A Macroeconomic Model with Bond Liquidity

by

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Dedication

This thesis is dedicated to my parents for their unconditional love, support, and understanding.

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Biographical Sketch

Tianchi Gao was born in Beijing, China. He received his Bachelor of Arts degree in Economics and Mathematics and Master of Arts degree in Economics from Northwestern University. He began his doctoral studies in Business Administration at the University of Rochester in 2014, and in 2017 he earned a Master of Science degree in Business Administration from the University of Rochester. He pursued his doctoral research in the area of macro-finance under the direction of Professor Alan Moreira and Professor Dmitry Orlov.

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Abstract

This thesis is about the macroeconomic implications of bond market liquidity. Using a two-country macro model with incomplete markets, it is shown that higher private bond liquidity not only leads to more growth from more efficient financing of investment opportunities, but also generates more instability by fueling equilibrium leverage and worsening the undercapitalized country's terms-of-trade. Numerical experiments, based on a parameterized version of the model, show that welfare costs from instability can outweigh gains from higher growth, and the instability effect of bond liquidity is felt more strongly by countries with low production volatilities. Moreover, private bond liquidity matters for the design and evaluation of macroprudential policies. In particular, bond liquidity can significantly relax the optimal tightness of leverage constraints, while also moderately increase the optimal tax on credit. Lastly, this paper distinguishes between public and private liquidity, and explores the model's fiscal policy implications. Public liquidity is shown to increase welfare through a seigniorage-redistribution channel, but the effectiveness of its crowding-out channel depends on the dynamic management of government balance sheet. Finally, effectiveness of its risk-premia channel depends on international coordination.

Contributors and Funding Sources

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Chapter 1

A Macroeconomic Model with Bond Liquidity

Following the 2007 financial crisis, a series of regulatory reforms have been designed and implemented to shore up financial stability (Claessens et al 2014; Greenwood et al 2017). Many market participants have since complained about such rules on the basis of its adverse impact on secondary market liquidity. In particular, the argument was that post-crisis financial regulations have affected dealers' capacity and willingness to intermediate (Adrian, Boyarchenko, and Shachar 2017), and the impacts are particularly felt in markets of prime assets such as corporate bonds¹. Partly as a response to this perceived decline in corporate bond liquidity, policy makers around the world have in recent years implemented various initiatives designed to enhance secondary market liquidity of corporate bonds².

¹Whether and to what extent regulations negatively impacted bond market liquidity is a contentious issue with mixed evidence (Anderson and Stulz 2017).

²In 2018 the National Stock Exchange (NSE) and the Bombay Stock Exchange (BSE) of India launched an electronic platform for repos with corporate bond as eligible underlying assets (see the Crisil 2018 re-

A natural question that arises is whether and when we need to be concerned with the unintended consequences of improvements in bond liquidity. In particular, how do changes in bond liquidity impact allocative efficiency and ultimately societal welfare?

In this paper, I address this question by building an international macro model where bond³ is more liquid than physical capital, based on Brunnermeier and Sannikov (2015) (henceforce BruSan15). I focus on the implications of bond liquidity in an international setting as corporate borrowers have steadily increased the proportion of bond issuances on international markets. This trend is global in nature and is especially pronounced after the financial crisis, as global banks became more constrained in part due to post-crisis regulations (Tendulkar and Hancock 2014; Shin 2013)⁴. I show that enhancing bond liquidity leads to increases in both growth and instability for the global economy, and hence is not always an unconditional blessing. In particular, the growth effect increases welfare, while the instability effect decreases welfare. Which effect wins? Numerical experiments, based on a parameterized version of the model⁵, show that the instability effect tends to dominate in environments in which fundamentals have low volatilities and that credit mar-

port), which is one of the many measures advocated in the 2016 Reserve Bank of India (RBI) report on developing India's corporate bond market. In Europe, one motivation for the introduction of MiFID 2 in the European Union in 2018 is to increase corporate bond market liquidity by increasing post-trade transparency, similar to the introduction of the Electronic Trading Platform (ETP) by the government of Malaysia back in 2008. Also, the ECB's sizable corporate sector purchasing programme which started in June 2016 provided additional source of liquidity to current and potential holders of eligible EU corporate bonds (Abidi and Miquel-Flores 2018). Finally, many institutions around the world have started using corporate bond ETFs which some argue are important in facilitating market liquidity of its underlying assets (Blackrock 2016a, b; US Treasury OFR 2019).

³Here bonds are interpreted as investment-grade corporate bonds, or debt issued by the financial sector, as long as the financial sector ultimately lends a sizable portion of its borrowing to productive but risky businesses.

⁴In addition, the mechanism of this paper depends crucially on imperfect aggregate risk sharing, which is more pronounced in an international context as evidenced by the persistence of equity home bias.

⁵The parameters are chosen similar to that in the roughly "calibrated" model of Chen and Phelan (2017). Results of this paper are not sensitive to alternative choices of parametrization.

ket frictions are low (as are true in developed countries). In contrast, the growth effect dominates in environments in which fundamentals have high volatilities and that credit market frictions are high (as are true in emerging markets). Roughly speaking, this is because leverage, and hence instability, are more sensitive to changes in bond liquidity in low volatility and low market friction environments.

In particular, The model is a two-country two-goods DSGE model in which financial frictions limit cross-border credit flows to be in the form of (riskless) short-term debt⁶. Because of limited equity issuance, aggregate risk sharing is imperfect and global output is an endogenous function of the relative wealth share of each country. When the two goods are imperfect substitutes in generating the final consumption goods, international lending leads to a better static allocation of capital, but also undermines the natural terms-of-trade hedge for the debtor country. In particular, debtor country's agents do not internalize the pecuniary externality that by leveraging up and producing more, they lower the price of their output and hence the whole country recapitalizes at a slower pace (in expectation). A slow recapitalization process lowers global welfare by increasing the fraction of time the global economy spends with misallocated capital. It is with this dynamic global stability in consideration that even complete capital control (shutting down the international credit market) can be welfare-enhancing as shown in BruSan15.

