FINANCIAL DEVELOPMENT, GROWTH, AND
THE DISTRIBUTION OF INCOME

by

Jeremy Greenwood
and
Boyan Jovanovic

C. V. STARR CENTER
FOR APPLIED ECONOMICS

NEW YORK UNIVERSITY
FACULTY OF ARTS AND SCIENCE
DEPARTMENT OF ECONOMICS
WASHINGTON SQUARE
NEW YORK, N.Y. 10003
FINANCIAL DEVELOPMENT, GROWTH, AND THE DISTRIBUTION OF INCOME

Jeremy Greenwood
University of Western Ontario
and
Rochester Center for Economic Research
and
Boyan Jovanovic
New York University

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ABSTRACT

A paradigm is presented where both the extent of financial intermediation and the rate of economic growth are endogenously determined. Financial intermediation promotes growth since it allows a higher rate of return to be earned on capital, and growth in turn provides the means to implement costly financial structures. Thus, financial intermediation and economic growth are inextricably linked in accord with the Goldsmith–McKinnon–Shaw view on economic development. The model also generates a development cycle reminiscent of the Kuznet hypothesis. In particular, in the transition from a primitive slow-growing economy to a developed fast-growing one, a nation passes through a stage where the distribution of wealth across the rich and poor widens.

Jeremy Greenwood
Department of Economics
University of Western Ontario
Social Science Centre
London, Ontario N6A 5C2
Canada

Boyan Jovanovic
Department of Economics
New York University
269 Mercer St. 7th Floor
New York, NY 10003
USA
I. INTRODUCTION

Two themes are salient in the growth and development literature. The first is Kuznets' (1955) hypothesis on the relationship between economic growth and the distribution of income. On the basis of somewhat slender evidence Kuznets (1955) cautiously offered the proposition that during the course of an economy's lifetime income inequality rises during the childhood stage of development, tapers off during the juvenile stage and finally declines as adulthood is reached. While far from being incontrovertible, other researchers have found evidence in support of this hypothesis. For example, Lindert and Williamson (1985) suggest "British experience since 1688 looks like an excellent advertisement for the Kuznets Curve, with income inequality rising across the Industrial Revolution, followed by a prolonged leveling in the last quarter of the nineteenth century" (p. 344). Using cross-country data Paukert (1973) finds evidence of intra-country income equality rising and then declining with economic development. Finally, inter-country inequality is examined by Summers, Kravis and Heston (1984). They discover that income inequality fell sharply across industrialized countries from 1950 to 1980, declined somewhat for middle income ones, and rose slightly for low income nations. Of related interest is their finding that between 1950 to 1980 real per capita income grew at about half the rate for low income countries as it did for high and middle income nations.

The second major strand of thought prevalent in the growth and development literature, often associated with the work of Goldsmith (1968), McKinnon (1973) and Shaw (1973), stresses the connection between "a country's financial superstructure and its real infrastructure". Simply put by Goldsmith (1968), the financial superstructure of an economy "accelerates economic growth and improves economic performance to the extent that it facilitates the migration of funds to the best user, i.e. to the place in the economic system where the funds will yield the highest social return" (p. 400). There also exists some evidence, although again not decisive, establishing a link between financial structure and economic
development. For instance, Goldsmith (1968) presents data showing a well-defined upward secular drift in the ratio of financial institutions' assets to GNP for both developed and less developed countries for the 1860–1963 period. As he notes, though, it is difficult to establish "with confidence the direction of the causal mechanism, i.e. of whether financial factors were responsible for the acceleration of economic development or whether financial development reflected economic growth whose mainsprings must be sought elsewhere" (p. 48). And indeed Jung (1986) provides postwar econometric evidence for a group of 56 countries of causality (in the Granger sense) running in either and both ways. Finally, historical case studies such as undertaken in Cameron (1967) have stressed the key importance of financial factors in the economic development of several European countries.

The current analysis focuses on the nexus between economic growth, institutional development, and the distribution of income. Economic growth fosters investment in organizational capital which in turn promotes further growth. In the framework presented, institutions arise endogenously to facilitate trade in the economy. First, the emergence of trading organizations allows for a higher expected rate of return on investment to be earned. In particular, in the environment modelled, information is valuable since it allows investors to learn about the aggregate state of technology. Through a research type process intermediaries collect and analyze information that allows investors' resources to be allocated to their most profitable use. By investing through an intermediary, individuals gain access, so to speak, to a wealth of experience of others. It should be noted that Boyd and Prescott (1986) also stress the role that intermediaries can play in overcoming information frictions, although the nature of these frictions is different. Second, trading organizations also play the traditional role of pooling risks across large numbers of investors. Thus, by investing through intermediated structures individuals obtain both a higher and safer return.

As in Townsend (1978), investment in organizational capital is taken to be costly. Consequently, high income level economies are better disposed to undertake such financial
superstructure building than are ones with low income levels. The development of financial superstructure, since it allows a higher return to be earned on capital investment, in turn feeds back on economic growth and income levels. In this latter regard the current analysis could be viewed as a close cousin of Townsend (1983), which also examines the relationship between financial structure and economic activity, although within the context of framework where the extent of financial markets is exogenously imposed and that abstracts away from the issue of growth. Also, in the spirit of recent work by Lucas (1985), Romer (1986), and Rebelo (1987) growth is modelled as an endogenous process, i.e. does not depend on exogenous technological change.

The dynamics of the development process are reminiscent of the Kuznets' (1955) hypothesis. In the early stages of economic development an economy's financial markets are virtually nonexistent and it grows slowly. Financial superstructure begins to form as the economy approaches the intermediate stage of the growth cycle. Here both the growth and savings rates of the economy increase, with the distribution of income across the rich and poor widening. By maturity the economy has developed an extensive structure for financial intermediation. In the final stage of development the distribution of income across agents stabilizes, the savings rate falls and the economy's growth rate converges (although perhaps nonmonotonically) to a higher and more stable level than that prevailing during its infancy.¹ According to Lindert and Williamson (1986) "it is exactly this kind of correlation—rising inequality with rising savings and accumulation rates during Industrial Revolutions—that encouraged the trade-off belief (between growth and inequality) among classical economists who developed their growth models while the process was underway in England" (pp. 342–3).

