SOME FURTHER ASPECTS OF SCATTER-FRINGE INTERFEROMETRY

by

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I. Introduction

The phenomena associated with the use of scattering as a means of beam-splitting were first described by Newton¹ who used a system similar to that shown in Figure 1. The principle is illustrated in Figure 2. Some of the light incident normally upon the front surface is scattered at O and strikes the back surface at B instead of A. The scattered light is reflected at B and emerges back into the air at D while some of the light reflected back from A is scattered at O. Since the two scattered beams emerge parallel, the optical path difference (OPD) between them is

$$OPD = 2t(1 - \cos \phi) \quad (1)$$

and they interfere to give fringes at infinity. By making both surfaces of the mirror concentric about C (Figure 1) Newton obtained his colored circular fringes on screen S. While experimenting with these fringes the Duke de Chaulnes² discovered that by permanently tarnishing the front surface of the mirror with a mixture of milk and water the brilliancy of the fringes was greatly increased, this due to the fact that the globules of dried milk were more effective

Figure 1. Newton's method of obtaining fringes.
The center of a small hole in screen S is placed at C, the center of curvature of both surfaces of mirror M. The back surface of M is silvered and the front surface has pits in it (left from polishing) that scatter the light.

scatterers than the small pits left by polishing. The first explanation of these fringes on the basis of wave theory was a brief one given by Young³. He pointed out the application of the newly discovered principle of interference to the explanation of the phenomena. The complete explanation of these fringes on the basis of the wave theory was first given by Sir John Herschel in his treatise on Light⁴. Whewell observed a set of colored fringes while viewing the image of a candle held near the eye by reflection from a back silvered plane mirror. This observation was communicated to M. Quetelet who published it⁵. Stokes⁶, in a paper given before the Cambridge Philosophical Society in 1851 traces the history of these phenomena (outlined above) and gives a detailed explanation of them, showing that the "Quetelet" fringes are a special case of the fringes first observed by Newton. More recently Shoenberg and Bauchwitz⁷ mention that by using a small white light source to illuminate an air film between heavily silvered surfaces they were able to observe the circular scatter-fringes in transmission, something Stokes was unable to do.

This method of beam-splitting was not practically employed until recently when Burch^{8,9} suggested using two identical scatter plates to make an interferometer which can be used for testing optical systems. It is worth mentioning that Burch was the first to suggest the

Figure 2. Diagram illustrating the principle of the scatter-fringes. O is a scatterer on the front surface of the mirror whose thickness is t . ϕ is the scatter angle inside the glass.

idea of replication in order to obtain two identical scatter plates. His interferometer is one of several types^{10,11,12} (classified according to method of beam-splitting) which have subsequently been referred to as "common path interferometers" by Dyson¹³ because the two interfering wavefronts travel in essentially the same direction (one inside the other) and through the same optics. The common path interferometers exhibit definite advantages over the Twyman interferometers used for testing¹⁴ because in the Twyman the reference and test wavefronts travel widely separated paths and are therefore affected differently by vibrations and thermal effects. In addition the beam-splitter and reference plane mirror in the Twyman must be at least as large as the optical system under test. The size of the beam-splitter can be reduced by using diverging light in both beams but the reference concave mirror (or plane mirror and corrected lens) must still be as large as the system under test.^{15,16} Still another advantage is that since the two wavefronts in the common path interferometers traverse the same optics a white light source can be used.

Common path interferometers are based on the assumption that for all practical purposes the paraxial region of any optical system is free from aberration. By suitable means two coherent wavefronts are obtained,

one filling the full aperture of the optical system under test and the other restricted to the paraxial region. When these wavefronts are recombined the resulting interference pattern is that which would be obtained using a Twyman interferometer. The general method suggested by Burch is shown in Figure 3. P_1 and P_2 are two identical scatter plates. (P_2 has been rotated 180 degrees in azimuth so that each light scatterer on P_2 is at the conjugate of the corresponding scatterer on P_1 . Thus light coming from A on P_1 will pass through A' on P_2 . Since A and A' are identical, light passing through each is affected in the same manner.) Light from source S is focused by lens L on mirror M being tested. Scatter plate P_1 scatters part of the light and fills the aperture of mirror M. The light which is not scattered by P_1 is focused on the paraxial region of mirror M. The light from mirror M is returned to scatter plate P_2 where part of the light which passed undeviated through P_1 is scattered, and part of the light which was scattered by P_1 passes through undeviated. There are essentially four wavefronts present after P_2 :

