A TAXONOMY OF GENERATIVE ACTIVITY DESIGN
SUPPORTED BY NEXT-GENERATION CLASSROOM NETWORKS

Walter M. Stroup, The University of Texas at Austin, wstroup@mail.utexas.edu
Nancy Ares, University of Rochester, Nancy.Ares@rochester.edu
Andrew C. Hurford, The University of Texas at Austin, hurfor@mac.com

Never ask a question with only one right answer.
– Judah Schwartz

Abstract: Previous work has examined how new theoretical, methodological, and design frameworks for engaging classroom learning are provoked and supported by the highly interactive and group-centered capabilities of a new generation of classroom–based networks. This network-supported interactivity, coupled with generative design, allows the environment of the classroom itself to be thought of as a group-oriented “manipulative” or mediating tool for teaching and learning. Given that this level of technologically-supported interactivity and group-oriented design is new to classrooms, new challenges for teachers, researchers, and curriculum developers relative to how to think about designing for, and working in, these types of environments need to be addressed. We present a taxonomy of generative activity design that has emerged from our work in developing and then working with next generation systems.

1.0 Introduction

Due to the group-focused interactivity and data collection capabilities of next-generation networking, we have a new tool to explore the dynamics of – and designs for – classroom learning. A number of projects supported by the National Science Foundation have responded to the challenge of advancing learning in these highly interactive networks by using mathematical/scientific ideas to organize and analyze classroom activity. This use of domain-related “big ideas” to study group learning has been discussed as “mathematics structuring the social sphere” [MS3] (Stroup, et. al. 2002; Stroup, Ares & Hurford, in press). Many of the network-based projects began by focusing on student learning (Kaput & Hegedus, 2002; Roschelle & Vahey, 2003; Wilensky & Stroup, 1999). Recently, however, significant efforts have emerged that focus more directly on issues related to supporting, and learning from, teachers’ developing understandings of how best to design for network supported classroom activity. We suggest that there is a kind of “resonance” between technological affordance and generative forms of teaching and learning. Generative design, as we use the term, develops from and supports the core commitments of the reform movements in mathematics and science education and it is this resonance that we look to advance. Toward this end, we present a new taxonomy of kinds of generative activity design useful both in clarifying the relations between kinds of generative activities and in clarifying internal aspects of generative design, especially as supported by next-generation classroom networks.

2.0 Technology and Moving to Group-Oriented Generative Design

There is a significant literature related to generative approaches to teaching and learning. Generative teaching, as discussed by Wittrock, is “a model of the teaching of comprehension and the learning of the types of relations that learners must construct between stored knowledge, memories of experience, and new information for comprehension to occur” (1991, p. 170). What Wittrock means by the learners’ active construction of new “relations” is close to what might be
called constructivist teaching pedagogy. Consequently, generative learning in his framework involves students’ ability to create artifacts that embody their constructed understandings. In a closely related way researchers from the Learning Technology Center at Vanderbilt emphasize aspects of creating “shared environments that permit sustained exploration by students and teachers” in a manner that mirrors the kinds of problems, opportunities, and tools engaged by experts (1992, p. 78). Teaching involves “anchoring or situating instruction in meaningful, problem-solving contexts that allow one to simulate in the classroom some of the advantages of apprenticeship learning” (1992, p. 78). Although many of these previous theories are generative at the level of the individual learner or even at the level of a small group (Lesh, et al., 2000) there is not enough of a picture of how to structure the cross-individual or cross-sub-group learning. Our efforts are directed at extending and reconceptualizing these earlier analyses of generativity to engage the issue of how to design for the diversity and multiplicity of learners’ ideas and insights supports the emergence and development of mathematical and scientific reasoning in group contexts.