II. THE ECONOMIC ENVIRONMENT

Consider an economy populated by a continuum of agents distributed over the interval [0,1] with Lebesgue measure 1/\Gamma. An agent's goal in life is to maximize his expected lifetime
utility as given by

\[ E[ \sum_{t=0}^{\infty} \beta^t \ln c_t ] \quad \text{with } 0 < \beta < 1, \]

where \( c_t \) is his period-\( t \) consumption flow and \( \beta \) the discount factor.

Each individual has access to his own production technology. In particular, an agent's production is governed by the following linear technological process:

\[ y_t = (\theta_t + \varepsilon_t)k_t, \]

where \( y_t \) is period-\( t \) output of goods, \( k_t \) the input of goods into the production process at the beginning of \( t \), and \( (\theta_t + \varepsilon_t) \) a composite technological shock. The output produced from this process in any given period \( t \) can either be consumed in \( t \) or costlessly stored for period-\( t+1 \) consumption or production. At the start of time, each agent is endowed with a certain amount of goods or capital, \( k_0^* \). Individuals are heterogeneous in the sense that their initial endowment of capital may differ. The initial distribution of wealth in the society is represented by the cumulative distribution function \( \Phi_0(\cdot) : \mathbb{R}^+ \to [0,1] \).

The period-\( t \) technological shock has two components. The first component, \( \theta_t \), represents an aggregate disturbance and thus is common across technologies while the second, \( \varepsilon_t \), portrays an individual (or project) specific shock. All that an agent can costlessly observe is the realized composite rate of return \((\theta_t + \varepsilon_t)\) on his own project. The stochastic structure of the economic environment will be delimited in the following way:

(A1) The aggregate shock \( \theta_t \) is governed by the time-invariant distribution function \( F(\theta) \). Let \( \Theta = [\underline{\theta}, \bar{\theta}] \subset \mathbb{R}^+ \) and \( F(\theta) : \Theta \to [0,1] \). Furthermore, suppose \( E[\ln \theta] = \int \ln \theta dF(\theta) > -\ln \beta \); by Jensen's inequality this implies \( E[\theta] > \beta \).

(A2) For each individual \( i \in [0,1] \) the idiosyncratic shocks \( \varepsilon_i(t) \) are drawn from the distribution function \( G(\varepsilon_i) \). Let \( \mathcal{E} = [\underline{\varepsilon}, \bar{\varepsilon}] \subset \mathbb{R} \) and \( G(\varepsilon) : \mathcal{E} \to [0,1] \). Additionally, assume that \( E[\varepsilon] = \int \varepsilon dG(\varepsilon) = 0 \) and \( \theta + \varepsilon > 0 \).
Given the assumed form of uncertainty in the economic environment, there are potential gains to establishing trading arrangements between agents. Following Townsend (1978), it will be assumed that the setting up of trading arrangements is costly. Specifically, a bilateral trading arrangement between any two individuals can be consummated at the once-and-for-all cost of \( \alpha \) (in terms of goods). Nothing precludes an agent from undertaking agreements with many different individuals; if a person has bilateral trading relationships with \( N \) others then the cost of setting up these ties is \( N\alpha \).

The potential benefits from establishing trading networks are twofold. First, information has a public good aspect to it. Each entrepreneur longs for information on the realized returns on the projects of others. This would allow his production decisions to be better made since such realized returns contain useful information about the magnitude of the aggregate shock. But even if such information was public knowledge no individual entrepreneur would want to produce first since by waiting he could gain the experience of others. Thus, there is a coordination problem inherent in individual entrepreneur's production planning which trading agreements may be able to overcome. Second, trading arrangements could potentially be used to diversify away the idiosyncratic risk associated with individual production projects. The emergence of such trading arrangements is the subject of the next section.

III. COMPETITIVE EQUILIBRIUM

In competitive equilibrium institutions will arise which collect and process information, coordinate production activity and spread risk. Since setting up organizational structures is costly, institution formation will be economized on. As is highlighted in Townsend (1978), a role arises for a subset of individuals to intermediate economic activity. To see this, consider a group of \( N+1 \) agents establishing some form of trading network amongst themselves. If possible it would be best if one agent could act as an intermediary organizing the exchange
process for the group as a whole via a series of $N$ bilateral arrangements between himself and each of the other agents. The total setup cost of such an organizational structure would be $N\alpha$ which amounts to $(N/N+1)\alpha$ per agent. Compare this with an institutional arrangement where each of the $N+1$ agents has a bilateral link with everybody else. Here the cost of this institutional design would be $(N+1)N\alpha$, amounting to $N\alpha$ per member. Note that having one agent act as an intermediary for a group minimizes the number of bilateral agreements that must be consummated. The question to be asked is whether a single individual can intermediate in an efficient manner the collective's economic activity.

**Intermediation Strategy**

Let an agent in the economy assume the role of an intermediary for some (measurable) set of agents, $A$. This go-between adopts the following strategy for intermediation: In exchange for a once-and-for-all-fee of $\alpha$, plus the rights to operate an individual’s project, the intermediary promises a (gross) return of $\max(1,\theta_t)$ on each unit of capital invested in any given period $t$. Note that this intermediation strategy, if feasible, offers agents a rate of return on their investments that is (i) completely devoid of the idiosyncratic component of production risk and (ii) safeguarded from the potential losses that could occur when the aggregate return on production falls below the opportunity cost of the resources committed. Also, investors are only charged a lump-sum fee which exactly compensates the go-between for the once-and-for-all cost of establishing a business arrangement with them.