I extend the BruSan15 framework by adding liquidity services from bond holdings. In particular, I assume investment opportunity realizes before output from production real-

⁶The production side of the model is taken from the two-country two-goods model a la Beckus, Kehoe and Kydland (1994). I restrict attention to the special case where final consumption goods is a Cobb-Douglass aggregate of the two intermediate goods from each country (unit elasticity of substitution between the two intermediate goods). Also I assume there is no consumption home-bias so that there is a universal consumption basket and hence a clear real interest rate relevant for both countries.

izes, so agents have to raise investment inputs externally⁷. Capital holdings are illiquid at this stage, but bond holdings can be sold quickly without adverse price impact. Thus, anticipation of ex-post investment opportunities leads to ex-ante incentives to hoard financial liquidity via lending (i.e., holding bonds). In particular, I model bond liquidity in the form of positive resaleability a la Kiyotaki and Moore (2018): with some probability an agent can sell (at most) a fraction of her bond holdings quickly with no adverse price impact⁸. I show that bond liquidity in this framework translates into an endogenous convenience yield to creditors, who then drive down funding costs. As a result, the economy features higher leverage in equilibrium as bond liquidity increases. While ex-post (at the time of investment financing) bond liquidity always increases welfare, ex-ante (at the time of portfolio allocation) bond liquidity further exacerbates the pecuniary externality which leads to excessive international credit. That is, in my framework the tradeoff of bond liquidity is that while it generates higher growth by allowing more efficient financing of investment opportunities in the creditor country, it also leads to higher instability by worsening the terms-of-trade hedge of the debtor country, so that the global economy spends a higher fraction of time with misallocated capital.

In terms of global welfare, which effect dominates depends on the parameters. Numeri-

⁷The approach currently used in the model follows from Benigno and Nistico (2017). It is important to note that internal liquidity ultimately comes from agents' output realization within a period, that is, investment inputs do not come from exogenous sources on net. This can be seen more clearly from the consumption goods market clearing condition below. Alternatively, we could assume that that every instant, a fraction of agents random receive investment opportunities, which allows (only) them to create capital from consumption goods. Then in equilibrium those agents will trade with others who do not have investment opportunities (by trading bonds). This approach is adopted in Gertler and Kiyotaki (2010) and Li (2017). Results of this paper are robust to this alternative formulation.

⁸For simplicity, capital holdings cannot be sold at this stage. This may arise from an asymmetric information problem a la Kurlat (2013), as shown in Appendix C. Results in this paper remain as long as there is a large enough friction in selling physical capital so that in equilibrium bond holdings will be (optimally) sold at the investment stage to generate additional liquidity.

cal experiments, based on parameters similar to that in Chen and Phelan (2017) (qualitative results of this paper are robust across a range of parameters), show that global welfare decreases with bond liquidity when fundamental volatilities (in terms of production) and/or credit market friction (in terms of transaction costs in the international bond market) are low⁹. Intuitively, this is because in such environments the change in leverage is larger per unit of change in interest rate¹⁰, and hence the stationary distribution of the economy is very sensitive to changes in bond liquidity. Thus, a small increase in bond liquidity has large welfare costs due to significant increases in global instability, in the sense that the economy spends much higher fraction of time with misallocated capital. As production volatility and market friction go up, enhancing bond liquidity first improves and then lowers welfare, leading to an endogenous optimal level of liquidity. For high levels of fundamental volatility and credit market friction, equilibrium leverage is not sensitive to interest rates and enhancing bond liquidity always improves welfare.

Using the same set of parameters, I then show that bond liquidity matters for the design of macroprudential policies. In particular, a moderate amount of bond liquidity significantly increases the optimal level of leverage constraint (i.e., social planner tolerating more private credit when bond has liquidity), irrespective of whether private bond liquidity increases or lowers welfare. This is because a binding leverage constraint fixes the terms-of-trade hedge, and thus only the benefit of bond liquidity needs to be considered. However, bond liquidity is shown to also increase the optimal level of tax on credit (i.e., social planner tolerating less private credit when bond has liquidity). This is because tax

⁹This transaction cost is meant to capture frictions in primary bond markets, such as those arising from costly state verification.

¹⁰Interest rate sensitivity to bond liquidity, in contrast, is not sensitive to fundamental volatilities or credit market frictions.

on credit actively facilitates the recapitalization process of the debtor country. Since higher bond liquidity induces higher equilibrium credit, marginal benefit from imposing credit tax increases as it generates higher amount of seigniorage profits to be redistributed to debtor country agents.

I then discuss the effects of bond liquidity in asymmetric environments. I consider two types of asymmetry – production volatility and size, with the latter referring to the importance of the intermediate good a country is better at producing in the final goods production process. It is shown that effects of bond liquidity is relatively symmetric for countries with different sizes, but are significantly asymmetric for countries with different volatilities. In particular, the distortionary effect of bond liquidity on stationary distribution is felt more strongly in countries with low-production volatilities, as agents in such countries react more strongly to lower interest rates (which result from increases in their debt liquidity). This may have direct implications on the negotiation process of international macroprudential policy coordination. As Chen and Phelan (2018) show, in a symmetric-volatility environment, both countries would gain from coordination in setting macroprudential policies, relative to the Nash equilibrium where each country chooses how tight the control of capital inflow is, given the policy stance of the other country. In contrast, in asymmetric-volatility environments there can be large redistribution effects from such coordinations – welfare of low-volatility countries decreases significantly from Nash equilibrium to coordination optimum (this is accompanied by a (Pareto-weighted) larger welfare gain in high-volatility countries). As such, Chen and Phelan (2018) argue that this gives the low-volatility countries more bargaining power in the negotiation process of international macroprudential policies, as they have a credible threat to not co-

operate. An increase in bond liquidity in this environment would further exacerbate this redistribution effect as low-volatility countries would experience a further welfare loss under coordination when their debt has liquidity. Thus, bond liquidity may give low-volatility countries even higher bargaining power in negotiating a tight constraint on credit flows.