Generativity as we discuss it below and elsewhere (Stroup, 1997; Stroup, et. al. 2002; Stroup, Ares & Hurford, in press) focuses on design for the group. The emphasis is on supporting the interactions of teacher and students together in a classroom and natural subsets of this classroom-based grouping. Additionally, we are interested in how mathematical and scientific content itself can frame the design of classroom activities and supportive technologies. Content in this sense is an organized body of knowledge developed over time, which while enacted through activity, still retains coherence and structures the group activity. The diversity of forms of student participation – including linguistic and socio-cultural diversity – is seen as the “engine” that drives the emergence and development of content in this group-oriented approach to generativity (e.g., see Ares & Stroup in this volume). Diversity creates the space of possibilities that students and teachers can use to advance teaching and learning.

The group focus of next-generation classroom networks develops from, and is consistent with, this expanded sense of generativity. These systems are typically designed “from the ground up” with the classroom in mind. Rather than constraining the learning experience to be somewhat narrowly individualistic, these technologies support socially situated interaction and investigation. Typically, each student has a device that allows him or her to participate either synchronously or asynchronously in group-oriented activities. Often the devices have local input, display, and analysis capabilities (e.g., those of a graphing calculator) and can interact in the network with each other and even with other kinds of devices (e.g., data-collection tools like Calculator-based Rangers™ motion detectors as used in the People-Molecules activity developed by Wilensky and Stroup [in review]). The interactions and emergent results can be projected on a public display space – often in real-time – using a computer or calculator projection system. Using this infrastructure and generative design, the learning trajectories and the processes of knowledge construction are owned by the group itself. Software and hardware work together in supporting this “group-oriented” design. The classroom or group, as supported by the network infrastructure, becomes a kind of participatory manipulative for teaching and learning challenging mathematical and/or scientific content.

A significant number of these networks are about to become widely available and are poised to become a major presence in classroom learning. It behooves us, then, to begin to support approaches to optimizing the generative potential of these networks. Although many of the forms and kinds of activities discussed using the pathways and endpoints discussion below can be done without next generation network technology and, indeed, have been used in pre-service teacher
education (cf., Stroop, 1997), the sense is that the highly interactive, and group-focused capabilities of these next-generation systems can support, extend, and add to this kind of activity design for classrooms.

3.0 What are Generative Activities?

Generative activities in group contexts are seen as space creating activities that extend the ideas of co-operation and emergent structure to classroom based activity. Learners create a space – or coordinated collection – of expressive artifacts and actions in relation to some shared task or set of rules. The structures that are created or embodied are not determined in advance but are co-constructed by learners as their sense-making evolves and develops. Students can be asked to create objects or outcomes that are the same or alike in some mathematically or scientifically significant sense (see taxonomy below) and these responses can be arrived at or built from the underlying sameness in a wide range of ways. The sameness gives coherence to the task. Generativity requires that the activity produce or give origin to a diversity of responses. This diversity is then used by the group to explore patterns in the responses and in the structural ways in which the responses might be seen as relating to one another, or of-a-whole. Ideally the space of responses will be large enough so that the kinds of behaviors or expressive artifacts students create give the teacher significant insights into the ways the students are thinking about the task. The activity should be “thought revealing” in this sense (Lesh, et al., 2000), and it should also be capable of giving rise to additional rounds of generative exploration and/or detailed investigation.

4.0 Taxonomy of Generative Activities

The following taxonomy of generative activities is organized in terms of pathways and endpoints. Pathways are intellectual and/or behavioral routes for arriving at a given endpoint. Endpoints are outcomes or artifacts created by learners that represent some form of completion of the given generative task (See Figure 1.).

Figure 1. The taxonomy centers on a pathways and endpoints analysis of generativity.

4.1 Nominally Generative

Using a pathways-and-endpoints representation, the form of generative activity shown in Figure 2 is barely, or nominally, generative because the structure of the activity has an agreed upon endpoint and a single pathway to this endpoint. An example would be asking students to simplify the expression $2x + 3x$. The endpoint would be $5x$ and the path to getting to this endpoint would be the application of a specific rule for simplifying.
Figure 2. Nominally generative activities have one pathway to get to a single endpoint. The “space” created by nominally generative activity in a classroom is a discussion of “right” and “wrong” answers and of how to correctly apply the given rule or procedure. Nominally generative activity accounts for much of traditional, or pre-reform-based, classroom practice. As a design approach it is relatively ineffective in putting students’ ideas at the center of classroom discourse and learning.