Such a policy is feasible when supported by the period-$t$ production plan outlined below. To begin with let $k_t(i)$ represent the period-$t$ capital stock of person $i$. Then the aggregate amount of capital that the intermediary has at its disposal to invest in $t$ is $\int_A k_t(i)d\Gamma(i)$, where again $\Gamma$ is a Lebesgue measure. Now, let the intermediary randomly select some finite number of projects, say $\tau$, from the set $A$; denote this set of projects by $A^C$. Each of the "trial" projects selected are funded with the amount $K_t = [\int_A k_t(i)d\Gamma(i)]/[\int_A d\Gamma(i)]$. The intermediary then calculates the average realized rate of return, $\hat{\theta}_{t\tau}$, on these projects where
formally \(^2\)

\[
\hat{\theta}_{tt} = \frac{1}{\tau} (\theta_t \tau + \sum_{j=1}^{\tau} \varepsilon_j).
\]

Now, if the "test statistic" \(\hat{\theta}_{tt}\) is greater than unity then the remaining projects operated by the intermediary are each funded with \(K_t\) units of capital, otherwise the resources the go-between has access to are withheld from production.

Note that relative to the size of intermediary's portfolio of projects the number of production technologies chosen for research purposes is negligible. More precisely, the set of experimental projects, \(A^c\), being countable has (Lebesgue) measure zero. Consequently, other than the important informational role these test projects play, they have a negligible impact on the profits earned by the intermediary. Thus, the rate of return on the intermediary's production activities, or \(z(\theta_t, \hat{\theta}_{tt})\), will be given by

\[
z(\theta_t, \hat{\theta}_{tt}) = \begin{cases} 
\{ \int_{A^c} c[\theta_t + \varepsilon_t(i)] d\Gamma(i) + \hat{\theta}_{tt} \int_{A^c} c d\Gamma(i) \}/[\int_{A} c d\Gamma(i)] = \theta_t, & \text{if } \hat{\theta}_{tt} > 1, \\
or & \text{if } \hat{\theta}_{tt} \leq 1.
\end{cases}
\]

The following lemma can now be stated:

**Lemma 1**: As \(\tau \to \infty\), \(z(\theta_t, \hat{\theta}_{tt}) \stackrel{a.s.}{\to} \max(1, \theta_t)\).

**Proof**: For \(x \in (-1, \infty)\), let \(I(x) = 1\) if \(x > 0\) and \(I(x) = 0\) otherwise. Then \(z(\theta_t, \hat{\theta}_{tt})\) can be
expressed as
\[ z(\theta_t, \hat{\theta}_{tt}) = I(\hat{\theta}_{tt} - 1)\theta_t + [1 - I(\hat{\theta}_{tt} - 1)]. \]

Clearly, if \( \theta_t = 1 \) then \( z(\theta_t, \hat{\theta}_{tt}) = 1 \), regardless of the value of \( \hat{\theta}_{tt} \). Therefore trivially here \( z(\theta_t, \hat{\theta}_{tt}) = \max(1, \theta_t) \). Suppose alternatively that \( \theta_t \neq 1 \). Now as \( \tau \to \infty \), \( \hat{\theta}_{tt} \) a.s. by assumption (A2) and the Strong Law of Large Numbers. This though implies that \( I(\hat{\theta}_{tt} - 1) \) a.s. \( \to I(\theta_t - 1) \), since \( I(\cdot) \) is a continuous function on \((-1, 0) \cup (0, \infty)\). Hence in the case where \( \theta_t \neq 1 \), it follows that \( z(\theta_t, \hat{\theta}_{tt}) \) a.s. \( \to \max(1, \theta_t) \) as \( \tau \to \infty \).

Not all agents may find the terms of the investment contract offered currently attractive. In particular, for some agents it may not be worthwhile now to pay a lump-sum fee of \( \alpha \) in order to gain permanent access to the intermediation technology paying a random return of \( \max(1, \theta_t) \) in each \( t \). Thus, it is natural at this point to examine the determination of participation in the exchange network. To do this consider the decision-making of an individual in period \( t \) who is currently outside of the intermediated sector. His actions in this period are summarized by the outcome of the following dynamic-programming problem

\[
\begin{align*}
w(k_t) = \max_{s_t} (\delta h(k_t - s_t) + \beta \max & w(s_t(\theta_{t+1} + \varepsilon_{t+1})) \\
& v(s_t(\theta_{t+1} + \varepsilon_{t+1}) - \alpha) dF(\theta_{t+1}) dG(\varepsilon_{t+1})),
\end{align*}
\]

(P1')

where \( s_t \) is the agent's period-\( t \) saving level and \( v(s_t(\theta_{t+1} + \varepsilon_{t+1}) - \alpha) \) represents the expected lifetime utility the agent could expect to realize in \( t+1 \) if he then entered the intermediated sector with \( s_t(\theta_{t+1} + \varepsilon_{t+1}) - \alpha \) units of capital at his disposal. It can be demonstrated that \( w(\cdot) \) is an increasing, concave, and differentiable function for any function \( v(\cdot) \) sharing these properties; it will be uniquely determined as well—see Lucas, Prescott and Stokey (1985). Note that the above programming problem presumes that in \( t+1 \) the agent will enter or remain outside of intermediated depending upon which choice then yields the highest expected utility.
Hence, \( w(k_t) \) gives the maximum lifetime utility on individual with \( k_t \) units of capital can expect in period \( t \) if he chooses not to participate in the exchange network just then.

Likewise, the dynamic-programming problem for any agent currently within the intermediated sector is given by

\[
v(k_t) = \max_{s_t} \left\{ \ln(k_t - s_t) + \beta \int \max[w(s_t \max(1, \theta_{t+1})), v(s_t \max(1, \theta_{t+1}))]dF(\theta_{t+1}) \right\}
\]

(P2')

If \( w(\cdot) \) is an increasing, concave, and differentiable function then \( v(\cdot) \) inherits these traits as well. Thus, (P1') and (P2') jointly define the pair of functions \( w(\cdot) \) and \( v(\cdot) \).