Finally, I discuss the role of public liquidity in this framework. It is often the case that government bond is more liquid than private bond, and some have suggested using government debt to partially crowd out the creation of private debt (e.g., Greenwood, Hanson, and Stein 2015). However, I show that a simple crowding-out policy where the government targets public debt issuance to be of a constant fraction of private debt level can be particularly ineffective. The intuition is that while a larger amount of government debt serves to increase demand for credit and hence increases equilibrium interest rate, government's debt management policy also matters crucially in determining agents' equilibrium leverage. When a positive shock translates into a state of the economy closer to the stochastic steady state such that private debt level goes down, public debt level according to this simple policy mechanically goes down as well, meaning higher taxes as government rolls over a smaller amount its debt. Agents can hedge their tax exposure by taking on additional leverage, so that they earn higher realized risk premia when a positive shock hits. The latter force completely overwhelms the former – in equilibrium debtor country's agents borrow slightly more at a higher interest rate, compared with the case without government debt issuance. Crowding-out is only effective when the debtor country agents are poor enough such that the amount they borrow increases when a positive shock hits, since their risk bearing capacity improves as their wealth recovers. In this region government balance sheet is expected to go up as well, and agents automatically get some exposure

to fundamental shocks through the tax policy, leading to a reduction in their optimal private leverage. However, in this region terms-of-trade hedge is fixed, so the benefit of less leverage greatly diminishes. Altogether, this particular kind of crowding-out policy does not correct the pecuniary externality when it is particularly harmful, and in general lowers debtor country's welfare.

Instead, governments can make the global economy more stable and increase welfare through a seigniorage-redistribution channel. In particular, to the extent that government bond has higher liquidity than private bond, the debtor country's government can issue public bond to buy private bond (similar to an asset purchase program), and redistributes its seigniorage profit to help its agents recapitalize faster. It is worth pointing out that the creditor country's government also has an incentive to engage in such asset purchase programs. While it does not contribute to stability – the seigniorage profit is redistributed to creditor country's agents, it does lead to an even higher global welfare because it pumps more liquidity into the system without generating more private debt. That is, terms-of-trade is not affected by the creditor country's asset purchase program. Thus with this redistributive channel, public liquidity works through the intensive margin (how much liquidity is generated per dollar of private debt) rather than the extensive margin (how much private debt is generated in total), and in general leads to higher welfare.

Lastly, I propose a risk-premia channel of government debt. Empirically, government bond is not only valuable for its liquidity in normal times but also for its unconditional liquidity during crisis times, as evidenced by various flight-to-liquidity episodes. Indeed, Longstaff (2002) finds that investors are willing to pay a significant premium for the crisis-time liquidity of Treasury bonds which he refers to as the “flight-to-liquidity premium”. To

the extent that debt issuers anticipate the occurrences of such flight-to-liquidity episodes, they may decide to hold a certain amount of government bond ex-ante as insurance against inefficient fire-sales when lenders tighten credit conditions (e.g., lenders refusing to roll over short-term lending or demanding higher haircuts). This insurance comes at a (opportunity) cost which is positively correlated with the government bond yield spread – a high premium of government bond increases the opportunity cost of insurance, and as a result agents choose lower leverage in equilibrium. By controlling the supply of its debt, a government can affect risk premia in the economy and hence its equilibrium leverage. This is what I call the risk-premia channel of government debt, which is identical in spirit to the risk-premia channel of monetary policy in Drechsler et al (2018). In a two-country setting, however, lack of international coordination can limit the effectiveness of this channel, as the two countries compete to earn seigniorage profits and issue debt at a higher level than the socially optimal target.

Related Literature

Pozsar (2011) documents the rise of institutional cash pools whose primary focus is on safety and liquidity, instead of yield. Pozsar (2014) explains how its scale may have contributed to the rise of shadow banking prior to the financial crisis. The story of this paper has a similar dimension, but instead it is the corporate sector rather than the banks issuing higher amount of debt due to access to cheaper credit resulting from liquidity demands. Goldstein et al (2019) document supportive evidence of this channel for the U.S. corporate bond markets. In particular, they show that the expected secondary market

liquidity of corporate bonds has a positive impact on their yields at issuance. A similar pattern is documented in other developed countries in Nozawa, Shi, and Huang (2019). On the regulatory side, Nyborg (2015) cautions that collateral framework of the central bank encourages the production of eligible assets, while Carpinelli and Crosignani (2017) document the effect of central bank actions on credit supply to corporations. Shin (2013) documents the trend in emerging economies where (international) bond financing is substituting for bank financing, and such shift in corporate funding structure has also been observed to some extent in Europe and the U.S. as well. This again may reflect that as banks are constrained by regulations post-crisis, some of the demand for safety and liquidity from money managers are partly being met with non-financial commercial papers or prime repos with (investment-grade) corporate bonds as underlying assets ¹¹.

Recently, a growing number of papers have studied the implications of bank debt providing liquidity services. Phelan (2016) shows how privately optimal leverage chosen by the financial sector is excessive because agents do not internalize its effect on systemic risk, using a similar framework. However, liquidity service enters directly into the utility function in his model, while liquidity service is microfounded with investment needs here. Moreira and Savov (2017) interpret shadow banking as the creation of shadow money, which is liquid but in a fragile way. Similar to this paper, there is a tradeoff between growth and fragility, and a rise in uncertainty can lead to the collapse of shadow banking as we observed during the financial crisis. Li (2017) proposes a monetary theory of financial instability where again the tradeoff involving bank debt issuance is growth and stability. Quadrini (2018) proposes a model where bank liabilities provide insurance against

¹¹This is also related to an “insufficient supply” of Treasuries, or in general debt that provides monetary services and explicitly and credibly backed by sovereigns. See Pozsar (2014).

idiosyncratic shocks, and illustrates how financial disturbances transmit real shocks via self-fulfilling expectations from this channel.