In our work with teachers we have sometimes talked about nominally generative activity as deriving from approaches associated with one-on-one tutoring. Whatever the efficacy of asking questions like “What is $2x + 3x$?” is in the context of tutoring individuals, tutoring-type questions tend to “break” as they are pushed to scale beyond one-on-one situations. In a classroom if we are asking one student the $2x + 3x$ question, other students are left with little to do or to contribute and this kind of questioning makes nearly no use of the group itself. As the number students moves from a few to a whole class, the limitations of tutoring-type questions becomes more profound; at any given moment most of the students, of necessity, are not included in the core activity. Rather than critiquing the usefulness of nominally generative questions in group contexts, too often teachers simply use the technology of the copy machine to compensate for, and artificially scale to the group, “tutor type” of questions. Individual copies of nominally generative questions are handed to each student (or, similarly, written on the overhead projector or chalkboard), and each student can now “participate” individually in answering the questions, but the group itself has no constructive role. Moreover the teacher is still confronted with the task of figuring out what, if anything, to do with all these worksheet-based or individual responses.

Computer-assisted instruction (CAI) environments, supported by traditional forms of networking, have attempted to address the issues related to teacher load but they do so in ways that preserve the limited focus on right and wrong answers. In addition, network-supported CAI systems make no use of the group itself as a community of inquiry and insight. “Individualized instruction” is somehow understood to mean the learning must take place “alone” and the responses must be evaluated in terms of right and wrong. More recent “cognitive” tutoring environments, while representing significant improvements over traditional CAI, still retain this very limited sense of what individualized means, and what it might represent as a purported ideal of instruction (cf., Carnegie Learning, 2003). Generative design, as supported by group-supportive networking, moves in a very different direction. Individualization is associated with seeing the unique-ness and diversity of each student’s participation as making an essential contribution to the emergent sense-making taking place in the classroom. Space-creating play (Stroup, Ares, et. al, 2002; Stroup, Ares & Hurford, in press), not item-response convergence, is a central feature of generative instructional designs.
Nominally generative tasks can often be made significantly more generative by simply reconsidering the form of the question. For example, rather than asking students to simplify \( \frac{2}{4} \), ask them to create ten fractions that are the same as \( \frac{1}{2} \). Rather than asking students to simplify \( 1x + 3x \), ask them to find five functions that are the same as \( 4x \). Better use is made of the uniqueness and creativity of each student in the class (everyone is involved in creating new functions) and attention to mathematical structural issues is highlighted. With all of the student responses, or more often a selection of each student’s favorite, projected in front of the class, structure-related questions like the following can be asked: “How can all these functions that look so different be the same?” or “If I added another example, how would you know if it was the same as or different from \( 4x \)?” Inviting students to extend the patterns they’ve observed to work in one context to a novel context also highlights mathematical structure. After completing the \( 4x \) activity, we’ve asked late elementary aged students to create functions that are the same as \( 4\sin(x) \). Attention is now drawn to the form or structure of mathematics that “works” across contexts.

Asking students to pursue multiple paths-agreed upon endpoint tasks allows a larger mathematical space to be explored than would be the case with nominally generative tasks and supports participation in a way that has the potential to significantly advance the group’s engagement with mathematical structure. Additionally, the teacher gets a quick snapshot of where student thinking is. For example, if none of the equivalent expressions involves decimals or negative numbers, the teacher is able to both see the space of kinds equivalence they are comfortable with and areas they may need to explore more. When this happened in some of the classrooms we worked with in Roxbury, Massachusetts the teacher could immediately adjust the flow of the activity in the classroom. One teacher simply asked his students, who were supposed to be familiar with these ideas, “How come?” in a playful/joking way. He then went on to encourage them to include, in the next “round” of expressions, examples of equivalent expressions involving decimals and negative terms.