- 1) one due to light which is scattered by P_1 but passes undeviated through P_2 , the test wavefront; 2) one due to light which passes undeviated through P_1 and is scattered by P_2 , the reference wavefront; 3) one due to light which passes undeviated through both P_1 and P_2 ; and 4) one due to

Figure 3. Schematic arrangement for testing a concave mirror using an interferometer with two scatter plates. M is the concave mirror; P_1 and P_2 are identical scatter plates; L is the lens used to focus source S on M ; and E is the position of the eye or camera.

light which is scattered by both P_1 and P_2 . Wavefront 3 (the direct-direct light) gives rise to a bright spot at the center of the field which can be annoying. However, since this wavefront serves no purpose a stop may be placed in the system after P_2 to eliminate it. This is especially desirable for photographic work since it eliminates a badly overexposed spot in the center of the field. Wavefront 4 (the scattered-scattered light) contributes to the background light and therefore degrades the contrast of the fringes.

An alternate method to that shown in Figure 3 which has been suggested by Dyson¹⁷ is to replace P_2 by a plane mirror, thus reimaging P_1 back on itself point for point (see Figure 4). This system is useful for testing long focus optical systems since it has a magnification of +1 and is therefore extremely insensitive to vibration. However, since the effect of the plane mirror is to rotate the wavefront through 180 degrees in azimuth all the odd order terms in the wavefront expression cancel out. Thus any coma present in the system will not be observed. An even greater disadvantage is that it is not possible to introduce tilt between the reference and test wavefronts by a lateral displacement of the scatter plate and plane mirror. In both cases mentioned above refractive systems can also be tested as shown in Figures 5 and 6. In practice the two scatter

Figure 4. Schematic arrangement for testing a concave mirror using an interferometer with one scatter plate and a plane mirror. M is the concave mirror; P_1 is the scatter plate; P_0 is the plane mirror; L is the lens used to focus source S on M ; and E is the position of the eye or camera.

Figure 5. Method of testing refractive optics using two scatter plates. P_1 and P_2 are the two scatter plates in the focal region of lens L under test; M is a good flat mirror used to return the incident wavefront back to P_2 .

Figure 6. Method of testing refractive optics using one scatter plate and a plane mirror. P_1 is the scatter plate in the focal region of the lens under test L ; P_2 is the plane mirror in the focal region of the well corrected lens O .

plates (or one scatter plate and plane mirror), the source, and the focusing lens are all rigidly mounted on a three dimensional motion to facilitate investigation of the region near the paraxial image.

The phenomena associated with scattering have been thoroughly explained in the literature. This, however, is not the case concerning the best conditions under which scattering can be used for testing. For example, Burch, Dyson, and Martin¹⁸ explain the principles and some of the advantages and disadvantages of the scatter-fringe interferometer but go no further. Scott¹⁹ has published a photograph taken with a scatter-fringe interferometer but hasn't given any particulars concerning the instrument. The purpose of this thesis is to describe a simple method of testing based on scattering as a means of beam-splitting. The objectives of this work were: 1) to obtain a system which is insensitive to vibration and therefore well suited for testing long focus optical systems; 2) to obtain a system with high contrast in the fringe patterns (this along with objective 1 make the system well suited for photographic work); and 3) to obtain a system which is at least as sensitive to the aberrations of the system being tested as the Twyman interferometer.

It is shown that a system using one scatter plate and a plane mirror is a system well suited for testing.

As previously mentioned since it has a magnification of +1 it is extremely insensitive to vibration. Although coma cannot be observed with this system this is not a disadvantage when testing mirrors. By introducing a thin plane parallel plate of glass into the system it is possible to introduce tilt between the reference and test wavefronts, thus making it possible to examine in detail a nearly perfect spherical wavefront. Since the test wavefront passes through the system twice this method is twice as sensitive as the Twyman interferometer for testing mirrors. It is also shown that by using the proper size grinding particles to roughen the scatter plate a contrast of 0.7 can be obtained. For the present work only scatter plates made by roughening one side of a small glass plate by grinding are considered because they are easy to make, they are very permanent, and they give adequate contrast in the fringe patterns.