This paper abstracts from banks and look at the effect of credit conditions on ultimate borrowers. However, the mechanism of this paper applies to bank debt, such as financial ABCPs or commercial papers – if they become more liquid, banks would have a cheaper cost of financing and expand their balance sheet by extending more credit to corporations (Adrian and Shin 2013)¹². Graham, Leary, and Roberts (2019) find supportive evidence suggesting established and financially sound non-financial corporations provide liquidity when they borrow (directly) from investors, which then affects their capital structure and investment policies. On the externality side, Greenwood, Hanson, and Stein (2015) show how government should internalize the fire-sale externality by issuing more short maturity debts to crowd out private sector’s creation of short-term debt, which has monetary services to its holders. Diamond, Hu, and Rajan (2018) show how easy monetary policy in source funding countries contributes to the boom and bust cycles in credit-receiving countries.

This paper is also related to the literature on macroprudential policies. Earlier works such as MacDougall (1960) and Kemp (1962) show that a large country can use capital control to exert monopoly or monopsony power over intertemporal prices. In a recent paper along this dimension, Costinot, Lorenzoni, and Werning (2014) derive the optimal time path of capital controls assuming commitment power, and show how relative prices in goods markets are distorted. Importantly, such distortions may not all be ill-intentioned. Indeed, a series of recent papers illustrate the effectiveness of capital control when markets

¹²The model could be viewed as the limiting case where there is no friction in the contracting between banks and firms.

are incomplete. For example, Brunnermeier and Sannikov (2015) show that capital control can improve welfare in an international setting by correcting pecuniary externalities in export markets. Korinek (2007, 2010) also illustrate the benefit of capital control in an international setting, focusing on the amplification mechanisms of foreign currency debt based on exchange rate dynamics. Bianchi (2011) shows how capital control can be beneficial in the presence of occasionally binding borrowing constraints. Bengui (2014) studies a case where national regulators attempt to distort the price of liquidity (ex-post) in a crisis event as a form of beggar-thy-neighbor policies. In contrast, my paper studies the ex-ante incentive to hoard liquidity in preparation for liquidity events (investment opportunities in this model).

Methodology wise, this paper follows the stochastic continuous-time macro literature, pioneered by Brunnermeier and Sannikov (2014) and He and Krishnamurthy (2012, 2013), among others¹³. Such modeling approach is ideal for studying non-linear global dynamics, as opposed to linear approximations of a small perturbation around the deterministic steady state in classic papers on financial frictions such as Bernanke, Gertler, and Gilchrist (1999) and Kiyotaki and Moore (1997). Global dynamics is crucial for some of the results in this paper. In a deterministic version of the model considered in this paper (where I change aggregate shocks to idiosyncratic shocks similar to that in Brunnermeier and Sannikov (2016a)), for reasonable parameters, bond liquidity always enhances welfare, and more importantly, both borrowers and lenders benefit from it. With aggregate shocks, however, capital (mis)allocation is stochastic and global welfare as well as the current debtor country's welfare may decrease after bond liquidity is introduced.

¹³Brunnermeier and Sannikov (2016b) provides a manual for this approach.

Outline

The remainder of this chapter is organized as follows. Section 1.1 presents the baseline model and derives its equilibrium characteristics. Section 1.2 provides a numerical example to illustrate the tradeoff of growth and stability regarding private liquidity creation. Section 1.3 presents comparative statics results regarding an economy's fundamental volatility and its degree of financial development in the form of credit market transaction cost. Section 1.4 considers how bond liquidity affects the evaluation and design of macroprudential policies. Section 1.5 discusses the effect of bond liquidity in asymmetric environments. Section 1.6 explores the role of government debt. Section 1.7 concludes.

1.1 The Model

The baseline model extends the two-country model of Brunnermeier and Sannikov (2015) by adding private liquidity creation from debt. In the model, agents belong to one of two countries, A and B . Agents can use capital to produce two types of intermediate goods, a and b , which are then aggregated (by a competitive final goods producer) into a final goods. The final goods can be used for consumption or investment and will serve as numeraire. To introduce varying degrees capital misallocation, I assume agents in country A are better at producing goods a than b , and vice versa for agents in country B . For simplicity, there are no trade barriers to the two intermediate goods, and there is a frictionless competitive market for capital (at the portfolio rebalancing stage), which is the only durable asset in this economy. However, financial frictions limit cross-country credit flows to be in the form of short-term (and hence risk-free) debt. Without such frictions, agents would freely

issue equity which separates aggregate risk taking and production scale choices, and the economy can attain its first-best outcome. The results in this paper hold as long as there is some limitation to issuing state contingent contracts.

1.1.1 Setup

Technology. Time is infinite and continuous. There is a single type of capital in this economy that can be used to produce either of the two intermediate goods a and b . The final goods, which we take as numeraire, is a CES aggregate of the two intermediate goods, and can be used for consumption or investment (to produce new capital). Aggregation takes the form of:

$$y = \left[\frac{1}{2}(y^a)^{\frac{s-1}{s}} + \frac{1}{2}(y^b)^{\frac{s-1}{s}} \right]^{\frac{s}{s-1}},$$

where y^a , y^b , and y denote quantity of goods a , b , and the final goods, respectively. $s \in [0, \infty]$ is a measure of substitutability of the intermediate goods. When $s = 1$, final goods production is Cobb-Douglas and is written as:

$$y = (y^a)^{\frac{1}{2}}(y^b)^{\frac{1}{2}}.$$

Agents can produce goods a and b with capital in a linear fashion (AK technology), but agents in country A are superior in producing goods a in the form of higher productivity. In particular, given k_t units of capital being employed, an agent in country A can produce goods a at rate of $\bar{a}k_t$ and goods b at rate $\underline{a}k_t$, where $\bar{a} > \underline{a} > 0$. Symmetrically, agents in country B can produce goods a at rate $\underline{a}k_t$ and goods b at rate $\bar{a}k_t$.