Presumably, in any period \( t \) a given endowment of capital, \( k_t \), is worth more to an agent operating within the intermediated sector than to one outside of it; that is \( v(k_t) > w(k_t) \). This should transpire since exchange with the go-between yields a better distribution of returns per unit of capital invested than autarky does. If this is so, then once an individual enters the intermediated sector he will never leave it.

**Lemma 2:** \( v(k_t) > w(k_t) \).

**Proof:** Define \( \bar{s}_t = \bar{s}(k_t) \) as the optimal policy function associated with problem (P2'); this decision-rule will in general be sub-optimal for (P1'). Thus,

\[
v(k_t) - w(k_t) \geq \ln(k_t - \bar{s}_t) + \beta \int \max[w(\bar{s}_t \max(1, \theta_{t+1})), v(\bar{s}_t \max(1, \theta_{t+1}))]dF(\theta_{t+1})
\]

\[
- \ln(k_t - \bar{s}_t) - \beta \int \max[w(\bar{s}_t(\theta_{t+1} + \epsilon_{t+1})), v(\bar{s}_t(\theta_{t+1} + \epsilon_{t+1}) - \alpha)]dF(\theta_{t+1})dG(\epsilon_{t+1})
\]

\[
\geq \beta \int \max[w(\bar{s}_t \max(1, \theta_{t+1})), v(\bar{s}_t \max(1, \theta_{t+1}))]dF(\theta_{t+1})
\]

\[
- \beta \int \max[w(\bar{s}_t(\theta_{t+1} + \epsilon_{t+1})), v(\bar{s}_t(\theta_{t+1} + \epsilon_{t+1}))]dF(\theta_{t+1})dG(\epsilon_{t+1}),
\]

where the last inequality follows from the fact that \( v(\bar{s}_t(\theta_{t+1} + \epsilon_{t+1})) > v(\bar{s}_t(\theta_{t+1} + \epsilon_{t+1}) - \alpha) \) since \( v(\cdot) \) is an increasing function. Finally, by Jensen's inequality, if \( w(\cdot) \) and \( v(\cdot) \) are
conca ve functions then

\[ v(k_t) - w(k_t) \geq \beta \int \max[w(s_t, \max(1, \theta_{t+1})), v(s_t, \max(1, \theta_{t+1}))] dF(\theta_{t+1}) \]

\[ - \beta \int \max[w(s_t, \theta_{t+1}), v(s_t, \theta_{t+1})] dF(\theta_{t+1}) \]

\[ \geq 0 \]

\[ \Box \]

The above lemma allows the functional equation (P2) to be simplified so that \( v(\cdot) \) can be defined without reference to \( w(\cdot) \). Specifically, (P2') can now be written as

\[ v(k_t) = \max \{ \ln(k_t - s_t) + \beta \int v(s_t, \max(1, \theta_{t+1})) dF(\theta_{t+1}) \} \]  

(P2)

Furthermore, given the logarithmic form of the utility function it is straightforward to establish that the value function \( v(k_t) \) and the policy–rule \( s_t = s(k_t) \) have the following simple forms:

\[ v(k_t) = \frac{1}{1-\beta} \ln(1-\beta) + \frac{\beta}{(1-\beta)^2} \ln(\beta) + \frac{\beta}{(1-\beta)^2} \ln \int \max(1, \theta) dF(\theta) + \frac{1}{1-\beta} \ln k_t \]  

(1)

and

\[ s(k_t) = \beta k_t. \]  

(2)

Thus, agents within the intermediated sector save a constant fraction of their wealth each period. Given the above solution for \( v(\cdot) \), problem (P1') then implies a solution for \( w(\cdot) \).

The extent of participation in the exchange network is now easily characterized. Consider some arbitrary set of agents for whom it was not in their individual interests to engage with the intermediary up until the current period \( t \). (This set of agents could be all or none of the actors in the economy.) Each of these individuals must now decide on whether or not to join the exchange network. Given that the cost of accessing the intermediary is lump–sum, it seems likely that agents with a capital stock falling below some minimal level \( k > 0 \) will remain outside of the exchange network while those having an endowment exceeding...
some upper threshold level $k \geq k$ will join.

**Lemma 3:** There exist $k$ and $k'$, with $0 < k < k'$, such that

$$v(k_t - \alpha) < w(k'_t) \text{ for } 0 < k_t < k, \text{ and } v(k_t - \alpha) > w(k_t) \text{ for } k_t > k.$$ 

**Proof:** Since both $w(k)$ and $v(k)$ are continuous functions in $k$ it is enough to demonstrate that (i) $\lim_{k \to \infty} [w(k) - v(k-\alpha)] > 0$ and (ii) $\lim_{k \to \infty} [w(k) - v(k-\alpha)] < 0$. To show (i) note on the one hand that from equation (1) $\lim_{k \to \infty} v(k-\alpha) = -\infty$. On the other hand, though, it is feasible never to join the condition and pursue the dynamic-program shown below.

$$w^0(k) = \max_s \{ \ln(k-s) + \beta \int w^0(s(\theta+\epsilon))dF(\theta)dG(\epsilon) \}. \quad \text{(P3)}$$

It is easy to show that the value function $w^0(k)$ and the policy-rule $s = s(k)$ have the following simple forms:

$$w^0(k) = \frac{1}{1-\beta} \ln(1-\beta) + \frac{\beta}{(1-\beta)^2} \ln\beta + \frac{\beta}{1-\beta} \int \ln(\theta+\epsilon)dF(\theta)dG(\epsilon) + \frac{1}{1-\beta} \ln k \quad \text{(3)}$$

$$s(k) = \beta k \quad \text{[cf. (1) and (2)].} \quad \text{(4)}$$

Clearly, $w(k) > w^0(k) > -\infty$ (by A(1) and A(2)) for all $k > 0$.

