesis due to errors in estimation of the ZPD values, perhaps caused by errors in WIIT, or due to additional aberrations.

Figure 7.6. Comparison between the spectral averages of (a) the synthesized phase reference sources after correcting OPD measurements and (b) the ideal dirty image of the phase reference sources for a baseline length of 66 mm
for which we have not accounted. We think our inability to synthesize the expected image is related to the unanticipated asymmetry in the fringe packets of the phase reference sources, which is observed, for example, in Fig. 7.3. The asymmetry could possibly be related to residual errors from the fluctuations in CHIP’s illumination or possible unknown chromatic aberrations. We leave identification of this unanticipated error as future work, mentioned in Chapter 8.2.3. If the asymmetry is not due to CHIP’s illumination, it is possible that the chromatic aberrations could also have a spatial dependence, which would further complicate the required correction. We think this possibility could exist because the nature of the fringe asymmetry is not consistent for all phase reference sources in a single measurement.

7.4 Conclusion

We described an approach to preprocessing WIIT data to account for known systematic errors, including unwanted background illumination and fluctuations in the strength of the illumination from CHIP’s light source. We also applied our image registration algorithm to put all measurements in the same orientation and account for unwanted image shifts during data collection. We outlined an approach to perform phase referencing using known point sources within a scene, allowing us to relate relative OPD measurements between baseline lengths and obtain estimates for the absolute OPDs for WIIT measurements. Unfortunately, our phase referencing approach was not enough to obtain the anticipated dirty images of the phase reference sources, suggesting that there are additional unknown errors in WIIT measurements that must be determined.
7.5 References


Chapter 8

Conclusion and Future Work

8.1 Key Contributions

This thesis has made substantial contributions to various aspects of wide-field spatio-spectral interferometry through both theoretical groundwork and image processing techniques. We have also expanded the capabilities of the Wide-field Imaging Interferometer Testbed (WIIT) and improved the quality of the signals in testbed measurements through postprocessing. The most significant contributions are summarized as follows:

1. We derived the measurement model for a dual-aperture wide-field double-Fourier interferometer using Fresnel propagations, showing that, similar to the van Cittert-Zernike theorem, this technique is applicable to source distances shorter than allowed by the Fraunhofer approximation. We derived an image synthesis algorithm for double-Fourier measurements and used it to discuss imaging properties of the interferometer. Understanding
the imaging properties of the interferometer makes it obvious that deconvolution or further processing beyond the image synthesis algorithm is necessary to account for gaps and redundancies in the effective OTF of the interferometer.

2. We applied the image synthesis algorithm to simulated data to show that the result is comparable to convolving each spectral component of the source hyperspectral image with the effective wavelength-dependent OTF of the interferometer. We also showed that the effective OTF has a large impact on the synthesized spectra and that deconvolution can be used to get a more accurate representation of the source spectra.

3. Although the CZT rotation algorithm has existed for quite some time, we have extended the capabilities of the technique to perform image registration and deconvolution when paired with a nonlinear optimization algorithm. We also showed that CZT resampling is beneficial for the simulation of wide-field double-Fourier interferometric measurements where source or interferometer rotations are needed. The CZT algorithm can also provide a substantial acceleration to simulations when the source hyperspectral image has a much larger spatial resolution than the measured images.

4. We have helped improve the complexity, quality, and understanding of WIIT measurements. A simple phase retrieval experiment provided an estimate of the underlying aberrations of WIIT’s imaging camera, which is important for understanding and processing WIIT measurements. We developed an NMF approach to decomposing complicated hyperspectral scenes into a basis set that is capable of being projected by CHIP within physical time constraints set by the delay line stage and the CCD camera in WIIT. Image preprocessing procedures were improved by taking advantage of the CZT registration. Data preprocessing was also improved by recognizing that CHIP’s light source fluctuates during data collection,
requiring that the low frequency fluctuations be compensated for in the measured data. We also developed a phase referencing algorithm that takes advantage of known point sources within a scene in order to relate the relative OPD measurements from any particular baseline length and orientation to another, as well as determine the absolute OPDs from the relative OPD measurements.

8.2 Future Work

Before wide-field spatio-spectral interferometry is adopted for space-based applications, there are many more aspects of the technique that require investigation. Also, the ability to synthesize hyperspectral images from WIIT measurements will require a better understanding of the experimental limitations of WIIT.

8.2.1 On-the-fly Measurements and Pointing Errors

In this thesis, we have assumed that interferometric measurements are taken using the stop-and-stare approach, where the orientation of the source/interferometer is stationary so that the baseline orientation is fixed throughout each delay line scan. A dual-aperture space-based interferometer would likely rotate continuously during data collection causing a rotational smearing. The limitations on how fast an interferometer would be able to spin while maintaining the ability to provide useful data will need to be understood. The possibility of including knowledge about the interferometer’s rotation into the image synthesis algorithms will also need to be investigated.

Other practicalities of on-the-fly measurements need to be investigated as well. For example, it might be useful to have the interferometer rotate faster at shorter baseline lengths where the SNR of the interferometric signal is larger and slower at longer baselines where the interferometric signal is fainter. The camera’s integration time could be related to the speed of
rotation in order to achieve better SNR at longer baselines and take advantage of the dynamic range of the camera that would not be used if the integration time were the same for all baseline lengths. This would be ideal because the angular momentum would cause the two telescopes to rotate faster as they are brought closer together.