Denote the aggregate amount of capital in this economy at time $t \in [0, \infty)$ as K_t , and the fraction of which used by agents in country $I \in \{A, B\}$ to produce goods $i \in \{a, b\}$ by ψ_t^{Ii} , we have, by definition:

$$\psi_t^{Aa} + \psi_t^{Ab} + \psi_t^{Ba} + \psi_t^{Bb} = 1.$$

Denote intermediate goods production level as $Y_t^i, i \in \{a, b\}$, and final goods quantity as Y_t , we have:

$$Y_t^a = (\bar{a}\psi_t^{Aa} + \underline{a}\psi_t^{Ba})K_t, Y_t^b = (\underline{a}\psi_t^{Ab} + \bar{a}\psi_t^{Bb})K_t,$$

$$Y_t = \left[\frac{1}{2}(Y_t^a)^{\frac{s-1}{s}} + \frac{1}{2}(Y_t^b)^{\frac{s-1}{s}} \right]^{\frac{s}{s-1}}.$$

Finally, the prices of intermediate goods a and b reflect their marginal value in final goods production and can be written as:

$$P_t^a = \frac{1}{2} \left(\frac{Y_t}{Y_t^a} \right)^{\frac{1}{s}}, P_t^b = \frac{1}{2} \left(\frac{Y_t}{Y_t^b} \right)^{\frac{1}{s}}.$$

For tractability, there is one type of capital in this economy, and I model productivity shocks as aggregate shocks to quantity of capital, with the interpretation that a negative shock reduces the “effective capital” stock. While there is a single type of capital, shocks are assumed to be independent for each country. This is a simple way of capturing country specific shocks as well as redistributive shocks. In this model, all agents have access to the same investment technology, where final goods can be turned into capital on a one-to-one basis. However, investment opportunity is short-lived, and cannot be costlessly financed externally. In particular, we assume that financing frictions result in an effective “adjustment cost” for capital creation, similar in form to that in BruSan15¹⁴. The interpretation

¹⁴Costly external financing is not crucial for any results. We could assume there is no external financing available and all investment, if any, must be financed with internal liquidity. All results will remain (and become amplified).

is different, however: in their paper it is assumed to capture a form of technological illiquidity – it is costly to create (or destroy) capital in large amounts within a short period of time. Here, the adjustment cost is capturing funding illiquidity – it is costly to raise a large amount of (uncollateralized) resources externally as investment inputs within a short period of time. Overall, capital employed in country I evolves according to:

$$\frac{dk_t}{k_t} = (\iota_t^{internal} + \Phi(\iota_t^{external}) - \delta)dt + \sigma^I dZ_t^I,$$

where $\iota_t^{internal}$ is the investment rate of the aggregate goods per unit of capital, financed with internal liquidity (to be specified below), and $\iota_t^{external}$ is that financed with external liquidity. For function Φ to capture the (marginally increasing) external financing premium, we assume $\Phi(0) = 0$, $\Phi'(0) = 1$, $\Phi'(\cdot) > 0$, and $\Phi''(\cdot) < 0$. In contrast, investment financed with internal liquidity is always one-to-one (There is no technological illiquidity in this model for simplicity). The results are not sensitive to alternative specifications, as long as there is a wedge between return to capital creation financed internally and externally in some states of the world.

Internal Liquidity. I assume that investment opportunities during period t arise somewhere between t and $t + dt$, before production outputs realize, and that physical capital holdings are illiquid at this time and cannot serve as collateral as well¹⁵. Bond holdings, however, can potentially be sold quickly to generate internal liquidity to their holders. In particular, I assume bond holders have a positive probability to be matched with an “outside” bond buyer, to whom she can sell a fraction of her bond holdings¹⁶. The matching

¹⁵See Appendix. C for a microfoundation.

¹⁶The outside bond buyers do not “inject” resources into this economy on net, which can be seen from the consumption market clearing condition below. As in Benigno and Nistico (2017), we do not model these buyers explicitly, except assuming that they have deep pockets and are competitive so that they pay

probability¹⁷ is exogenous and is denoted by χ . Once matched, the rate at which bond holdings can be sold by agents in country I is denoted by ϕ_t^{-I} (it is the debt of country $-I$ that agents in country I are selling), which can potentially vary with the state of the economy. In the baseline case, we assume constant resaleability: $\phi_t^I = \phi^I$, for $I \in \{A, B\}$.

Preferences. Agents in country I have log utility described by:

$$\mathbb{E}_t \left[\int_t^\infty e^{-\rho^I(s-t)} \log(c_s^I) ds \right],$$

where c_s^I is the consumption rate of the final goods at time s , and ρ^I is the discount factor for agents in country I for $I \in \{A, B\}$.

Market for Physical Capital and the Short-Term Bond. Capital trades in a competitive and frictionless market. Denote the equilibrium market price of capital by q_t , I look for an equilibrium where the law of motion for q_t takes the form:

$$\frac{dq_t}{q_t} = \mu_t^q dt + \sigma_t^{qA} dZ_t^A + \sigma_t^{qB} dZ_t^B,$$

where the drift and volatility terms will be determined endogenously in equilibrium.

There is also a frictional market for short-term (hence risk-free in continuous time) bond, which is in zero net supply. I assume this market to be competitive, and that exogenous frictions lead to a transaction cost in this market (specified below), which is paid by lenders¹⁸. As discussed in the Appendix, without transaction costs, natural borrowers

the fair price of the bonds. Furthermore, these outside bond buyers do not have to come from outside the model. They could be interpreted as agents whose production output realized early and who do not have an investment opportunity during the current period. See Footnote 6.

