~~Please return to
P. W. Baumeister~~

2M INTERFERENCE FILTERS
FOR THE ULTRAVIOLET

by

Robert L. Maier

Submitted in Partial Fulfillment
of the
Requirements for the Degree
MASTER OF SCIENCE

Supervised by Dr. P.W. Baumeister
Institute of Optics

The University of Rochester
Rochester, New York

1966

ACKNOWLEDGEMENTS

The suggestion of the topic, as well as the advice and encouragement of Professor P.W. Baumeister is gratefully acknowledged by the author.

The author wishes to express his appreciation to designer Robert Horner and instrument maker Patrick Borelli for their helpful suggestions in the design of some of the internal components for the vacuum chamber.

Special thanks is due Miss Gail Whitmore for her assistance in typing of the final draft.

Financial support came from the National Aeronautics and Space Administration and from the National Science Foundation.

ABSTRACT

The problems of designing all dielectric bandpass filters for the 2000 Å to 3000 Å region of the ultraviolet spectrum are discussed. Alternative designs using absorbing layers, particularly two absorbing layers, are investigated.

The theory is outlined by which the maximum transmission at a desired wavelength can be obtained, for a Fabry-Perot type interference filter composed of two given metal layers separated by a dielectric spacer. A systematic procedure is presented for the design of a Fabry-Perot type filter of optimum transmission. The peak transmission for such a filter is twice as great as that for a conventional Fabry-Perot filter. In addition, the bandwidth is considerably narrower.

Experimental transmission curves are presented for comparison with the maximum attainable transmission predicted by theory.

TABLE OF CONTENTS

<u>Chapter</u>	<u>Page</u>
I. INTRODUCTION	1
Design Problems in the Ultraviolet.....	1
Transmission-Optimized 2M Filters.....	2
II. THEORY	4
Potential Transmittance.....	4
Calculation of $\Psi_{\max}$	5
Finding the Admittance at $\Psi_{\max}$	7
Kard Calculator.....	9
III. CONVENTIONAL 2M FILTER FOR 2536 Å	13
Choice of the Metal Layers.....	13
Choice of the Spacer Layer.....	13
An Attempt to Improve the Design.....	20
IV. A 2M FILTER OF OPTIMIZED TRANSMISSION	21
Design Procedure.....	21
Design of the Matching Stacks.....	23
V. EXPERIMENTAL RESULTS	32
Conventional 2M Filter.....	32
T-Optimized 2M Filter.....	35
VI. SUGGESTIONS FOR FURTHER STUDY	42
VII. REFERENCES	43

LIST OF FIGURES

	Page
1. Potential Transmittance for a Single Absorbing Layer	5
2. Geometry for a Dielectric Stack of m Films	11
3. Geometry Used to Compute the Internal Phase Shifts Upon Reflection for a Conventional 2M Filter	15
4. Computed Internal Phase Shifts for a Conventional 2M Filter	16
5. Computed Reflectance, Transmittance, and Potential Transmittance for a Conventional 2M Filter	19
6. Geometry for Computing the Admittances of the Matching Stacks	22
7. Geometry of a Multilayer Filter	24
8. Matching Stacks for the Half-Filters	26
9a. Design of a Matching Stack Using a Kard Calculator	27
9b. Design of a Matching Stack Using a Kard Calculator	28
10. Geometry Used to Compute Internal Phase Shifts Upon Reflection for a T-Optimized 2M Filter	29
11. Computed Transmittance and Potential Transmittance for a T-Optimized 2M Filter	31
12. Computed and Measured Transmittances for a Conventional 2M Filter	34
13. Computed Front Side and Back Side Reflectances for a T-Optimized 2M Filter	36
14. Measured Front Side and Back Side Reflectances for a T-Optimized 2M Filter	37
15. Computed Transmittance for a T-Optimized 2M Filter	39
16. Measured Transmittance for a T-Optimized 2M Filter	40

I. INTRODUCTION

This thesis is concerned with bandpass filters which function by interference. Such filters are commonly divided into two categories: those composed of all dielectric layers, and those composed of both dielectric and absorbing layers. A typical design of the all dielectric type consists of two highly reflecting quarter wave stacks, separated by a spacer layer tuned to the appropriate wavelength. Examples of this type are given by Heavens.¹ The result is a filter having high peak transmission, narrow bandwidth, but suffering from transmission "leaks" at longer wavelengths. A typical dielectric-metal type of bandpass filter consists of two thin highly reflecting metal layers separated by a dielectric spacer of appropriate thickness. Examples are given by Schroeder.² Such filters suffer from relatively low peak transmissions and relatively wide bandwidths. The desirable feature of this type of filter is its "self-blocking" nature, that is, the long wavelength transmission is low due to the high reflectance and high absorptance of the metal layers.

Design Problems in the Ultraviolet

In theory, both of the above mentioned types of bandpass filters can be designed for any region of the spectrum; however in practice, many limitations are imposed. Bandpass filter design for the ultraviolet spectrum is complicated by the lack of suitable evaporant materials, especially those of high refractive index. To construct a highly reflecting quarter wave stack, as for example the stack which is used in the all dielectric type of bandpass filter, alternate quarter

wave layers of high and low refractive index are required. Due to the fact that transparent materials of high index are not available in the ultraviolet, and due to the problem of long wavelength transmission leaks, the all dielectric type of bandpass filter is commonly not used at wavelengths below $3000 \text{ \AA}$.