To establish (ii) observe that equation (P1') and Lemma 2 imply

$$w(k) \leq \max_s \{ \ln(k-s) + \beta \int v(s(\theta+\epsilon))dF(\theta)dG(\epsilon) \},$$
which together with (P2) yields that
\[
w(k) - v(k-\alpha) \leq \max_s \left\{ \ln(k-s) + \beta \int \nu(s(\theta+\epsilon))dF(\theta)dG(\epsilon) \right\} \\
- \max_s \left\{ \ln(k-\alpha-s) + \beta \int \nu(s(\max(1,\theta)))dF(\theta) \right\}.
\]

Since \( \nu(\cdot) \) is concave, by Jenson's inequality
\[
w(k) - v(k-\alpha) \leq \max_s \left\{ \ln(k-s) + \beta \int \nu(s(\theta))dF(\theta) \right\} \\
- \max_s \left\{ \ln(k-\alpha-s) + \beta \int \nu(s(\max(1,\theta)))dF(\theta) \right\}.
\]

Next given the logarithmic form of the value function, \( \nu(\cdot) \), the first term in braces is maximized by setting \( s = \beta k \); this saving rule is also a feasible choice for \( s \) for the second term in braces providing that \( k \geq \alpha/(1-\beta) \). Thus, for \( k \geq \alpha/(1-\beta) \),
\[
w(k) - v(k-\alpha) \leq \ln(k(1-\beta)) + \beta \int \nu(\beta k \theta) dF(\theta) \\
- \ln(k(1-\beta)-\alpha) - \beta \int \nu(\beta k \max(1,\theta)) dF(\theta).
\]

Using the expression for \( \nu(\cdot) \) provided by (1) the above can be rewritten as
\[
w(k) - v(k-\alpha) \leq \ln(k(1-\beta)) - \ln(k(1-\beta)-\alpha) \\
+ \left[ \beta/(1-\beta) \right] \left[ \ln(\beta k \theta) - \ln(\beta k \max(1,\theta)) \right] dF(\theta) \\
= \ln[k(1-\beta)/(k(1-\beta)-\alpha)] + \left[ \beta/(1-\beta) \right] \int_1^{\ln(\theta)} dF(\theta).
\]

Consequently, \( \lim_{k \to \infty} [w(k) - v(k-\alpha)] < 0 \), since \( \lim_{k \to \infty} [k(1-\beta)/(k(1-\beta)-\alpha)] = 0 \) and \( \beta \int_1^{\ln(\theta)} dF(\theta) < 0 \) as \( \ln \theta \leq 0 \) for \( 0 < \theta \leq 1 \).
Remark: If \( v(k - \alpha) - w(k) \) is strictly increasing in \( k \) then \( k = \bar{k} \). In general, though, this result doesn’t appear to transpire.

Now define the sets \( B^C \) and \( B \) in the following manner:

\[
B^C = \{ k_t : v(k_t - \alpha) < w(k_t) \} \quad \text{and} \quad B = \{ k_t : v(k_t - \alpha) \geq w(k_t) \}
\]

By Lemma 3 the sets \( B^C \) and \( B \) are nonempty. Also, \( \underline{k} = \inf B \) and \( \bar{k} = \sup B^C \). Clearly, it is in the interest of those individuals having a capital stock \( k_t \in B \) to establish a trading link with the go-between, and likewise not so for those agents with an endowment \( k_t \in B^C \). Equally as evident, it is possible to have a competitive equilibrium prevailing in period \( t \) where some agents choose to participate in the exchange organization and others pick to remain outside; this will depend on the distribution of capital across individuals who were outside of the trading network in \( t-1 \).

IV. SAVINGS, GROWTH, DEVELOPMENT AND INCOME DISTRIBUTION

Some of the model’s predictions about savings, growth, development and income distribution will now be presented. To begin with, it will be demonstrated that economies in phases of development where institutional infrastructure building is occurring will tend to have high rates of savings. This occurs since the construction of economic organization is expensive; specifically, each trading link costs \( \alpha \) to establish. Recall that those agents participating in the intermediary-coalition save the amount \( s_t = \beta k_t \). Individuals outside of the trading network are saving in accord with the following dynamic-program [see (P1) and (5)]:

\[
w(k_t) = \max \left\{ \min(k_t - s_t) + \int_{D^C(s_t)} w(s_t(\theta_t + \varepsilon_{t+1}))dF(\theta_{t+1})dG(\varepsilon_{t+1}) \right. \\
+ \left. \int_{D(s_t)} v(s_t(\theta_t + \varepsilon_{t+1}) - \alpha)dF(\theta_{t+1})dG(\varepsilon_{t+1}) \right\} (P1)
\]

where \( D^C(s_t) = \{(\theta_{t+1}, \varepsilon_{t+1}): s_t(\theta_{t+1} + \varepsilon_{t+1}) \in B^C \} \) and \( D(s_t) = \{(\theta_{t+1}, \varepsilon_{t+1}): s_t(\theta_{t+1} + \varepsilon_{t+1}) \in B \}. \)
Now, denote the decision-rule governing optimal savings in the above problem by \( s_t = s(k_t) \). These individuals will lend to save an amount \( s(k_t) \) which is greater than \( \beta k_t \), since they expect at some future date to incur the lump-sum cost \( \alpha \) of developing a link with the exchange system.