II. Aberrations

A. General

The general equation for the OPD (for primary aberrations only) between the reference and test wavefronts in the exit pupil of the system under test is²⁰

$$\begin{aligned} \text{OPD} = & B(x^2 + y^2)^2 + F(x^2 + y^2)y + C(x^2 - y^2) \\ & + \frac{\bar{x}}{f}x + \frac{\bar{y}}{f}y + \frac{\bar{z}(x^2 + y^2)}{2f^2} \end{aligned} \quad (2)$$

where: B = Spherical aberration coefficient

F = Coma coefficient

C = Astigmatism coefficient

$\bar{x}$ = Lateral displacement of the scatter plates in the x direction from the paraxial image

$\bar{y}$ = Lateral displacement of the scatter plates in the y direction from the paraxial image

$\bar{z}$ = Longitudinal defocusing of the scatter plates in the z direction from the paraxial image

f = Focal length of system (radius of curvature for a mirror)

For the case when a mirror is being tested the deviation of the test wavefront from the reference wavefront is double the deviation of the mirror surface from a true sphere; therefore, if the coefficients in Eq. (2) are to

apply directly to the mirror the right side of the equation must be multiplied by two. It is more convenient to work with Eq. (2) if the right side is normalized by multiplying by $(r_{\max}/r_{\max})^4$, where $r_{\max}$ is the maximum radius of the exit pupil. For constructive interference Eq. (2) becomes

$$\begin{aligned} \text{OPD} = 2B_0(u^2+v^2)^2 + 2F_0(u^2+v^2)v + 2C_0(u^2-v^2) \\ + \frac{\bar{x}}{f^\#} u + \frac{\bar{y}}{f^\#} v + \frac{\bar{z}(u^2+v^2)}{4(f^\#)^2} = m\lambda \end{aligned} \quad (3)$$

where: $B_0 = B(r_{\max})^4$ units of wavelength
 $F_0 = F(r_{\max})^3$ units of wavelength
 $C_0 = C(r_{\max})^2$ units of wavelength
 $u = x/r_{\max}$
 $v = y/r_{\max}$
 $f^\# = f/2r_{\max}$
 $m = 0, 1, 2, \dots$ (order of interference)
 $\lambda = \text{wavelength of light}$

B. Case When Two Scatter Plates Are Used

1. Spherical Aberration

If spherical aberration alone is present in the wavefront then Eq. (3) becomes

$$2B_0(u^2+v^2)^2 + \frac{\bar{x}}{f^\#} u + \frac{\bar{y}}{f^\#} v + \frac{\bar{z}(u^2+v^2)}{4(f^\#)^2} = m\lambda \quad (4)$$

At the paraxial focus ($\bar{x} = \bar{y} = \bar{z} = 0$) this reduces to

$$u^2 + v^2 = \left(\frac{m\lambda}{2B_0} \right)^{\frac{1}{2}} \quad (5)$$

For a small longitudinal defocusing ($\bar{x} = \bar{y} = 0$)

$$u^2 + v^2 = \left[\frac{m\lambda}{2B_0} + \left(\frac{\bar{z}}{16B_0(f\#)^2} \right)^2 \right]^{\frac{1}{2}} - \frac{\bar{z}}{16B_0(f\#)^2} \quad (6)$$

For a small lateral displacement ($\bar{z} = 0$)

$$2B_0(u^2 + v^2)^2 + \frac{\bar{x}}{f\#} u + \frac{\bar{y}}{f\#} v = m\lambda \quad (7)$$

Large amounts of spherical aberration are readily detected as shown by Eqs. (5) and (6) because of the spacing and number of fringes in the field. To detect small amounts of spherical aberration it is necessary to introduce tilt between the reference and test wavefronts by a small lateral displacement as shown in Eq. (7).

2. Coma

If coma alone is present Eq. (3) reduces to

$$2F_0(u^2 + v^2)v + \frac{\bar{x}}{f\#} u + \frac{\bar{y}}{f\#} v + \frac{\bar{z}(u^2 + v^2)}{4(f\#)^2} = m\lambda \quad (8)$$

At the paraxial focus

$$v^3 + u^2 v = \frac{m\lambda}{2F_0} \quad (9)$$

For small longitudinal defocusing

$$V^3 + \frac{\bar{z}}{8F_0(f\#)^2} V^2 + u^2 V + \frac{\bar{z}}{8F_0(f\#)^2} u^2 = \frac{m\lambda}{2F_0} \quad (10)$$

For small lateral displacement

$$V^3 + \left(u^2 + \frac{\bar{y}}{2F_0 f\#}\right) V + \frac{\bar{x}}{2F_0 f\#} u = \frac{m\lambda}{2F_0} \quad (11)$$

Small amounts of coma are easily detected because of the asymmetry of the fringe pattern for lateral displacement.

3. Astigmatism

If astigmatism alone is present in the wavefront Eq. (3) reduces to

$$2C_0(u^2 - v^2) + \frac{\bar{x}}{f\#} u + \frac{\bar{y}}{f\#} v + \frac{\bar{z}(u^2 + v^2)}{4(f\#)^2} = m\lambda \quad (12)$$

For small longitudinal displacement Eq. (12) becomes

$$\left[2C_0 + \frac{\bar{z}}{4(f\#)^2}\right] u^2 - \left[2C_0 - \frac{\bar{z}}{4(f\#)^2}\right] v^2 = m\lambda \quad (13)$$

Astigmatism is easily identified by its either hyperbolic or elliptical fringe patterns (depending on the relative values of $\bar{z}$ and C).