This thesis also assumed that both apertures of the interferometer are pointed in the desired direction. It is possible that the whole interferometer or each aperture individually could have pointing errors [1]. We also assumed that the beams from both arms of the interferometer overlap perfectly at beam combination, which might not be true for an interferometer launched into space. The robustness of image synthesis to these types of realistic errors needs investigation.

8.2.2 Improvements to Image Synthesis

Although we have demonstrated that the image synthesis algorithm from Chapter 2 combined with deconvolution techniques demonstrated in Chapter 6 can be quite effective, there is still much more to be understood. The possibility of using more sophisticated approaches to image synthesis, such as the use of nonlinear optimization algorithms and various regularization techniques, could provide advantages in the presence of noise and sparsely sampled baselines. There is a plethora of research in image reconstruction for ground-based interferometry, so it would be important to investigate the possibility of modifying the state-of-the-art algorithms [2] for the application of wide-field spatio-spectral interferometric image synthesis. It might also be useful to investigate post-processing techniques for conventional Fourier transform spectroscopy (FTS) for applicability; however, one of the most common post-processing techniques [3,4] for conventional FTS involves removing a linear phase term from the spectra resulting from interferograms not being centered where the OPD is zero, which would not be good for double-
Fourier interferometry because the linear phase provides information about the spatial location of the source. These FTS post-processing techniques might be useful for phase referencing or diagnosing error in WIIT measurements, which we discuss in Sect. 8.2.3.

In addition to improving upon image synthesis, a thorough understanding of the impact of missing low spatial frequencies is necessary. Synthesis of a variety of test scenes is needed both with and without conventional FTS. Those results need to be compared to estimation of low spatial frequencies through the gray-world approximation, as was done in Chapter 6.3. It would also be interesting to attempt using low-spatial-resolution spectral measurements from imaging modalities other than FTS to fill in the missing low spatial frequencies.

8.2.3 Phase Referencing and WIIT Preprocessing

We have made many advances to the preprocessing of WIIT data, but there are still aspects that could use improvement. For example, we used postprocessing techniques to remove the fluctuations in the interferograms due to fluctuations in the power emitted by CHIP’s light source. This postprocessing would greatly benefit from independent measurements of CHIP’s emitted power during data collection. We also notice that there is a spatially dependent illumination induced by CHIP. This is caused by nonuniform illumination on CHIP’s spatial engine. This creates an effect where as we rotate the source digitally, the source strength at a given location within the scene is changing as a function of rotation angle when ideally it would be constant with rotation angle. In order the mitigate this problem, the nonuniform illumination in CHIP could be estimated and calibrated out of the measurements.

Although we have demonstrated a phase referencing algorithm on WIIT measurements, we are still currently unable to successfully apply image synthesis to a WIIT dataset. It is unclear whether this is a limitation with our current phase referencing approach or an unknown source of
error in WIIT itself. A peculiar aspect of the interferograms associated with the phase reference sources is that some of the fringe packets are asymmetric. We are currently uncertain what is causing this asymmetry, but it makes estimating the ZPD location of the fringe more difficult and error prone. We suspect that there are ways of improving our ZPD estimates and the quality of the fringes using techniques that exist for FTS. For example, if we subtract off the ZPD estimate from our OPD measurements for each phase reference source, we can apply methods used in FTS used to center the fringe, which would refine our ZPD estimate. At the same time, we would be able to remove any additional phases in the spectral domain that prevent the Fourier transform of the fringe from producing an entirely real result. It would be particularly interesting to see if the aforementioned phases are consistent for all reference sources in a scene or even across an entire dataset. We believe that such an approach could improve the existing phase referencing approach.

If phase referencing is performed properly on a dataset that is free from other unknown errors, we should be able to produce dirty images of the reference sources from the phase referencing results. It is possible that we might be able to further refine our phase referencing results by comparing the recovered dirty images to the expected dirty images, given known baseline sampling, through nonlinear optimization. Another approach could be to use a sharpness metric to improve the sharpness of the dirty images of the reference sources. However, for the case of WIIT measurements, we need to first understand why our current phase referencing procedure is not allowing us to produce sufficient dirty images of all reference sources simultaneously, as previously mentioned.

We have been developing phase referencing procedures with WIIT in mind, but it is important to consider that phase referencing will likely be necessary in a space-borne
interferometer. The approach of using estimated ZPD values will likely still be useful; however, the sources that are used will likely be different. We currently embed reference sources outside of the field-of-view (FOV) of the desired scene in WIIT measurements. This convenience will likely not exist for most astronomical sources. A different approach would be to use interferometric measurements in a different spectral band where isolated sources are known and are potentially stronger, such as near-infrared measurements of stars in the science FOV of a far-infrared interferometer. The intricacies of performing phase referencing at a different spectral band will need investigation. It will also be interesting to perform studies that show just how many reference sources are needed in a particular scene to perform phase referencing. Space observatories often have different instruments occupying different portions of the entire telescope’s FOV, so it would be useful to explore the opportunity of exploiting a different portion of the FOV for phase referencing than that used for science measurements.
8.3 References