¹⁷In the continuous time limit, the bond-buyer-matching shock is difficult to define. For interpretation, we could follow the argument in Brunnermeier and Sannikov (2016a) and treat the results as an approximation for a discrete time setup where the time interval shrinks to zero. Also, in characterizing equilibrium it is only the product, $\chi \cdot \phi^b$, that matters, so we could take $\chi = 1$ and all results will remain.

¹⁸All results remain if this transaction cost is paid by borrowers, or shared in anyway between the borrowers and the lenders.

(those who have a better production technology due to favorable relative prices of their goods) will want to first lend to the natural lenders, and finance their production with all borrowed money. While theoretically interesting, it is less empirically realistic so I focus on the more standard equilibrium in which the asset side of borrowers consist of only physical capital. For simplicity, the transaction costs are assumed to be paid to some global intermediaries who redistribute the proceeds on a pro rata basis¹⁹.

Return from Holding Capital. Return from capital depends on which country the agent holding the capital belongs to, as well as what goods that agent uses the capital to produce. The return from holding capital comes in two parts: dividend yield in the form of output, and capital gains coming from the law of motion for price as well as capital²⁰. Formally, the return to an agent in country $I \in \{A, B\}$ from using capital to produce goods $i \in \{a, b\}$ can be written as:

$$dr_t^{Ii} = \left(\frac{a^{Ii} P_t^i - \iota_t^{Ii, external}}{q_t} \right) dt + \frac{d(q_t k_t^{Ii})}{q_t k_t^{Ii}}.$$

Plugging in the law of motion for the price of capital, we have:

$$\begin{aligned} dr_t^{Aa} = & \left(\frac{\bar{a} P_t^a - \iota_t^{Aa, external}}{q_t} + \Phi(\iota_t^{Aa, external}) + \mu_t^q - \delta + \sigma^A \sigma_t^{qA} \right) dt \\ & + (\sigma^A + \sigma_t^{qA}) dZ_t^A + \sigma_t^{qB} dZ_t^B, \end{aligned}$$

and analogous expressions can be derived for dr_t^{Ab} , dr_t^{Ba} , dr_t^{Bb} .

The optimal investment rate via external financing is determined from a static problem:

¹⁹Alternative assumptions on how the revenues are distributed matter only slightly for some of the results. The revenues could also be assumed to be wasted. This would reduce the benefit of liquidity in general, but does not change the qualitative results of this paper.

²⁰Per standard procedure, I write return from capital investment via external financing inside the capital holding return.

$$\text{Max}_{\iota_t^{external} \geq 0} q_t \Phi(\iota_t^{external}) - \iota_t^{external}$$

We see that the optimal investment rate is the same for all agents and is given by the first-order condition:

$$\Phi'(\iota_t^{external}) = q_t^{-1}.$$

Similarly, the optimal investment rate via internal financing (selling bonds) takes the form of:

$$\iota_t^{internal} = \begin{cases} \iota_t^{max} & \text{if } q_t > 1 \\ 0 & \text{if } q_t \leq 1 \end{cases},$$

where I assume that physical capital cannot be shorted. ι_t^{max} denotes the maximum amount of investment (per unit of capital) possible. For an agent i in country I with bond holdings b_t^i and capital holdings k_t^i , it is given by:

$$\iota_t^{max} = \begin{cases} \frac{\phi_t^{-I} b_t^i \cdot \mathbf{1}_{b_t^i \geq 0}}{q_t k_t^i} & \text{if matched with a bond buyer} \\ 0 & \text{otherwise} \end{cases},$$

The formula is simply saying the following: if price of capital is higher than 1, an agent with investment opportunity as well as internal liquidity can generate positive profits without any risk (creating capital at a low price and instantly selling it at a higher price). Internal liquidity is only generated when that agent has positive amount of bond ($b_t^i \geq 0$) and is matched with a bond buyer, in which case she sells the maximum fraction of bond holdings possible, denoted by $\phi_t^{-I} dt$ ²¹. Throughout the paper I restrict attention to

²¹There is a dt term because investment is a flow variable while bond holding is a level variable. In a discrete time setup, we have $dt = 1$ and $\phi_t \in (0, 1)$ stands for the fraction of bond sold.

parameters such that the price of capital is always strictly greater than 1, i.e., capital is sufficiently productive that no disinvestment ever occurs except for natural depreciation²².

Incomplete and Frictional Markets. There are three frictions in this model. First, incomplete markets prevent agents from issuing equity to each other, so that agents can only borrow through risk-free debt to buy capital. Second, financing frictions prevent agents from fully utilizing investment opportunities²³. Lastly, there is a transaction cost to the bond market – in particular, we assume that creditors pay a proportional cost for lending, at rate γ^I per dollar, for $I \in \{A, B\}$. We will assume γ^I is big enough such that in equilibrium there is no cross borrowing and lending. This assumption allows us to focus on the equilibrium that is empirically realistic and also closer in spirit to that in BruSan15²⁴.

Equilibrium. For any initial allocation of wealth, an equilibrium is a map from histories of shocks $\{Z_s^A, Z_s^B, s \in [0, t]\}$ to the allocations of capital $(\psi_t^{Aa}, \psi_t^{Ab}, \psi_t^{Ba}, \psi_t^{Bb})$, aggregate consumption choices (C_t^A, C_t^B) , as well as prices q_t and dr_t^F for capital and bond respectively, such that

1. Agents maximize utility subject to budget constraints and internal liquidity constraints, and
2. All markets clear.

²²Due to lack of technological adjustment costs, price of capital will never fall below 1 as agents will always convert capital back to consumption goods.

²³This friction is essential for non-trivial law of motion for price of capital, as in Brunnermeier and Sannikov (2015).