In each case above the fringe patterns have exactly

the shape and magnitude of the fringe patterns obtained using a Twyman interferometer.

C. Case When One Scatter Plate And A Plane Mirror Are Used

When one scatter plate and a plane mirror are used the test wavefront passes through the system twice. The effect of the plane mirror is to rotate the wavefront through 180 degrees in azimuth so that light striking the aperture at $P(x,y)$ on the first pass strikes the aperture at $P(-x,-y)$ after being reflected by the plane mirror. For this case then Eq. (3) becomes

$$4B_0(u^2+v^2)^2 + 4C_0(u^2+v^2) + \frac{\bar{z}(u^2+v^2)}{2(f\#)^2} = m\lambda \quad (14)$$

The major disadvantage in this case is shown by the absence of the $\bar{x}$ and $\bar{y}$ terms in Eq. (14). This means that it is not possible to introduce a tilt between the reference and test wavefronts by a simple lateral displacement of the scatter plate and plane mirror. Consequently it is not possible to examine a nearly perfect spherical wavefront in detail. For example, it is not possible to visually detect amounts of spherical aberration less than $B_0 = 1/4 \lambda$.

1. Perfect Optical System

For a perfect optical system $B_0 = C_0 = F_0 = 0$ in the returning wavefront, therefore Eq. (14) reduces to

$$\frac{\bar{z} (u^2 + v^2)}{2(f\#)^2} = m\lambda \quad (15)$$

For $\bar{z} = 0$ this represents one bright fringe filling the aperture. For $\bar{z} \neq 0$ Eq. (15) can be rewritten as

$$u^2 + v^2 = \frac{2(f\#)^2 m\lambda}{\bar{z}} \quad (16)$$

This is just the equation of a family of concentric circular fringes. In Figure 7 are photographs showing the fringe patterns for no longitudinal defocusing (corresponding to the paraxial focus) and for a small longitudinal defocusing for the case of a perfect wavefront.

2. Spherical Aberration

If spherical aberration alone is present in the wavefront Eq. 13 reduces to

$$4B_0(u^2 + v^2)^2 + \frac{\bar{z}(u^2 + v^2)}{2(f\#)^2} = m\lambda \quad (17)$$

which can be rewritten as

$$u^2 + v^2 = \left[\frac{m\lambda}{4B_0} + \left(\frac{\bar{z}}{8B_0(f\#)^2} \right)^2 \right]^{\frac{1}{2}} - \frac{\bar{z}}{8B_0(f\#)^2} \quad (18)$$

Figure 7. Photographs showing the fringe patterns due to a perfect spherical wavefront for no longitudinal defocusing (corresponding to the paraxial focus) and for a small longitudinal defocusing.

For no longitudinal defocusing this becomes

$$u^2 + v^2 = \left(\frac{m\lambda}{4B_0} \right)^{\frac{1}{2}} \quad (19)$$

In Figure 8 are photographs showing the fringe patterns due to spherical aberration for no longitudinal defocusing and for a small longitudinal defocusing.

3. Astigmatism

If astigmatism alone is present in the wavefront Eq. (14) reduces to

$$4C_0(u^2 - v^2) + \frac{\bar{z}(u^2 + v^2)}{2(f^*)^2} = m\lambda \quad (20)$$

which can be rewritten as

$$\left[4C_0 + \frac{\bar{z}}{2(f^*)^2} \right] u^2 - \left[4C_0 - \frac{\bar{z}}{2(f^*)^2} \right] v^2 = m\lambda \quad (21)$$

For C_0 positive, $\bar{z}$ positive and $\frac{\bar{z}}{2(f^*)^2} > 4C_0$ the fringes are elliptical with the v axis as the major axis. For

$$\frac{\bar{z}}{2(f^*)^2} = 4C_0$$

the fringes are straight and parallel to the v axis and the spacing between fringes decreases as $|u|$ increases. This position corresponds to the sagittal focus. For

$$\frac{\bar{z}}{2(f^*)^2} < 4C_0$$

the fringes are hyperbolic with a family of rectangular

hyperbolas at $\bar{U} = 0$. For $\frac{-\bar{Z}}{2(\bar{S}^*)^2} = 4C_0$ the fringes are

again straight but parallel to the u axis with the spacing decreasing as v increases. This position corresponds to the tangential focus. For $\left| \frac{-\bar{Z}}{2(\bar{S}^*)^2} \right| > 4C_0$

the fringes are again elliptical with the u axis the major axis. In Figure 9 are photographs showing the fringe patterns for the various values of $\bar{Z}$.