**Proposition 1:** \( s(k_t) > \beta k_t \).

**Proof:** The proof proceeds by induction. Consider the sequence of functions \( \{w^j\}_{j=0}^{\infty} \) and \( \{s^j\}_{j=0}^{\infty} \) generated from the mapping \( w^j = Tw^{j-1} \), with the operator \( T \) defined by

\[
Tw^{j-1} = \max_{s} \left\{ (\ln(k-s^{j-1}) + \beta \int_{s}^{j-1} (s^{j-1}(\theta+\varepsilon))dF(\theta)dG(\varepsilon) \right. \\
\left. + \beta \int_{D(s)} \nu(s^{j-1}(\theta+\varepsilon) - \alpha)dF(\theta)dG(\varepsilon) \right\}. 
\]

The efficiency condition governing the optimal choice of \( s^{j-1} \) in the above mapping is shown below

\[
\frac{1}{k-s^{j-1}} = \beta \int_{D(s)} (\theta+\varepsilon)w_k^{j-1}(s^{j-1}(\theta+\varepsilon))dF(\theta)dG(\varepsilon) \\
+ \beta \int_{D(s)} (\theta+\varepsilon)\nu_k(s^{j-1}(\theta+\varepsilon) - \alpha)dF(\theta)dG(\varepsilon). 
\]

It is easy to show that the operator \( T \) is a contraction whose fixed point defined by \( w = Tw \) is characterized by \((P1).^{3}\) Thus given any initial function \( w^0 \), \( \lim_{j} w^j = w \) and \( \lim_{j} s^j = s \). Now, first it will be demonstrated that if \( w_k^j > w_k^{j-1} \) then \( s^j > s^{j-1} \) and \( w_k^{j+1} > w_k^j \). Second, to start the induction hypothesis off a \( w^0 \) will be chosen so that \( w_k^1 > w_k^0 \) and \( s(k) > \beta k \).
Consequently, \( s(k) = \lim_{j \to \infty} s^j(k) > \beta k \) since \( s^j(k) > \beta k \) for all \( j \).

Assume that \( w^j_k > w^{j-1}_k \). From (7) the first-order condition governing the optimal choice of \( s^{j-1} \) is

\[
\frac{1}{k-s^j} = \beta \int_{D(s)} (\theta + \epsilon) w^j_k(s^j(\theta + \epsilon)) dF(\theta) dG(\epsilon)
\]

\[ + \beta \int_{D(s)} (\theta + \epsilon) v_k(s^j(\theta + \epsilon) - \alpha) dF(\theta) dG(\epsilon). \tag{8}\]

By comparing (8) with (7) observe that if \( s^j = s^{j-1} \) then the right-hand side of the above expression would exceed the left-hand side since \( w^j_k > w^{j-1}_k \). To restore equality \( s^j \) must be increased since the right-hand side is decreasing in \( s^j \) while the left-hand side is increasing.

Next, note that

\[
w^{j+1}_k = \frac{1}{k-s^j},
\]

so that if \( s^j > s^{j-1} \) then \( w^{j+1}_k > w^j_k \).

Finally, let \( w^0 \) be specified as in (3). Then using (1), (3) and (7) the efficiency condition governing the optimal choice of \( s^0 \) can be written as

\[
\frac{1}{k-s^o} = \beta \int_{D(s)} \frac{(\theta + \epsilon)}{D^c(s) [(1-\beta) s^o(\theta + \epsilon) - \alpha]} dF(\theta) dG(\epsilon)
\]

\[ + \beta \int_{D(s)} \frac{(\theta + \epsilon)}{D(s) [(1-\beta) s^o(\theta + \epsilon) - \alpha]} dF(\theta) dG(\epsilon). \]

It is easy to see that \( s^0(k) > \beta k \), since when \( s^0(k) = \beta k \) the right-hand side of this expression
(which is decreasing in $s^O$) exceeds the left-hand side (which is increasing in $s^O$). Last, it immediately follows that $w^1_k > w^O_k$ as

$$w^1_k = \frac{1}{k-s^O} > \frac{1}{(1-\beta)k} = w^O_k.$$ 

Agents who are members of the intermediary coalition save the amount $s_t = \beta k_t$ and earn a per unit rate of return of $\max(1, \theta_{t+1})$ on this savings. Consequently, their wealth grows at the expected rate $E[k_{t+1}/k_t] = \beta \int \max(1, \theta_{t+1}) dF(\theta_{t+1}) > 1$ (by (A1)). Individuals outside of the exchange network save $s_t = s(k_t) \geq \beta k_t$, earning a rate of return of $(\theta_{t+1} + \epsilon_{t+1})$. Thus, they accumulate wealth at the expected rate $E[k_{t+1}/k_t] = [s(k_t)/k_t] \int \theta_{t+1} dF(\theta_{t+1}) > 1$ (by (A1) and Proposition 1). It's unclear whose wealth is growing faster on average. While on the one hand noncoalition members face an inferior distribution of returns on their investments, on the other they tend to save more.

It seems reasonable to suspect, though, that very poor agents have a low savings rate. That is, for the very poor $s(k_t) \approx \beta k_t$. If so, then poor individuals will accumulate wealth at approximately the expected rate $\beta \int \theta_{t+1} dF(\theta_{t+1}) < \beta \int \max(1, \theta_{t+1}) dF(\theta_{t+1})$. Consequently, there will be an increase in equality across the very rich and very poor segments of the population. The rationale underlying this conjecture is that very poor agents are likely to remain outside of the intermediary-coalition for some time to come and consequently are heavily discounting the future cost of developing a link with the exchange network. Additionally, from (P3) it is known that in circumstances where an agent never will transact with the go-between the amount $s_t = \beta k_t$ is saved.

**Proposition 2:** For all $\epsilon > 0$ there exists a $K$ such that

$$\sup_{k \in [0, K]} |s(k) - \beta k| < \epsilon.$$
Proof: Consider the dynamic—programs (P1) and (P3) defining the value functions \( w(k) \) and \( w^0(k) \), respectively, and the associated policy—rules \( s(k) \) and \( \beta k \). Since these value and policy functions are unique it suffices to demonstrate that

\[
\lim_{k \to 0} \{\sup_{k \epsilon [0, \bar{k}]} |w(k) - w^0(k)|\} = 0. \tag{9}
\]