The Fabry-Perot interference filter, employing metal layers, has become a widely used type of bandpass filter for the $2000 \text{ \AA}$ to $3000 \text{ \AA}$ region of the ultraviolet spectrum. Such a filter will be referred to as the conventional "MDM" type, that is, "metal-dielectric-metal," or more simply as the "2M" type, to indicate the number of metal layers present.

Thin aluminum films exhibit better transmission properties in the ultraviolet, than for longer wavelengths. This desirable feature results in the self-blocking characteristic of a 2M filter. For example, a conventional 2M filter composed of two aluminum layers each of physical thickness $150 \text{ \AA}$, separated by a cryolite ($3\text{NaF} \cdot \text{AlF}_3$) spacer tuned for $2536 \text{ \AA}$, and deposited on a quartz substrate, will transmit less than 1% for all wavelengths longer than $3500 \text{ \AA}$, while at $2536 \text{ \AA}$, a transmission of 43% will result. A plot of transmission vs. wavelength for such a filter is shown in Figure 5.

Transmission Optimized 2M Filters

An ultraviolet interference filter which exhibits a narrower bandwidth and a greater peak transmission, would be of much greater utility than the conventional 2M type. Berning and Turner³ have suggested that by the proper choice of external environment, the peak transmission of the conventional 2M filter can be greatly increased. In particular, they show that the proper choice of dielectric stacks allows the elimination of the spacer, so that a single metal layer filter results. They have called such a filter an "induced transmission" type.

The phrase "transmission-optimized," or simply "T-optimized" is used here to describe the 2M filter, situated in external environments such that the peak transmission is significantly increased over that for the conventional 2M filter. In addition, the bandwidth is significantly reduced.

While Berning and Turner have shown that transmission-optimized filters of the 2M type can be designed, a systematic procedure for designing them has never been published. It is the purpose of this investigation to establish such a procedure. While the results are applied to bandpass filters for the ultraviolet using aluminum films, the procedure is applicable to longer wavelength regions using silver and gold for the metal layers.

II. THEORY

Several computational methods⁴ are known for calculating the reflectance, transmittance, and other parameters, of a dielectric stack. If absorbing layers are included, the concept of potential transmittance⁵ is quite useful. Potential transmittance provides a method of calculating the admittance of a metal film, at the point where the transmission is a maximum. Once this admittance is known, a transmission optimized 2M filter can be designed by the technique of admittance matching. The design is carried out most easily using a Kard⁶ calculator.

This section contains a discussion of potential transmittance and admittance matching, as well as a brief explanation of the Kard calculator.

Potential Transmittance

The potential transmittance ψ , of a film, is the ratio of the net flux (energy density) flowing out of the film, to that flowing into the film. If $(1-R)$ is the net flux flowing into the j th interface, and T is the net flux flowing out of the $(j+1)$ st interface, then:

$$\begin{aligned}\psi &= \frac{T}{1-R} \\ &= \frac{T}{A+T}\end{aligned}\tag{1}$$

Figure 1. Potential Transmittance for a Single Absorbing Layer

From this definition, we note that the potential transmittance is unity for a dielectric layer, and that the conservation of energy requires:

$$0 \leq \psi \leq 1.0$$

For a stack of m films:

$$\begin{aligned} \psi_{\text{TOTAL}} &= \prod_{i=1}^m \psi_i \\ &= \prod_{i=1}^m \left(\frac{T}{1-R} \right)_i \end{aligned} \quad (2)$$

Calculation of ψ_{max}

For a normally incident plane wave of the form:

$$\vec{E} = E_0 e^{i(\omega t - 2\pi z \tilde{n}_j t_j)} \quad (3)$$

an expression for ψ_j can be conveniently obtained by computing Poynting's Vector at both interfaces of the j th medium. A derivation of the equation for ψ_j is given by Costich,⁷ but only the equation is given here:

$$\psi_j = \frac{n_j - n_j |r_j|^2 - 2K_j |r_j| \sin(\rho_j)}{n_j e^{4\pi\delta K_j t_j} - n_j |r_j|^2 e^{-4\pi\delta K_j t_j} - 2K_j |r_j| \sin(\rho_j - 4\pi\delta n_j t_j)} \quad (4)$$

where:

$$\tilde{n}_j = n_j - iK_j$$

$$r_j = |r_j| e^{i\rho_j}$$

t_j is the physical thickness of the j th layer. Costich also shows that contours of constant ψ_j are circles, when plotted in the complex r plane,⁸ and that the degenerate circle of zero radius corresponds to the point where:

$$\psi_j = \psi_{j\max}$$

In this case, eq.(4) takes the form:

$$\psi_{j\max} = \mu_j \pm \sqrt{\mu_j^2 - 1} \quad (5)$$

where:

$$\mu_j = \frac{\cosh(4\pi\delta K_j t_j) + (K_j/n_j) \cos(4\pi\delta n_j t_j)}{1 + (K_j/n_j)^2} \quad (6)$$