Figure 9. Photographs showing the fringe patterns due to astigmatism for various amounts of longitudinal defocusing.

III. Effect Of Lateral Separation Of Scatter Plate And Plane Mirror On The Aberration In The Wavefront

In using Eq. (2) to represent the OPD between the reference and test wavefronts it is assumed that the scatter plate and the plane mirror have no lateral separation, an impossibility. The effect of this separation is to introduce astigmatism into the wavefront. The amount of astigmatism introduced is easily calculated using Coddington's equations (Eqs. (22) and (23)).

$$\frac{n'}{s'} = \frac{n' \cos \theta' - n \cos \theta}{r} + \frac{n}{s} \quad (22)$$

$$\frac{n' \cos^2 \theta'}{t'} = \frac{n' \cos \theta' - n \cos \theta}{r} + \frac{n \cos^2 \theta}{t} \quad (23)$$

where: n = Index of refraction

θ = Angle of incidence of principle ray

r = Radius of curvature of surface

s = Sagittal image or object distance
from the surface along the principle
ray

t = Tangential image or object distance
from the surface along the principle
ray

The unprimed terms refer to object space
and the primed terms refer to image space

Using Figure 10 Eq. (22) becomes

$$s' = \frac{r}{2 \cos \theta - 1} \quad (24)$$

and Eq. (23) becomes

$$t' = \frac{r \cos \theta}{2 - \cos \theta} \quad (25)$$

The longitudinal astigmatism is

$$LA' = (s' - t') \cos \theta = \frac{2r \cos \theta (1 - \cos^2 \theta)}{5 \cos \theta - 2 \cos^2 \theta - 2} \quad (26)$$

which reduces to

$$LA' = \frac{d^2}{2r} \quad (27)$$

upon substituting $(1 - \theta^2/2)$ for $\cos \theta$, d/r for θ , and neglecting terms of order higher than two in θ . Now

$$OPD = - \frac{LA' y^2}{2r^2} \quad (28)$$

so that the OPD introduced for one pass by a separation d is

$$|OPD| = \frac{d^2}{16r(f\#)^2} \quad (29)$$

Figure 10. Diagram illustrating the various parameters in Coddington's equations. M is the concave mirror under test; r is the radius of curvature of M ; d is the separation between scatter plate and plane mirror; and s and t , and s' and t' are the sagittal and tangential object and image distances from M .

or rearranging Eq. (29) it is seen that the permissible separation of scatter plate and plane mirror for a given amount of astigmatism is

$$d \leq 2 f^{\#} [2r |OPD|]^{\frac{1}{2}} \quad (30)$$

For example, if it is desired to test to $1/10 \lambda$ then the OPD introduced due to the separation of scatter plate and plane mirror should ideally be less than $1/10 \lambda$. For $f^{\#} = 3$ and $r = 750$ millimeters (mm)

$$d \leq 1.6 \text{ mm}$$

Such a small separation is not strictly necessary because due to the nature of the fringe patterns even for relatively large amounts of astigmatism small amounts of spherical aberration can readily be detected. Further, since the amount of astigmatism introduced can be easily calculated using Eq. (29) it can be taken into account when evaluating the fringe patterns.

IV. Method Of Obtaining Tilt Between Test And Reference Wavefronts

Tilt between reference and test wavefronts can be obtained by introducing a small tilt to a thin plane parallel plate which has been placed into one half of the cone of light which makes up the reference and test wavefronts as shown in Figure 11. The effect of the tilted plate is to introduce a lateral displacement, $\bar{x}$ of the image of the scatter plate falling on the plane mirror. Since the tilt angle α is small (see Figure 12) $\bar{x}$ can be approximated by

$$\bar{x} = \frac{t\alpha}{n} \quad (31)$$

Since the wavefronts pass through the plate twice this corresponds to an OPD of

$$\text{OPD} = \frac{\bar{x}}{f\#} u = \frac{t\alpha}{n f\#} u \quad (32)$$

As shown in Figure 13a when the plate is introduced into the field there is a longitudinal focal shift and it is necessary to adjust the system so that the plane mirror is coincident with the image of the scatter plate for $\bar{z} = 0$. When a small tilt of the plate is introduced the image of the scatter plate due to the top half of the wavefront is displaced a distance $-\bar{x}$ to B as shown in

Figure 11. Diagram illustrating method of introducing a plane parallel plate into the system to obtain tilt between reference and test wavefronts. M is the concave mirror under test; P is the position of the scatter plate and plane mirror; and PP is the plane parallel plate.