Note that under program (P1) the minimal capital stock for which it is potentially profitable to join the exchange network is \( \bar{k} \). Let the current period be \( t \) and consider an individual who has an initial endowment of capital \( k_t = k \) and is saving in line with this program. Now define \( P_{t+j}(k',k) \) as the probability that under the savings plan \( s_t = s(k_t) \) the agent’s capital stock will exceed \( k \) for the first time at \( t+j \) but then have a value less \( k' \); that is, more formally, \( P_{t+j}(k',k) = \text{prob}[k_{t+j} \leq k', k_{t+j} \geq k \text{ and } k_{t+i} < k \text{ for } 0 < i < j-1] \) with \( k_{t+j} \) being generated by the law of motion \( k_{t+j} = (\theta_{t+j} + \epsilon_{t+j})s(k_{t+j-1}) \). The savings plan \( s_t = s(k_t) \) is also feasible for an individual following the other program (P3). Note that while implementing this scheme is clearly suboptimal for (P3) it will yield the same time path of momentary utility as (P1) for the duration of time that the agent remains outside of coalition under the latter program. Thus,

\[
w(k) - w^0(k) \leq \sum_{j=1}^{\infty} \beta^j \int_{k} \{v(k') - w^0(k')\}dP_{t+j}(k';k) \tag{10}
\]

[recalling that \( v(k') \geq w(k') \) by Lemma 2].

Next, from (1) and (3) it is known that

\[
v(k') - w^0(k') = \frac{\beta}{(1-\beta)^2} \{\int \max(1,\theta)dF(\theta) - \int \max(\theta+\epsilon)dF(\theta)dG(\epsilon)\} = \delta > 0.
\]
implying
\[
    w(k) - w^0(k) \leq \delta \sum_{j=1}^{\infty} \beta^j \int dP_{t+j}(k';k) = \delta \sum_{j=1}^{\infty} \beta^j P_{t+j}(k),
\]
where \( P_{t+j}(k) = \int dP_{t+j}(k';k) \) is the marginal probability of crossing the threshold level of capital \( k \) for the first time at \( t+j \). Alternatively, imagine an individual who saves everything each period so that \( s_t = k_t \) and define \( Q_{t+j}(k';k) \) as the probability that under this extreme savings rule the agent's capital stock will exceed \( k \) for the first time at \( t+j \) but have a value then no greater than \( k' \). More formally, \( Q_{t+j}(k';k) \equiv \text{prob}[k_{t+j} \leq k', k_{t+j} \geq k \text{ and } k_{t+j} < k] \) for \( 0 < i < j-1 \), where \( k_{t+j} \) is generated from the equation \( k_{t+j} = \sum_{i=1}^{j} (\theta_{t+j} + \varepsilon_{t+j} k). \) Such a savings policy leads to the threshold level of capital stock being crossed for the first time at an earlier date. Therefore, by letting \( Q_{t+j}(k) = \int dQ_{t+j}(k',k) \) represent the marginal probability of crossing \( k \) for the first time at \( t+j \), it follows that the distribution of the \( P_{t+j}(k) \)’s stochastically dominates the distribution of the \( Q_{t+j}(k) \)’s, or that \( \sum_{j=1}^{m} P_{t+j}(k) \leq \sum_{j=1}^{m} Q_{t+j}(k) \) for all \( m \). Since \( \beta^j \) is a decreasing function in \( j \) this implies
\[
    w(k) - w^0(k) \leq \delta \sum_{j=1}^{\infty} \beta^j P_{t+j}(k) \leq \delta \sum_{j=1}^{\infty} \beta^j Q_{t+j}(k).
\]
Now given any \( \varepsilon, T > 0 \) a sufficiently small value for \( \bar{k} \), denoted by \( \bar{k}(\varepsilon,T) \), can be chosen so that \( \sum_{j=1}^{T} Q_{t+j}(k) \leq \varepsilon \) for all \( k \in [0,\bar{k}(\varepsilon,T)] \). Therefore
\[
    \sup_{k \in [0,\bar{k}(\varepsilon,T)]} \|w(k) - w^0(k)\| \leq \delta \sum_{j=1}^{T} \beta^j + \delta \sum_{j=T+1}^{\infty} \beta^j \leq \frac{\beta \varepsilon (1 - \beta^{T+1})}{1 - \beta} + \beta^T. \tag{11}
\]
Since this can be done for any \( \varepsilon \) and \( T \), the right–hand side of (11) can be made arbitrarily tiny by choosing a small \( \varepsilon \) and large \( T \). The desired result (9) now immediately obtains by letting \( \varepsilon \to 0 \) and \( T \to \infty \) in a manner such that \( \bar{k}(\varepsilon,T) \to 0. \)
To reiterate, Proposition 2 implies that the difference in relative wealth levels between members of the intermediated sector and the very poor will widen over time. This result obtains since both groups have the same savings rate while the former face a better distribution of returns on their investments.

Some of the long-run properties of the developed model will now be presented. To begin with, agents in the less-developed sector of the economy accumulate wealth according to

\[ k_{t+1} = (\theta_{t+1} + \epsilon_{t+1})s(k_t). \]

Now define \( \tilde{\psi}(k';k) \) as the law of motion, in cumulative distribution function form, governing the evolution of the capital stock that is implied by the above equation. Thus \( \tilde{\psi}(k';k) = \text{prob}[k_{t+1} = (\theta_{t+1} + \epsilon_{t+1})s(k_t) \leq k' | k_t = k] \). Note that those agents entering \( t \) with a \( k_t \in \mathcal{B} \) will join the intermediation–coalition, it not being worthwhile for the rest (\( k_t \in \mathcal{B}^c \)) to establish a link at that time. Therefore, \( \psi(k';k) = \int_{\mathcal{B}^c} \tilde{\psi}(z;k) \) represents the probability that an agent residing in period \( t \) in the less-developed sector of the economy with \( k \) units of capital will remain in this sector in \( t+1 \) with a capital stock in value no greater than \( k' \).