and from this it can be seen:

$$\psi_{j\max} = f(\hat{n}_j, t_j/\lambda)$$

Finding the Admittance at $\psi_{\max}$

It is shown by Costich,⁹ that if r_j has the form:

$$\begin{aligned} r_j &= |r_j| e^{i\rho_j} \\ &= U_j + iV_j \end{aligned}$$

then at $\psi_{j\max}$:

$$r_j = \frac{(k_j/n_j)\psi_{j\max} \sin(4\pi n_j t_j)}{\psi_{j\max} e^{-4\pi \delta k_j t_j} - 1} + i \frac{(k_j/n_j)[1 - \psi_{j\max} \cos(4\pi n_j t_j)]}{\psi_{j\max} e^{-4\pi \delta k_j t_j} - 1} \quad (7)$$

Also from eq.(7) it is noted:

$$r_j = f(\hat{n}_j, t_j/\lambda)$$

Therefore, knowing $\hat{n}_j$ and t_j/λ for a given film, $\psi_{j\max}$ can be computed using eq.(5) and eq.(6). Eq.(7) is then used to compute r_j at:

$$\psi_j = \psi_{j\max}$$

The admittance, rather than the reflectance, is the quantity used below in the design of a transmission optimized 2M filter. The admittance, Y_j in the j th medium, is defined as:

$$Y_j = \frac{H_j}{E_j} \quad (8)$$

where H_j and E_j are the amplitudes of the magnetic and electric fields respectively, in the j th medium. However, the admittance must be the same on both sides of an interface, due to the continuity of E and H . The reduced admittance y_j in the j th medium is defined as:

$$y_j = \frac{Y_j}{n_j} = \frac{H_j}{n_j E_j} \quad (9)$$

If $\vec{E}^+$ and $\vec{E}^-$ represent respectively the incident and reflected electric fields, and similarly for the magnetic fields, then the total fields are:

$$\begin{aligned} \vec{E} &= \vec{E}^+ + \vec{E}^- \\ \vec{H} &= \vec{H}^+ + \vec{H}^- \end{aligned} \quad (10)$$

Substituting into eq.(8), the admittance becomes:

$$Y = \frac{H}{E} = \frac{H^+ + H^-}{E^+ + E^-} \quad (11)$$

But from Maxwell's equations, there is a boundary condition that states:

$$\begin{aligned} H^+ &= n E^+ \\ H^- &= -n E^- \end{aligned} \quad (12)$$

The minus sign is introduced by a phase change of 180° between the fields in reflection. Eq.(12) can be substituted into eq.(11) to give:

$$Y = \frac{n(E^+ - E^-)}{(E^+ + E^-)} \quad (13)$$

or:

$$Y = n \frac{(1 - E^-/E^+)}{(1 + E^-/E^+)} \quad (14)$$

However, the complex amplitude reflectance r_j , measured in the medium of index n_j , can be expressed as:

$$r_j = \frac{E_j^-}{E_j^+} \quad (15)$$

so that eq.(14) becomes:

$$Y_j = n_j \frac{1 - r_j}{1 + r_j} \quad (16)$$

or, in terms of the reduced admittance:

$$y_j = \frac{1 - r_j}{1 + r_j} \quad (17)$$

From eq.(7), the real and imaginary parts of r_j at $\psi_{j \max}$ can be computed, given only $\tilde{n}_j$ and t_j/λ . Using these in eq.(17), we can compute the reduced admittance at $\psi_{j \max}$.

Kard Calculator

The Kard Calculator is an exact graphical means of calculating the reflectance and phase of a multilayer stack, given its admittance. It is quite useful in illustrating the design of a T-optimized filter of the 2M type, as well as numerous other filter types.

Basically, the conformal bilinear transformation of eq.(17), namely:

$$r_j = \frac{1 - y_j}{1 + y_j} \quad (18)$$

is used to transform the natural logarithm of points in the reduced admittance plane into points in the complex reflectance plane. The reduced admittance plane, in polar form, is first transformed to the log admittance plane.

$$y = |y| e^{i\phi}$$

$$\therefore \ln \gamma = \ln |\gamma| + i\phi = u = u_r + i u_i$$

This rectangular grid for u , is then transformed to the complex reflectance plane using the bilinear transformation. The resulting coordinate system for r , is known as the Kard Calculator.

On the Kard Calculator, the amplitude reflectance is found by taking a ratio of the radial distance of the final point plotted, to the maximum chart radius. The phase angle is measured counterclockwise from the positive real axis, to the vector drawn from the origin to the point of interest.

From the definition of admittance, given by eq.(8), it is seen that the admittance must be the same on either side of an interface, due to the boundary conditions regarding the continuity of $\vec{E}$ and $\vec{H}$.

$$\therefore Y_j = Y_{j-1}$$

or:

$$(\gamma n)_j = (\gamma n)_{j-1} \quad (19)$$

$$\gamma_{j-1} = \gamma_j \left(\frac{n_j}{n_{j-1}} \right) \quad (20)$$

and taking the natural log of both sides:

$$\begin{aligned} u'_{j-1} &= \ln \gamma_{j-1} = \ln \gamma_j + (\ln n_j - \ln n_{j-1}) \\ &= u_j + (\ln n_j - \ln n_{j-1}) \end{aligned} \quad (21)$$

Figure 2. Geometry for a Dielectric Stack of m Films

Therefore, if the admittance is known on one side of an interface, the reduced admittance, and then the natural log of the reduced admittance on the other side can be found using eq.(20) and eq.(21) respectively.