Figure 12. Diagram illustrating the amount of lateral displacement introduced by a tilted plane parallel plate. t is the thickness of the plate; α is the angle of tilt; α' the angle of refraction; and $\bar{x}$ is the lateral displacement.

Figure 13. Diagrams showing the displacement of the image of the scatter plate for the top and bottom halves of the wavefronts when the plane parallel plate is in the system.

Figure 13b, while the image of the scatter plate due to the bottom half of the wavefront is displaced a distance $+\bar{x}$ to B as shown in Figure 13c. (There is also an additional longitudinal shift of the focus but this is small and it is corrected by an additional longitudinal motion of the scatter plate and plane mirror.) The fringe pattern observed is actually two separate fringe patterns, one due to the top half of the wavefront and one due to the bottom half of the wavefront. Since the plate does not effect the symmetry of the fringe pattern all the information concerning the aberrations of the system under test are contained in each half of the wavefront. The expression for the OPD of the wavefront when the plate has a tilt α as defined in Figure 13 is

$$\begin{aligned} \text{OPD} = 4B_0(u^2+v^2)^2 + 4C_0(u^2-v^2) + \frac{\bar{z}(u^2+v^2)}{2(f\#)^2} \\ \pm \frac{t\alpha}{n f\#} u \end{aligned} \quad (33)$$

where the plus sign defines one half of the wavefront and the minus sign defines the other half. In Figure 14 are photographs showing the fringe patterns observed for a nearly perfect spherical wavefront and for a wavefront with a small amount of spherical aberration. Another way of looking at this is that the plate has the same effect as tilting one half of the mirror under test with respect to the other half so that the center of curvature of one

Figure 14. Photographs showing the fringe patterns for a nearly perfect spherical wavefront and for a wavefront with a small amount of spherical aberration when the plane parallel plate is used to obtain straight fringes.

half of the mirror is laterally displaced with respect to the center of curvature of the other half. This is predicted on the basis of Eq. (33) and demonstrated in Figure 15 by a photograph of the fringe pattern for a small longitudinal defocusing. It is seen that the centers of the two fringe patterns are displaced with respect to each other.

If it is desired to have ten fringes in the field for $f^\# = 5$, $n = 1.5$, and α one degree, then from Eq. (32) $t = 1$ mm. The amount of OPD introduced by the spherical aberration of the plate can be calculated using Eq. (34)²¹ below.

$$|OPD| = \frac{(n^2 - 1) t}{128 n^3 (f^\#)^4} \quad (34)$$

For the present case where $n = 1.5$ and $t = 1$ mm this becomes

$$|OPD| = \frac{5.8}{(f^\#)^4} \quad (\text{wavelengths}) \quad (35)$$

In Table I are the amounts of OPD introduced for various $f^\#$ s in this case.

Figure 15. Photograph demonstrating the effect of the plane parallel plate on the wavefronts of the system.

Table I

OPD introduced due to a 1 mm plane parallel plate of glass in a converging beam.

<u>f#</u>	<u>OPD Introduced</u>
1	5.800 Wavelengths
2	.370 Wavelengths
3	.071 Wavelengths
4	.023 Wavelengths
5	.009 Wavelengths

It is seen that for testing systems faster than about $f/3$ the plate introduces too much spherical aberration. This can be overcome in two ways. One way is to use a thinner plate. For most cases this is sufficient since it is possible to obtain parallel plates down to .25 mm in thickness of the necessary length and width (however, for the case of ten fringes in the field and equal to one degree for an $f/1$ system, $t = .22$ mm which introduces about one wavelength of OPD). Another method is suggested by Eq. (29). For an $f/1$ mirror of radius 500 mm, by separating the plane mirror and scatter plate by 2.8 mm, four wavelengths of astigmatism are introduced (i.e. $4C_0$ in Eq. (20) is equal to 4). Then by adjusting

$$\bar{z} \text{ until } \frac{\bar{z}}{2(f\#)^2} = 4C_0 \quad \text{or} \quad \frac{-\bar{z}}{2(f\#)^2} = 4C_0$$

straight fringes will appear in the field as shown by

Eq. (35) which is the expression for OPD when $\frac{\bar{z}}{2(f\#)^2} = 4C_0$

$$OPD = 4B_0(u^2 + v^2)^2 + 8C_0 u^2 = m\lambda \quad (36)$$

Any deviation from straightness is of course a measure of the aberration of the wavefront. The disadvantage of the last method is that as mentioned previously the fringe spacing decreases as the aperture increases, with a broad fringe in the center of the field. This makes it difficult to examine the center of the field in detail. In Figure 16 are photographs showing the fringe patterns obtained for a nearly perfectly spherical wavefront and for a wavefront with a small amount of spherical aberration.