²⁴In the numerical examples, γ^I can be pretty small – a fraction of the growth rate of the economy.

1.1.2 Equilibrium Characterization

I restrict attention to Markov equilibria with rational expectations in which the single state variable is the wealth share of country A . In particular, denote N_t^A as the aggregate wealth of country A agents, and $q_t K_t$ as the global wealth of the whole economy, the state variable is then denoted as:

$$\eta_t \equiv \frac{N_t^A}{q_t K_t}.$$

I conjecture and verify an equilibrium in which it can be divided into five regions: in the first region, η_t is small enough that country A is borrowing from country B and country B is producing both goods, despite their relative disadvantage at producing goods a . In the second region, η_t is larger so that each country specializes in producing what it is better at producing, but country A still borrows from country B . In the middle region, η_t is close to the stochastic steady state ($\eta = 0.5$ if the two countries are symmetric) and there is financial autarky due to transaction cost of the bond market. In the fourth region, η_t is big enough that country A is lending to country B . In the final region, η_t is close to 1 and agents in country A produce both goods. As mentioned before, I make an assumption so that in equilibrium the only active credit market is for resources (credit) to flow from the “natural borrower” country to the “natural lender” country²⁵:

ASSUMPTION 1. Transaction cost in the bond market satisfies the following inequality²⁶:

²⁵Here a natural borrower is an agent from a country whose wealth share is smaller than its wealth share in the stochastic steady state defined below. In a symmetric world, the stochastic steady state is at $\eta = 0.5$ so natural borrower refers to the currently poor country and natural lender refers to the currently rich country.

²⁶We could impose the less restrictive assumption that credit must be used for production, and not for financial investment (holding bond). With this assumption, part of the equilibrium will be a corner solution such that country A agents lend all their net worth to country B agents, and produce (buy capital) with only

$$\gamma^I > \chi \cdot \max_{\eta} \{ \max \{ \phi^A(\eta), \phi^B(\eta) \} \cdot [q(\eta) - 1] \}, I \in \{A, B\}.$$

Agent's problem²⁷ can then be written recursively as:

$$\begin{aligned} \rho^I V^I(n_t, \eta_t) &= \text{Max}_{c_t^I, \{x_t^{Ii}\}_{i \in \{a, b\}}} \log(c_t^I) + \mathbb{E}[dV^I(n_t, \eta_t)], \\ \text{s.t. } \frac{dn_t^I}{n_t^I} &= x_t^{Ia} dr_t^{Ia} + x_t^{Ib} dr_t^{Ib} + (1 - x_t^{Ia} - x_t^{Ib}) dr_t^{IF} - \frac{c_t^I}{n_t^I} dt + \tau_t dt. \end{aligned}$$

where the expectation is taken over shocks dZ_t^A, dZ_t^B , and the bond-buyer matching shock. Denote the equilibrium market risk-free rate as $\frac{dr_t^F}{dt}$, and the “effective interest rate” for $I \in \{A, B\}$ as $\frac{dr_t^{IF}}{dt}$. In particular:

$$\begin{aligned} \frac{dr_t^{AF}}{dt} &= \begin{cases} \frac{dr_t^F}{dt} - \gamma^A + \phi^B(q_t - 1) & \text{if } A \text{ is lending, and matched with a bond buyer} \\ \frac{dr_t^F}{dt} - \gamma^A & \text{if } A \text{ is lending, and not matched with a bond buyer} \\ \frac{dr_t^F}{dt} & \text{if } A \text{ is borrowing} \end{cases} \\ \frac{dr_t^{BF}}{dt} &= \begin{cases} \frac{dr_t^F}{dt} - \gamma^B + \phi^A(q_t - 1) & \text{if } B \text{ is lending, and matched with a bond buyer} \\ \frac{dr_t^F}{dt} - \gamma^B & \text{if } B \text{ is lending, and not matched with a bond buyer} \\ \frac{dr_t^F}{dt} & \text{if } B \text{ is borrowing} \end{cases} \end{aligned}$$

The intuition for $\frac{dr_t^{AF}}{dt}$ is simple: when an agent from country A is a net creditor, and not matched with a bond buyer, her effective return is just the market interest rate minus transaction cost, i.e., $\frac{dr_t^F}{dt} - \gamma^A$. But if she is matched with a bond buyer with whom she

borrowed money. Intuitively, because debt of country A has liquidity value which is reflected in the form of a liquidity premium, agents in country A have incentives to maximize the “production” of such debt. While such production is always bounded by their risk appetite in equilibrium, as long as their debt is still special on the margin, they are always better off using borrowed money to produce, instead of their own money.

²⁷Note that we already incorporated the optimal investment rates into the setup of the problem.

can transact to generate internal liquidity, bond holdings allow her to make riskless profits to the extent of the resaleability constraint of her bond holdings. Per unit of bond holding, she can sell $\phi_t^B dt$ fraction of it, which yields an extra profit of $\phi_t^B (q_t - 1)dt$ from capital creation (creating capital at the price of 1 and selling it instantaneously at the price of q_t). The interpretation for $\frac{dr_t^{BF}}{dt}$ is identical.

We characterize the equilibrium dynamics with the HJB equations and market clearing conditions. The goal is to characterize aggregate variables as functions of the stage variable η_t . The results are summarized in Proposition 1.

PROPOSITION 1: The state space can be divided into three regions²⁸. In the middle region, denoted as $\eta_t \in [\eta^a, \eta^b]$, production achieves full specialization – agents use capital to produce only the goods they have productive advantage of. In the left region, i.e., $\eta_t \in (0, \eta^a)$, country B agents also use capital to produce goods a , while in the right region, i.e., $\eta_t \in (\eta^b, 1)$, country A agents also use capital to produce goods b . Also, in equilibrium country A borrows when η_t is small, while country B borrows when η_t is large, and there is financial autarky when η_t is close to the economy's stochastic steady state.