Using eq.(21), the change in the amplitude of the reflectance can be plotted as an interface is encountered. In eq.(21), u'_{j-1} is primed to indicate that the phase shift due to the optical thickness of the $(j-1)$ st layer has not yet been accounted. This is accomplished by rotating the vector, u'_{j-1} about the origin, through a phase corresponding to $(nt)_{j-1}$, noting that half of a revolution on the Kard Calculator represents an optical thickness of one quarter wave.

Given an arbitrary point, $r = |r| e^{i\Delta}$ on the Kard Calculator, corresponding to the magnitude and phase of amplitude reflectance from a given surface, this surface can be anti-reflected to zero by choosing indices and layer thicknesses such that the final reflectance vector has zero amplitude. The effect of adding a particular layer to the surface, can be seen graphically on the Kard Calculator by applying eq.(21), and then rotating through the appropriate phase angle.

III. CONVENTIONAL 2M FILTER FOR 2536 Å

In this section, the method of design is illustrated for a conventional 2M interference filter. The wavelength of peak transmittance is chosen as 2536 Å, because this wavelength can be easily obtained for monitoring purposes, using a germicidal lamp, available from the General Electric Company.

Choice of the Metal Layers

For thin semi-transparent absorbing films, aluminum is the best material to use for the 2000 Å to 3000 Å spectral range¹⁰ because it has a more favorable k/n ratio than other metals. This means that the energy lost in transmission through a thin film of aluminum, is due more to reflection than to absorption. The thickness of the aluminum films is chosen such that the desired amount of long wavelength blocking is realized, while at the same time, sufficient transmission is obtained at 2536 Å. A reasonable compromise is a 2M filter composed of two aluminum layers, each of physical thickness 150 Å. An aluminum film 150 Å thick, transmits 17.4% at 2536 Å. At longer wavelengths, the transmission of a 2M filter is almost entirely dependent upon the total thickness of metal present in the complete filter. An aluminum layer of physical thickness 300 Å transmits only about 1% at 3600 Å, and less than 1% for all longer wavelengths.

Choice of the Spacer Layer

A design composed of two aluminum films, each of physical

thickness $150 \text{ \AA}$ is used. The substrate is quartz, of index 1.48 at $2536 \text{ \AA}$. The center spacer layer should be a suitable dielectric material. Higher peak transmission is obtained if the spacer layer has a low index, rather than a high index. For example, a 2M filter of design:

QUARTZ MDM AIR

has:

$$T_{\max} = 14.6\% \quad \text{if} \quad n_D = 2.2$$

$$T_{\max} = 29.5\% \quad \text{if} \quad n_D = 1.41$$

where M is an aluminum layer of physical thickness $200 \text{ \AA}$.

Cryolite, of index 1.36 is a good choice for the spacer material.

The thickness of the spacer layer will now be determined.

The ordinary Fabry-Perot type of filter, uses two highly reflecting dielectric mirrors separated by a spacer, to achieve a resonant cavity.

The phase thickness of the cavity should be $m\pi$ for transmission maxima, where m is the order number of interference. The equation to be satisfied is:

$$\frac{2\pi}{\lambda} nt = m\pi \quad (22.)$$

$$m = 1, 2, 3, \dots$$

However, if metal mirrors are used, the phase shifts are no longer 0° or 180° , as is the case for internal reflections off of dielectric mirrors. In this case, eq.(22) must be modified as:

$$\frac{2\pi}{\lambda} nt = \left(\frac{\epsilon_1 + \epsilon_2}{2} \right) + m\pi \quad (23)$$

where ϵ_1 and ϵ_2 are the phase shifts due to reflection off of the metal layers, computed from inside the spacer layer.

Figure 3. Geometry Used to Compute the Internal Phase Shifts Upon Reflection for a Conventional 2M Filter

The phase shifts ϵ_1 and ϵ_2 , can be computed by any of several well known methods.¹¹ In particular, all calculations contained in this thesis, were performed on the I.B.M. 7074 computer, using a thin films program based on the $\begin{pmatrix} E \\ H \end{pmatrix}$ matrix method. All designs submitted for computation contained two aluminum layers, each having a physical thickness of 150 Å. To compute ϵ_1 and ϵ_2 , two separate designs are used. For one of the designs, a quartz substrate supports the aluminum layer, while the incident medium is taken as cryolite. To compute ϵ_2 , the metal layer is assumed to be deposited on an air substrate, and cryolite is again the incident medium, as shown in Figure 3. The results of this calculation have been plotted, and are shown in Figure 4.