Figure 16. Photographs showing the fringe patterns for a nearly perfect spherical wavefront and for a wavefront with a small amount of spherical aberration when a separation of the scatter plate and plane mirror is used to obtain straight fringes.

V. Study Of Contrast Of Fringe Patterns

A. Source Size

The two parameters that most affect the contrast of the fringes are the size of the light source and the scattering properties of the scatter plate (it is assumed that stray light in the vicinity of the testing area has been minimized). The source should be such that its size does not introduce more than one quarter wavelength of OPD to the wavefront. In other words, if it is desired to have N equally spaced fringes in the field, then the size of the image of the source on the paraxial region should be

$$S = \frac{D}{4N} \quad (37)$$

where: D = Diameter of system under test

S = Diameter of image of source

For all practical purposes

$$S = 2\theta R \quad (38)$$

where: 2θ = Angular size of source

R = Radius of curvature of mirror
under test

Therefore the angular size of the source should be

(39)

If the fringes are not equally spaced then the size of the image of the source should be equal to or less than one fourth the width of the white light fringe.

B. Scatter Plates

In order to determine a method for making scatter plates which give high contrast fringe patterns an investigation of contrast using scatter plates with various degrees of roughness was made. Each scatter plate was made by grinding one side of a small glass plate with grinding powder whose particles were of a certain maximum size, and then polishing to give a smooth surface with pits in it. Plates were made using grinding powders with particles 12 microns, 16 microns, 23 microns, 30 microns, and 60 microns in size. Micrographs of the surfaces are shown in Figures 17, 18, 19, 20 and 21. (the magnification in each case is approximately 200). Scattering data were obtained using the set-up shown in Figure 22. Light from the tungsten source S is collimated by lens L and falls on the scatter plate P. A mask in the shape of a slit 5 mm long and 1/2 mm wide was placed on the scatter plate and over one end of tube

Figure 17. Scattering data for the scatter plate made with 12 micron grinding powder.
a) relative intensity versus scattering angle; b) micrograph of the scatter plate; c) photograph of the fringes.
The contrast of the first bright fringe is 0.7.

(a)

Figure 18. Scattering data for the scatter plate made with 16 micron grinding powder.
a) relative intensity versus scattering angle; b) micrograph of the scatter plate; c) photograph of the fringes.
The contrast of the first bright fringe is 0.63.

(a)

Figure 19. Scattering data for the scatter plate made with 23 micron grinding powder.
a) relative intensity versus scattering angle; b) micrograph of the scatter plate; c) photograph of the fringes.
The contrast of the first bright fringe is 0.64.

(a)

Figure 20. Scattering data for the scatter plate made with 30 micron grinding powder.
a) relative intensity versus scattering angle; b) micrograph of the scatter plate; c) photograph of the fringes.
The contrast of the first bright fringe is 0.44.

(a)

Figure 21. Scattering data for the scatter plate made with 60 micron grinding powder.
a) relative intensity versus scattering angle; b) micrograph of the scatter plate; c) photograph of the fringes.
The contrast of the first bright fringe is 0.29.

Figure 22. Schematic diagram showing set up used to obtain scattering data. S is the source; L is the collimating lens; P is the scatter plate; and T is the tube holding the detector D.

T. At the other end of the light tight tube was the detector (a 1P28 Photomultiplier). The tube was also baffled inside such that the collimated beam of light incident upon it was detected only over angle of 2 degrees when the scatter plate was not in place. The scatter plate was placed directly over the center of rotation of the arm to which the tube T was rigidly affixed and at a distance of 40 cm from the detector. The roughened surface of the scatter plate was placed toward the detector and was rendered perpendicular to the incident beam by auto collimation of the source. The values of the relative intensity versus θ that were recorded are shown in Table II. In order to plot these data it was convenient to normalize them with respect to the value for $\theta = 3$ degrees. The normalized values are plotted in Figures 17, 18, 19, 20, and 21. Each scatter plate was then used in the system shown in Figure 23 and photographs were taken of the fringe pattern obtained in each case. The scatter plate holder H was made so that one scatter plate could be removed and another put in without adjusting the system. Polaroid Type 55 Positive/Negative film was used with a Wratten Number 23 filter. The exposure in each case was twenty minutes. The relative intensities of the bright and dark fringes in each case were determined by using a Hilger Watts Model L50 Recording Microphotometer to scan the

Figure 23. Schematic diagram showing set up used to obtain contrast photographs. M is the concave mirror under test; H is the scatter plate holder; B is the beam-splitter; L is the lens used to focus light from the source S on M; and E is the position of the eye or camera.