Unlike the nominally generative tasks described earlier, a final observation we can make based on our work in classrooms is how mathematical creativity, flair, and insight are more likely to be acknowledged and celebrated with this type of task. \( 8x - 4x \) is certainly acceptable (and was praised by the teacher) as an example of an expression that is equivalent to \( 4x \), but \( 1,000,004x - 1,000,000x \) and \( 100x/25 \) are seen as more interesting by the students themselves. We know this, in part, because once these examples were projected for the whole class other students quickly worked on creating similar examples to share. Mathematics serves to structure the social activity of the group – students create, discuss, and share expressions that embody the
idea of equivalence – and at the same time, the social sense of knowing and legitimate participation serve to structure the mathematical activity (Stroup, Ares & Hurford, in press). Sixth graders, within ten minutes of working with graphing calculators for the first time, created these and many other examples and our experience is that the next time an activity like this is done, students want to be the ones creating more interesting examples to “show off” to their peers.

In collaborating with pre-service and in-service elementary and secondary teachers, we began working on this kind of design well before the latest generation of highly interactive classroom networks was developed (cf., Stroup, 1997). Teachers find it useful to think about turning answers (“4x” as in “1x+3x = 4x”) into questions (Can you find expressions that are the same as 4x?). Highly interactive networks support this kind of generativity by allowing expressive artifacts to be readily shared, displayed, recorded and aggregated. Students can submit responses anonymously and then decide later if they want to take ownership of a particular solution (Davis, 2002).

4.3 Modeling - Multiple Paths and Endpoints Where Fit with Data is Central

Modeling has the potential to be a multiple pathway and multiple endpoint activity. Learners can create different models that yield distinct outcomes. A central feature of the subsequent conversation is

![Figure 4. Modeling characterized as series of pathways, endpoints, and comparisons with data (experience).](image)

how well the outcomes of the model fit with the data (whether real or anticipated) or experience (broadly conceived). Unfortunately modeling in classrooms is often pursued as if it was nominally generative. Much of current laboratory work in science classrooms has this collapsed structure. Students are to use a prescribed model (single path) to create computed outcomes that are then mechanically compared to the actual data collected from using tightly scripted “lab” procedures. A similarly collapsed notion of modeling is also what gets discussed when the “application” of a particular mathematical idea is presented in textbooks or classroom presentations. Modeling at its best, however, would have a description closer to that represented in Figure 4. – learners would create a range of models and use them to create model outcomes (implications or predictions). These outcomes would then be discussed in terms of goodness of fit to data (real or anticipated). Structural conversations about the ways in which various models might be similar or distinct can and should occur. In addition, issues related to deciding what it means to fit data can be engaged. The double arrow over modeling indicates that models and outcomes interact iteratively in the sense-making of learners.

As other researchers have noted, in what can be seen as extending aspects of the pragmatists’ notion of “truth” to modeling, “models consist of conceptual systems that are expressed using a
variety of interacting media (concrete materials, written symbols, spoken language)” and are used to organize our experiences and action in the world (Lesh et.al. 2003, p. 214). While a generative sense of modeling can certainly be carried out without network technology, next generation network capabilities allow the students and teachers to make visible and act on the “interacting media” used to express a given set of models. Whether it is a drawing, a sketch in a network-enabled geometry environment, a finite-difference equation, text, voice, or a Logo program, next generation networks offer the potential for making the machine-based interacting media associated with mathematical ideas the coin of the realm in pursuing generative approaches to modeling. Models can be made more visible and can be acted upon directly in a network space. A perhaps more dramatic outcome, in terms of student learning, may be the ways in which network mediated role-playing (discussed below) begins to make visible to students that useful and informative modeling is often the negotiated product of groups of interacting modelers.

4.4 Design Tasks – Multiple Path and Endpoints where Satisfaction of Goal is Central
Design tasks are similar to modeling above in that both are multiple-path and multiple-outcome tasks.