Figure 4. Computed Internal Phase Shifts for a Conventional 2M Filter

Recalling eq.(23),

$$\frac{2\pi}{\lambda} nt = m\pi + \left(\frac{\epsilon_1 + \epsilon_2}{2} \right)$$

In order to eliminate transmission maxima at longer wavelengths, the lowest order of interference must be chosen, so take $m=1$. Then:

$$\begin{aligned} \frac{2\pi}{\lambda} nt &= \pi + \left(\frac{\epsilon_1 + \epsilon_2}{2} \right) \\ &= \pi + 123.82^\circ \end{aligned} \quad (24)$$

However, in measuring phase shifts, integral multiples of π can be added or subtracted, as long as the resultant phase is not negative. Hence, the lowest order of interference is actually obtained when:

$$\begin{aligned} \frac{2\pi}{\lambda} nt &= 123.82^\circ \\ \therefore nt &= \frac{\lambda}{360^\circ} (123.82^\circ) \end{aligned} \quad (25)$$

It is convenient to express the optical thickness of the cavity, nt , in terms of a multiple of quarter waves. The quarter wave optical thickness is defined by:

$$QWOT = 4nt \quad (26)$$

Substituting into eq.(25), we have:

$$\begin{aligned} QWOT = 4nt &= \lambda \left(\frac{123.82^\circ}{90^\circ} \right) \\ &= \lambda (1.375) \end{aligned} \quad (27)$$

Thus, the cavity should be 1.375 quarter waves in optical thickness, or since $\lambda_0 = 2536 \text{ \AA}$, the optical thickness of the spacer should be $872 \text{ \AA}$.

The following design, which has a maximum transmittance at $2536\text{ \AA}$, for the given thickness and type of metal layers, was submitted for computation:

QUARTZ MDM AIR

where: $M = 150\text{ \AA}$ of aluminum
 $D = 1.375$ quarter waves of cryolite
 $n_M = .195 - i(3.183)$ at $2536\text{ \AA}$

The optical constants for aluminum were obtained from Mr. W.R. Hunter¹² and the computations were performed dispersively. The computed results have been plotted, and are shown in Figure 5:

Figure 5. Computed Reflectance, Transmittance, and Potential Transmittance for a Conventional 2M Filter

An Attempt to Improve the Design

$\psi_{\max}$ does not occur at λ_o but rather at $2300 \text{ \AA}$. This might indicate that a better 2M filter could be designed, if $\psi_{\max}$ could be shifted to λ_o since:

$$\psi_{\max} = f(\tilde{n}_M, t/\lambda)$$

However, the complex index of aluminum, $\tilde{n}_M$, does not change very rapidly over $2300 \text{ \AA} \leq \lambda \leq 2600 \text{ \AA}$.

$$\therefore \psi_{\max} \approx f(t/\lambda)$$

But at $\psi_{\max}$:

$$\lambda = 2300 \text{ \AA}$$

$$t = 150 \text{ \AA}$$

so that:

$$t/\lambda = .065$$

This value can be maintained at longer wavelengths, if t is increased appropriately. If t is increased to $165 \text{ \AA}$, then at $2536 \text{ \AA}$:

$$t/\lambda = .065$$

If a 2M filter is designed for metal layers of physical thickness $165 \text{ \AA}$ each, the spacer layer must be increased to $880 \text{ \AA}$ of cryolite. Such a design shifts the position of $\psi_{\max}$ closer to $2536 \text{ \AA}$, but due to the increased thickness of the metal layers, the magnitude of $\psi_{\max}$ decreases to a value of .615, while $T_{\max}$ decreases to 38%.

Therefore, a 2M filter designed such that $\psi_{\max}$ occurs at λ_o requires such thick metal layers, that sufficient transmission does not result at λ_o . It is thus concluded that for a given metal layer thickness, a 2M filter of optimum design results when the spacer layer thickness is adjusted so that eq.(23) is satisfied.

IV. A 2M FILTER OF OPTIMIZED TRANSMISSION

The undesirable features of the 2M filter described previously, are the low peak transmittance and wide bandwidth. The transmittance at λ_0 can be increased by suitably anti-reflecting (admittance matching) the metal layers. The effect of admittance matching serves to reduce the amount of absorption, in addition. For example, a conventional 2M filter at $\lambda_0 = 2536 \text{ \AA}$ has a transmittance of 43%, a front side reflectance of 14%, and an absorptance of 43%. However, the same design, when suitably admittance matched, has a transmittance of 80.5%, a front side reflectance of 2%, and an absorptance of 17.5%. The decrease in absorption can be visualized as being due to a repositioning of the nodes of the electric field. The optical thickness of the spacer for the conventional 2M filter designed previously, was $872 \text{ \AA}$, while that for the T-optimized 2M filter (i.e. the admittance matched 2M filter) designed in this section, is $1083 \text{ \AA}$. The increased spacer width means that the position of the nodes has changed with respect to the metal layers, resulting in decreased absorption.

Design Procedure

In this section, the conventional 2M filter designed previously, is modified by the technique of admittance matching, such that a filter of maximized peak transmission results. Admittance matching means that the two aluminum layers are sandwiched between two dielectric stacks whose admittances match those of the aluminum layers.