Table II Relative intensity of the scattered
light as a function of the scattering
angle for the various scatter plates.

Particle Size	Scatter Angle	Relative Intensity				
		60 microns	30 microns	23 microns	16 microns	12 microns
0		0.5000	0.8250	0.7800	0.7100	0.3000
1		0.1400	0.2600	0.2700	0.2800	0.1400
2		0.0820	0.1600	0.1500	0.1600	0.0670
3		0.0620	0.1100	0.0860	0.0960	0.0440
4		0.0480	0.0860	0.0610	0.0690	0.0300
5		0.0380	0.0670	0.0450	0.0490	0.0220
6		0.0350	0.0530	0.0365	0.0370	0.0160
7		0.0300	0.0430	0.0290	0.0300	0.0120
8		0.0250	0.0360	0.0230	0.0235	0.0087
9		0.0190	0.0300	0.0180	0.0190	0.0069
10		0.0160	0.0240	0.0145	0.0150	0.0055
11		0.0140	0.0200	0.0120	0.0130	0.0043
12		0.0120	0.0165	0.0087	0.0094	0.0035
13		0.0100	0.0140	0.0087	0.0094	0.0035
14		0.0085	0.0120	0.0058	0.0063	0.0024
15		0.0036	0.0044	0.0024	0.0027	0.0010
20		0.0020	0.0023	0.0013	0.0015	0.0005
25						
30						

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negatives. The contrast was determined for the first bright fringe ($m = 1$) in each case. Since the intensity in the pattern is inversely proportional to the transmission of the negative the expression for contrast reduces to

$$\text{Contrast} = \frac{T_{\text{max}} - T_{\text{min}}}{T_{\text{max}} + T_{\text{min}}} \quad (40)$$

where: T = Transmission as recorded by the Microphotometer

The contrast in each case is as noted in Figures 17, 18, 19, 20, and 21. It is seen that by making a scatter plate as described above using a grinding powder with a maximum particle size of 12 microns a contrast of 0.7 can be realized. The pit density on this plate is about 8,000 pits per square millimeter and the pit sizes range up to 12 microns as shown in Table III.

Table III

Pit Size	Relative Population
12 Microns	5%
10 Microns	5%
8 Microns	10%
5 Microns	30%
3 Microns	50%

From the plots of the scattering data shown in Figures

17, 18, 19, 20, and 21, it is seen that as the percentage of the light scattered decreases the contrast of the fringes increases. However, for the three cases when the contrast is greater than 0.6 the plots are essentially identical even though there is a spread of 0.06 in contrast. For the purpose of the present thesis it is sufficient to say that a scatter plate made using 12 micron grinding powder as described above has the scattering properties necessary to give high contrast.

VI. Summary

In the foregoing it has been shown that an interferometer using one scatter plate and a plane mirror is well suited for testing long focus systems. The first requirement of an instrument used to test long focus systems is that it be insensitive to vibration. This is automatically achieved in the present system because of its $+1$ magnification. The $+1$ magnification also makes it extremely easy to align the system for testing. In order to test a nearly perfect system it should be possible to introduce a tilt between the reference and test wavefronts so that straight fringes appear in the field. In the present system this is achieved by introducing a slightly tilted plane parallel plate of glass into the system, thus displacing the image of the scatter plate with respect to itself. Alternately straight fringes can be obtained by separating the scatter plate and plane mirror a certain amount to introduce a given amount of astigmatism, which when viewed at either the sagittal or tangential focus gives straight fringes. The disadvantage with this method is that the fringe spacing is not constant (as it is in the first method) but decreases with increase in aperture, having a broad fringe in the center of the field. However, for very fast systems the amount of spherical aberration introduced is excessive and the second method is preferred.

Since the test wavefront passes through the optical system being tested twice, the sensitivity of this system is twice that of the Twyman interferometer for testing mirrors. The major disadvantage with this method is that coma cannot be observed. However, this is not a disadvantage when testing mirrors. Finally it has been shown that making a scatter plate by grinding one side of a small glass plate with a grinding powder whose maximum size is 12 microns and then polishing to give a smooth surface with pits in it, a contrast in the fringe pattern of 0.7 can be realized. So that with the exception that coma cannot be observed, all of the objectives of this work have been realized.

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