Figure 5. Generative design tasks are like modeling tasks except fit with a goal is central. The difference is that fit with the goal or design specifications replaces the analysis of fit with data. As is true with modeling, structural issues can be ignored in which case designing comes to be only about learners arriving at particular designs. Unfortunately, when design tasks are pursued in classrooms it is often the case that no larger discussions of the processes of design are engaged. Much of the richness and learning potential of design is lost. When these types of lessons are approached in a more generative way structural issues of design, esthetics and even aesthetics are engaged. This kind of task is closest to what the researchers at Vanderbilt refer to as generative teaching and learning. As noted earlier, teaching comes to be seen as being about “anchoring or situating instruction in meaningful, problem-solving contexts that allow one to simulate in the classroom some of the advantages of apprenticeship learning” (1992, p. 78). As with modeling, network supported design tasks can make the design artifacts and representations public and even interactive. Cross design analyses related to what makes a “good” design is more readily supported and apprenticeship begins to be seen as a shared, group, activity.

4.5 Emergent Group Activity
Participatory simulations (cf., Wilensky & Stroup, 1999) and other forms of mathematical and scientific role playing are examples of emergent group activities. Learners assume iconic (like-the-thing) roles in a system and through their interactions create emergent behaviors of that system. Using individual devices learners assume the roles of predators or prey in an ecology simulation, or control individual traffic lights in a simulated city and then work towards
improving traffic flow (Stroup & Wilensky, in review). The idea is that the emergent behavior created by these activities

Figure 6. A range of role-playing activities, including participatory simulations, are fully generative.

becomes the object of attention and analysis as modeling or design tasks (see Figure 4 or 5 above). As with problems raised above, if the sole outcome of participating in an emergent activity is that a certain behavior emerges – and there is no subsequent analysis of how the behavior might have developed, how it might be different in another iteration or under different conditions, or how it might be like or unlike other systems and so on – then little learning of a generative or structural nature is likely to occur. However if thoughtfully utilized, next-generation networking can play a particularly significant role relative to emergent group activities and learning about properties of emergent systems in general.

4.6 Explorations of Kind and Quality of Pathways

This is possibly the most challenging of the generative forms to describe. Using the pathways and endpoints depiction, the focus here is not so much on getting to an endpoint as exploring the “quality” of possible pathways (Figure 7). Not only would this kind of exploration involve the many ways, for example, to prove the Pythagorean Theorem, but it would also include broad issues related to how reasoning moves forward, what the nature of development or improvement is, establishing cross-context similarity in the structure of a given system, structural reasoning proper, generalization, reflective abstraction (as it appears in Piaget’s writings), and how situated-ness helps to determine the nature of reason and belief. Even the notions of domain and systems theory – as it includes complexity theory – exist as kinds of explorations into the nature of structural reasoning. This attention to forms

Figure 7. Exploring the kind and quality of pathways is generative.
of reasoning and the quality of pathways, can be engaged mechanically or by rote as is often the case with many students’ experience of geometric proof or many students’ experience of studying algorithmic design in computer science. But to the extent that these ideas can be engaged generatively, the potential of students attending to the forms of reasoning can assume greater significance in group-based learning settings.

The particular power of network-supported capabilities relative to this form of activity comes from making visible multiple instances of a particular kind of reasoning. This form of generativity is focused on rhetoric and logic, broadly conceived, and students and teachers are supported in engaging the senses in which the particular forms of reasoning they use are related. What kind of reasoning, for example, allows us to speak of this expressive artifact “3x+1x” as being the “same” as “2x + 2x”? How is this form of reasoning like or different from the form of reasoning that allows us to say that two pieces of music are both jazz?

5.0 Conclusion

The pathways and endpoints taxonomy of kinds of generative activity is intended to be useful both in clarifying the relations between kinds of generative activities and in exemplifying internal aspects of generative design, especially as supported by next-generation classroom networks. These approaches to generative learning and teaching can be integrated with new forms of functionality supported by next-generation classroom networks and result in significant improvements in mathematics and science teaching and learning.

References