The first step is to determine the admittances of the given metal layers at the point where the transmission is a maximum. Recall the definition of potential transmittance from eq.(1):

$$\psi = \frac{T}{1-R}$$

It is shown by Costich¹³ that the transmission through a given metal film is maximized only when the admittances of the matching stacks are equal, when measured as shown:

Figure 6. Geometry for Computing the Admittances of the Matching Stacks

The matching stack anti-reflects the metal-dielectric interface, so that eq.(1) becomes:

$$\psi = T = T_{\max} = \psi_{\max}$$

and the transmission must be the same in either direction. The reduced admittance at $\psi_{\max}$ (i.e. at $T_{\max}$) can now be calculated from eq.(7) and eq.(17).

The thin films computer program used for computations for this thesis, has the capacity to calculate the reduced admittance at $T_{\max}$ using the procedure outlined above. The reduced admittance for an aluminum layer of physical thickness $150 \text{ \AA}$, at the point where the transmission is a maximum, was computed to be:

$$y_M = 2.0146 - i(4.6278)$$

This point can be plotted on the Kard Calculator as follows:

$$\begin{aligned} y_M &= 2.0146 - i(4.6278) \\ &= |R| e^{i\Delta} = \sqrt{(2.0146)^2 + (4.6278)^2} e^{i\Delta} \end{aligned}$$

and:

$$\begin{aligned} \tan \Delta &= \frac{y_i}{y_r} = 2.2971 \\ \therefore \Delta &= 66^\circ 28' = 1.16 \text{ rad.} \end{aligned}$$

$$\therefore y_M = 5.047 e^{i(1.16)}$$

or:

$$\ln y_M = 1.619 + i(1.16)$$

These coordinates can now be plotted on the Kard Calculator, as shown in Figure 9a. and Figure 9b. Using the Kard Calculator, two dielectric matching stacks must be designed, one stack matching quartz to aluminum, the other matching air to aluminum.

Design of the Matching Stacks

The admittance y_M , at $T_{\max}$ for the metal layer, corresponds to a particular complex amplitude reflectance. It is desired to construct a dielectric matching stack having the same admittance. One way to accomplish this, is to begin at the point y_M and add layers such that

the reflectance is systematically reduced to zero, when the final layer and substrate are included. Conversely, the substrate can be chosen as the starting point, and dielectrics added until the desired admittance is achieved. This latter procedure is used here.

A highly refined quartz substrate of refractive index 1.48, serves as the starting point. The dielectric materials to be used are thorium fluoride of index 1.55, and cryolite of index 1.36. The design will be carried out on a Kard Calculator, according to eq.(21) and eq.(17). As a starting value, it is assumed that $r_s = 0$, that is, there is no returning wave in the substrate.

Figure 7. Geometry of a Multilayer Filter

From eq.(17):

$$y_s = \frac{1 - r_s}{1 + r_s} = 1.0$$

$$\therefore u_s = \ln y_s = 0$$

This is the starting point on the Kard Calculator. From eq.(21):

$$\begin{aligned} u_j' &= u_s + \ln n_s - \ln n_j \\ &= 0 + \ln(1.48) - \ln(1.36) \\ &= +.085 \end{aligned}$$

When the phase of the quarter wave optical thickness is included, this becomes:

$$u_j = -.085$$

The layer next to the substrate is taken as cryolite because the greater index mis-match allows for greater ease in optical monitoring of the first layer during actual production.

The process of adding layers alternately of high and low index according to eq.(21) is continued, and the results plotted on the Kard Calculator. The final equation of the type eq.(21), is of the form:

$$u_o = u_1 + \ln n_1 - \ln n_o \quad (28)$$

but $n_o=1.0$, and u_o is the point y_M , the reduced admittance of the metal layer at the point where the transmittance is a maximum:

$$u_o = 1.6188 + i(1.16)$$

Hence, eq.(28) becomes:

$$1.6188 + i(1.16) = u_1 + \ln n_1$$

However, n_1 can have only a real part, since all of the layers are dielectrics. Therefore, the imaginary part of u_1 must also be 1.16, that is:

$$\begin{aligned} u_1 &= u_r + i u_i \\ &= u_r + i(1.16) \end{aligned}$$

Hence:

$$1.6188 = u_r + \ln n_1$$

or:

$$u_r = 1.6188 - \begin{cases} \ln n_L \\ \ln n_H \end{cases}$$

Therefore, the effect of the final matching layer should be such that the reduced admittance of the complete matching stack is:

$$u_i = u_r + i u_i = 1.6188 - \begin{cases} \ln n_L \\ \ln n_H \end{cases} + i(1.16)$$

The ambiguity in the real part is resolved simply by plotting two points, one corresponding to a design having a final matching layer of high index, and one for a final layer of low index. Of these two designs, the one having an admittance closest to its particular desired value, is then chosen.

In the same manner, the other stack, matching air to the aluminum, is constructed. For this stack, an air substrate is assumed.

Figure 8. Matching Stacks for the Half-Filters

Figure 9a shows the form of the Kard Calculator used to design the quartz to metal matching stack, while Figure 9b shows that used to design the air to metal stack.