Next let \( \bar{\mathcal{H}}_0(k) \) represent the economy's initial time zero distribution of capital over people so that \( \bar{\mathcal{H}}_0(k) : \mathbb{R}_+ \rightarrow [0,1] \). The initial sizes of the developed and less-developed sectors of the economy will therefore be \( \int_{\mathcal{B}} d\mathcal{H}_0(k) \) and \( 1 - \int_{\mathcal{B}} d\mathcal{H}_0(k) \). Consequently, the distribution function governing the allocation of capital across individuals in the less-developed sector of the economy in period one will be given by \( H_1(k') = \int_{\mathcal{B}^c} \psi(k';k)d\bar{\mathcal{H}}_0(k) \).

In general it transpires that

\[ H_{t+1}(k') = \int \psi(k';k)dH_t(k), \]

where \( H_{t+1}(k') \) measures the expected size of the population in period \( t \) who are outside of
the intermediated sector and have a capital stock \( k_{t+1} \leq k' \). Since in any given period \( t+1 \) no agent has a capital stock \( k_{t+1} \geq k \) (Lemma 3) it follows that the expected \( t+1 \)-size of the less-developed sector is \( H_{t+1}(k) \). Given the assumed growth in the economy, \( \lim_{t \to \infty} H_{t+1}(k) = 0 \).

Finally, in any given period \( t \) those agents in the less-developed sector of the economy realize a rate of return of \( \theta_t + \varepsilon_t \) on their investment while those in the developed part obtain the yield \( \max(1, \theta_t) \). Therefore for any given realization of the aggregate shock \( \theta_t = \theta \) the expected return earned across individuals, denoted by \( r_t(\theta) \) is

\[
r_t(\theta) = H_t(k)\theta + [1 - H_t(k)]\max(1, \theta).
\]

Clearly, as the future time horizon is extended \( r_t(\theta) \) converges monotonically upward to the best technologically feasible expected return possible, \( \max(1, \theta) \), conditional on the aggregate state-of-the-world.

**Proposition 3:** \( \lim_{t \to \infty} \sup_{\theta} r_t(\theta) - \max(1, \theta) | = 0. \)

Furthermore, note that individuals outside and inside of the organized sector of the economy save the amounts \( s_t = s(k_t) \) and \( s_t = \beta k_t \), respectively. Consequently, as the less developed sector atrophies larger numbers of agents are accumulating wealth at the expected rate \( \beta E[\max(1, \theta)] \). Thus, asymptotically all agents' wealth will be growing at the same rate and a stable distribution of relative wealth levels, say as measured by a Lorenz curve, will attain.\(^6\) The economy's expected growth rate converges (though non-monotonically) to \( \beta E[\max(1, \theta)] \) with variance \( \beta^2 \text{var}(\max(1, \theta)) \).
V. CONCLUSIONS

Two themes have been prominent in the growth and development literature: the link between economic growth and the distribution of income, and the connection between financial structure and economic development. Both of these issues were addressed here within the context of a single model. Growth and financial structure were inextricably linked. Growth provided the wherewithall to develop financial structure while financial structure in turn allowed for higher growth since investment could be more efficiently undertaken. The model yields a development process consistent, at least, with casual observation. In the early stages of development where exchange is largely unorganized growth is slow. As income levels rise financial structure becomes more extensive, economic growth more rapid and income inequality across the rich and poor widens. In maturity an economy has a fully developed financial structure, attains a stable distribution of income across people, and has a higher and more stable growth rate than in its infancy.
FOOTNOTES

1. Some interesting evidence that countries' growth rates have actually tended to increase over time is reported in Romer (1986). Also, Baumol (1986) presents (graphically) some data in which it appears that dispersion in growth rates across countries with similar per capita incomes declines as per capita income rises.

2. For the purpose of taking sums, reindex the (countable) collection of agent in the set $A^c$ by the natural numbers.

3. Briefly, the Euler equation connected with problem (P1) is

\[
\frac{1}{k-s(k)} = \beta \int \frac{(\theta+\varepsilon)}{D^c(s(k)) \left[ s(k)(\theta+\varepsilon) - s(k)(\theta+\varepsilon) \right]} dF(\theta)dG(\varepsilon)
\]

\[
+ \beta \int \frac{(\theta+\varepsilon)}{D(s(k)) \left( 1-\beta \right) [s(k)(\theta+\varepsilon) - \alpha]} dF(\theta)dG(\varepsilon),
\]

with $s(k)$ denoting the optimal policy function. Now consider the fixed point associated with the mapping shown by (6). Here the sets $D^c(s(k))$ and $D(s(k))$ are fixed, as far as the implied maximization is concerned. The choice problem underlying this mapping has the following Euler equation:

\[
\frac{1}{k-s(k)} = \beta \int \frac{(\theta+\varepsilon)}{D^c(s(k)) \left[ s(k)(\theta+\varepsilon) - s(k)(\theta+\varepsilon) \right]} dF(\theta)dG(\varepsilon)
\]

\[
+ \beta \int \frac{(\theta+\varepsilon)}{D(s(k)) \left( 1-\beta \right) [s(k)(\theta+\varepsilon) - \alpha]} dF(\theta)dG(\varepsilon),
\]

where $\bar{s}(k)$ denotes the optimal policy function. Next, examine the unique solutions for the policy functions to each of these Euler equations; they are the same implying $\bar{s}(k) = s(k)$. Thus, the fixed point to (6) must be represented by (P1).

4. Strictly speaking, the $H_t$ functions are not proper cumulative distribution functions as in general $H_t(\infty) < 1$. 

By Proposition 1, $s(k_t) > \beta k_t$. Consequently, it follows that $\ln k_t > \ln k_o + \sum_{j=1}^{i} [\ln(\theta_j + \epsilon_j) + \ln \beta]$. The right-hand side of this expression is a random walk with positive drift, since $E[\ln(\theta_j + \epsilon_j)] + \ln \beta > 0$ by (A1) and (A2). Thus, $k_t$ must become absorbed into the set $[K_o, \infty)$ with probability one. For more detail see Feller (1971).

In a somewhat different context, Hart and Price (1956) present evidence on the tendency for Lorenz curves to stabilize over time.
REFERENCES