Figure 9a. Design of the Matching Stack Using a Kard Calculator

Figure 9b. Design of the Matching Stack Using a Kard Calculator

As in the case of the conventional 2M filter, the spacer thickness must be computed according to eq.(23). Therefore, the following two "half-filter" designs were submitted for computation, in order to determine ϵ_1 and ϵ_2 :

Figure 10. Geometry Used to Compute Internal Phase Shifts Upon Reflection for a T-Optimized 2M Filter

The computed values are:

$$\epsilon_1 = 153.48^\circ$$

$$\epsilon_2 = 154.14^\circ$$

Using these, eq.(23) becomes:

$$\frac{2\pi}{\lambda} nt = m\pi + 153.81^\circ$$

As in the case of the conventional 2M filter, the first order of interference spacer thickness is chosen, and it is found:

$$\begin{aligned} nt &= (1.71) \frac{\lambda}{4} \\ &= 1083 \text{ Å} \end{aligned}$$

It will be noted, that this spacer thickness is considerably greater than

that for the conventional 2M filter designed previously.

The completed design for the transmission-optimized 2M filter at $\lambda_0 = 2536\text{\AA}$ is as follows:

$$\text{QUARTZ (LH)}^8 \text{ LL}'_1 \text{ M LL}'_2 \text{ L}'_2 \text{ (LH)}^7 \text{ AIR}$$

where:

$$\begin{aligned} n_s &= 1.48 & L'_1 &= .825 \frac{\lambda}{4} \\ n_L &= 1.36 & L'_2 &= .816 \frac{\lambda}{4} \\ n_H &= 1.55 & \text{LL}' &= 1.71 \frac{\lambda}{4} \end{aligned}$$

$$M = 150\text{\AA} \text{ ALUMINUM}$$

This design was computed on the I.B.M. 7074. The results of the computation are given in Figure 11.

This filter is considerably improved over the conventional 2M filter of Figure 5, which employs the same metal layers. The addition of the matching stacks serves to double the peak transmission, while the bandwidth is reduced by a factor of 4.7.

Figure 11. Computed Transmittance and Potential Transmittance for a T-Optimized 2M Filter

V. EXPERIMENTAL RESULTS

It has been found¹⁴ that semi-transparent aluminum films for use in the ultraviolet have optimum performance characteristics if they are produced from high purity aluminum evaporated onto an unheated substrate, at a pressure of 1×10^{-5} mm. or lower, and at a deposition rate of about 300 Å per second.

The aluminum layers for the filters described here, were produced under near optimum conditions. High purity aluminum was evaporated onto unheated substrates, at pressures of 2×10^{-5} mm. during the evaporation of the metal. Otherwise, a pressure of 2×10^{-6} mm. was maintained, rising to 8×10^{-6} mm. during the evaporation of the dielectrics. Since only 150 Å of aluminum was desired, the suggested rate of evaporation of 300 Å per second was not feasible. Instead, a rate of 50 Å per second was used. Thus the aluminum layers were deposited in about 3 seconds, and an automatic shuttering system was employed to increase uniformity and reproducibility. The aluminum, in the molten state, was outgassed for about 20 minutes at a pressure of 1×10^{-5} mm. before the evaporation.

Substrates were cleaned in water and detergent, then rinsed thoroughly in distilled water. Finally they were cleaned for 20 minutes in distilled water at 190°F, using an ultrasonic cleaner. Drying was accomplished by blowing the water off of the substrate in a single sheet, using compressed dry nitrogen.

Conventional 2M Filter

Some conventional Fabry - Perot filters were produced at first, in an attempt to verify the calibration of the automatic shutter, with

regard to control of the metal layer thicknesses. Also, the technique for monitoring the fractional quarter waves, needed to be perfected. The dielectric layers were optically monitored in reflection, while transmission monitoring was used for the metal layers.

Figure 5 shows the theoretical transmission curve for a conventional 2M filter at $2536\text{\AA}$, constructed from two aluminum layers each of physical thickness $150\text{\AA}$. The spacer layer of cryolite is $872\text{\AA}$ in optical thickness. The theoretical transmission curve from Figure 5 is reproduced below, together with the measured transmission of an experimental filter. Transmission measurements were performed on the Cary 14 spectrophotometer.

Figure 12. Computed and Measured Transmittances for a Conventional 2M Filter

T-Optimized 2M Filter

The transmission-optimized 2M filter is much more difficult to fabricate than the ordinary 2M type. In particular, the phase shifts upon reflection off of the dielectric matching stacks are quite dispersive, and consequently, the anti-reflection is a very strong function of wavelength. For example, Figure 13 shows the computed reflectances for the T-optimized 2M filter designed above, while Figure 14 shows the measured reflectances for an experimental filter.

In constructing such a filter, it is important that the front side (coated side) and back side (substrate side) reflectance minima coincide at the same wavelength. Maximum peak transmission is not attained unless this wavelength also coincides with that for which the dielectric spacer is tuned. Therefore, to obtain a good T-optimized 2M filter, it is essential that the three fractional quarter wave layers, be monitored accurately.

Figure 13. Computed Front Side and Back Side Reflectances for a T-Optimized 2M Filter

Figure 14. Measured Front Side and Back Side Reflectances for a T-Optimized 2M Filter

The computed design for the T-optimized 2M filter, shown in Figure 11, indicates a secondary transmission peak at $2950 \text{ \AA}$. This wavelength coincides with that for which the front side and back side reflection curves have secondary minima, as shown in Figure 13. For incidence from the front side, computed values for the conventional 2M filter at $2950 \text{ \AA}$ are:

$$R = 67.7\% \qquad T = 5.9\% \qquad A = 26.4\%$$

For the T-optimized 2M filter, designed above, the computed values for incidence from the front side, at $2950 \text{ \AA}$, are:

$$R = 13.0\% \qquad T = 41.0\% \qquad A = 46.0\%$$

Thus, the effect of the secondary reflectance minima at $2950 \text{ \AA}$, due to the matching stacks, is to reduce the reflectance by 54.7%, for incidence from the coated side. Of this amount, 35.1% goes into increased transmission, and 19.6% goes into increased absorption.

The secondary transmission peak at $2950 \text{ \AA}$ is an undesirable feature of the design. It could be eliminated if matching stacks could be designed such that the reflectances from the front and back sides remained high, for wavelengths longer than $2536 \text{ \AA}$.

Several filters were produced according to the design of Figure 11. In general, the wavelength centering was not at $2536 \text{ \AA}$, but at $2710 \text{ \AA}$. This is attributed to the fact that the optical monitoring system that was used, did not operate at normal incidence, but at approximately 10° incidence. Figure 15 shows the computed design for transmittance plotted on a log scale, while Figure 16 shows the measured transmittance plotted on a log scale.

Figure 15. Computed Transmittance for a T-Optimized 2M Filter

Figure 16. Measured Transmittances for a T-Optimized 2M Filter and a Conventional 2M Filter

Figure 17. Computed and Measured Reflectances for a T-Optimized 2M Filter

The experimental filter has less peak transmission and a wider bandwidth than the computed design. It is superior to the computed design with respect to short wavelength cut-off, and with respect to the ratio of peak transmission to transmission at the secondary maximum. The high value of transmission in the secondary band of the computed design is explained with reference to Figure 13. It is seen that the secondary minima for the front side and back side reflectances, coincide at the same wavelength, $2950 \text{ \AA}$. This is also the wavelength at which the secondary transmission peak occurs.

From Figure 14, it is seen that the secondary minima for the front side and back side reflectances for the experimental filter do not coincide at the same wavelength. This results in a lower value of transmission at the secondary peak.

Therefore, for the computed design, the matching stacks almost match the admittances of the metal layers at $2950 \text{ \AA}$, whereas for the experimental filter, the secondary admittance matches are not as good.

VI. SUGGESTIONS FOR FURTHER STUDY

The most undesirable feature about the transmission-optimized 2M filter of Figure 11 is the transmission leak in the neighborhood of $3000\text{ \AA}$. It might be interesting to try to eliminate this leak, and in this connection, two suggestions appear worthy of further study.

The first item to investigate should be the effects of increasing the order of interference, with respect to the secondary transmission peak. During the course of the present work, it was determined that adding extra "half-waves" to the matching layers had little effect on the over-all transmission. However, when the optical thickness of the spacer layer was increased by a single half-wave, the secondary transmission at $2950\text{ \AA}$ was reduced from 40% to 13%, while the other transmission characteristics were not degraded.

The computer program used for all computations for the present work has the capacity to compute and plot contours of constant transmission in the admittance plane. This capability provides another method of analyzing the secondary transmission at $2950\text{ \AA}$.

Little attention has been given to induced transmission filters for wavelengths beyond $3000\text{ \AA}$. The possibility of using silver, gold, and copper for the absorbing layers could be investigated in the $3000\text{ \AA}$ to $18,000\text{ \AA}$ region. The generation of contours of constant transmission, plotted in the admittance plane, would be useful in determining the optimum conditions for such filters.

Experimentally, the complexity of a transmission-optimized 2M filter, requires accuracy in monitoring the fractional quarter waves. Thus if T-optimized filters are to be produced, some time should be spent in perfecting a good monitoring system and monitoring technique.

VII. REFERENCES

1. O.S. Heavens, Optical Properties of Thin Solid Films (Dover, 1965), p. 225.
2. D.J. Schroeder, Interference Transmission Filters for the Far Ultraviolet, J. Opt. Soc. Am. 52, 1380 (1962).
3. P.H. Berning and A.F. Turner, Induced Transmission in Absorbing Films Applied to Bandpass Filter Design, J. Opt. Soc. Am. 47, 230 (1957).
4. O.S. Heavens, op. cit., Chap. 4.
5. P.H. Berning and A.F. Turner, loc. cit.
6. P.G. Kard, A Graphical Method of Calculation for Multilayer Films, Proc. Acad. Sci. U.S.S.R., 108, 60 (1956).
7. V.R. Costich, PhD dissertation, University of Rochester, Rochester, New York, p. 26, 1965 (unpublished)
8. Ibid., p. 29.
9. Ibid., p. 37.
10. G. Hass and J.E. Waylonis, Optical Constants and the Reflectance and Transmittance of Evaporated Aluminum in the Visible and Ultraviolet, J. Opt. Soc. Am., 51, 719 (1961).
11. O.S. Heavens, op. cit., Chap. 4.
12. Private Communication from Mr. W.R. Hunter, U.S. Naval Research Laboratory, Washington, D.C.
13. V.R. Costich, op. cit., p. 38.
14. G. Hass and J.E. Waylonis, loc. cit.