For all three regions, law of motion for η_t can be written as:

$$\frac{d\eta_t}{\eta_t} = \mu_t^\eta dt + \sigma_t^{\eta A} dZ_t^A + \sigma_t^{\eta B} dZ_t^B,$$

where:

$$\begin{aligned} \mu_t^\eta = & \left(\frac{\psi_t^{Aa}}{\eta_t} \right) \left(\frac{\psi_t^{Aa}}{\eta_t} - \psi_t^{Aa} \right) [(\sigma^A + \sigma_t^{qA})^2 + (\sigma_t^{qB})^2] \\ & - (\psi_t^{Ba} + \psi_t^{Bb}) \cdot \left(\frac{\psi_t^{Ba} + \psi_t^{Bb}}{1 - \eta_t} \right) [(\sigma^B + \sigma_t^{qB})^2 + (\sigma_t^{qA})^2] - \end{aligned}$$

²⁸In this proposition, the middle region contains the three regions in the middle discussed at the beginning of this section (five-region characterization).

$$[\psi_t^{Aa}\sigma^A + \sigma_t^{qA}]\sigma_t^{\eta A} - [\sigma_t^{qB} + (\psi_t^{Ba} + \psi_t^{Bb})\sigma^B]\sigma_t^{\eta B} +$$

$$(1 - \eta_t)\{[\gamma^B - \chi\phi^A(q_t - 1)]\mathbf{1}_{A \text{ is debtor}} - [\gamma^A - \chi\phi^B(q_t - 1)]\mathbf{1}_{A \text{ is creditor}}\},$$

$$\sigma_t^{\eta A} = \frac{\frac{\psi_t^{Aa}}{\eta_t}(1 - \eta_t)}{1 - [\psi_t^{Aa} - \eta_t]\frac{q'(\eta_t)}{q(\eta_t)}}\sigma^A,$$

$$\sigma_t^{\eta B} = -\frac{\frac{\psi_t^{Ba} + \psi_t^{Bb}}{\eta_t}(1 - \eta_t)}{1 - [\psi_t^{Aa} - \eta_t]\frac{q'(\eta_t)}{q(\eta_t)}}\sigma^B.$$

In terms of asset pricing, in the left region, we have:

$$\frac{\bar{a}(P_t^a - P_t^b)}{q_t} + \sigma^A\sigma_t^{qA} - \sigma^B\sigma_t^{qB} + \chi\phi^A(q_t - 1) - \gamma^B$$

$$= \frac{\psi_t^{Aa}}{\eta_t}[(\sigma^A + \sigma_t^{qA})^2 + (\sigma_t^{qB})^2] - \frac{1 - \psi_t^{Aa}}{1 - \eta_t}[(\sigma^B + \sigma_t^{qB})^2 + (\sigma_t^{qA})^2],$$

and

$$\underline{a}P_t^a = \bar{a}P_t^b.$$

In the right region we have two symmetric equations. In the middle region, only the first equation holds.

In terms of consumption market clearing, in the left and middle region, we have (for the case where $s = 1$, i.e., Cobb-Douglass aggregation):

$$[\eta_t\rho^A + (1 - \eta_t)\rho^B]q_t = [(\bar{a}\psi_t^{Aa} + \underline{a}\psi_t^{Ba})\bar{a}\psi_t^{Bb}]^{\frac{1}{2}} - \frac{q_t - 1}{\kappa} - \chi\phi^A q_t(\psi_t^{Aa} - \eta_t),$$

and a symmetric equation for the right region.

Finally, capital market clearing is written as:

$$\psi_t^{Aa} + \psi_t^{Ab} + \psi_t^{Ba} + \psi_t^{Bb} = 1.$$

Figure 1.1: Effect of bond liquidity on leverage on prices

1.2 Numerical Example of Excessive Liquidity

In this section I provide a set of parameter values and use Proposition 1 to characterize the whole equilibrium dynamics²⁹. I assume $\Phi(\iota) = \frac{\log(1+\kappa\iota)}{\kappa}$, so that $\iota^* = \frac{q-1}{\kappa}$ and $\Phi(\iota^*) = \frac{\log(q)}{\kappa}$, and set $\bar{a} = 1, \underline{a} = 0.6, \delta = 0.05, \kappa = 5, \rho^A = \rho^B = 0.05, s = 1, \sigma^A = \sigma^B = 0.05$, similar to those in Chen and Phelan (2017). For bond liquidity, I set $\chi = 0.1$ and $\phi^A = \phi^B = 0.05$, that is, bond holders have 10% probability of being matched with a bond buyer, and if matched, can sell at most 5% of her bond holdings. Finally, I set $\gamma^A = \gamma^B = 0.01$, a 1% flow transaction cost to lenders.

Figure 1.1 plots some preliminary graphs to illustrate the equilibrium dynamics. I compare between our case and the case where bond has no liquidity at all, i.e., $\phi^A = \phi^B = 0$.

When bond has liquidity, equilibrium leverage is higher because creditors have an extra incentive to lend – bond holdings are their financial liquidity for capital investment

²⁹Computational algorithm is similar to that in BruSan15.

opportunities³⁰. The price of capital is lower because capital becomes less scarce due to more investment, a supply-side effect³¹. Lower price of capital, together with higher slope of capital price dynamics (i.e., $q'(\eta_t)$), generate stronger amplification results³², as seen from the formulae

$$\sigma_t^{\eta A} = \frac{\frac{\psi_t^{Aa}}{\eta_t}(1-\eta_t)}{1 - [\psi_t^{Aa} - \eta_t] \frac{q'(\eta_t)}{q(\eta_t)}} \sigma^A, \sigma_t^{\eta B} = - \frac{\frac{\psi_t^{Ba} + \psi_t^{Bb}}{\eta_t}(1-\eta_t)}{1 - [\psi_t^{Ba} - \eta_t] \frac{q'(\eta_t)}{q(\eta_t)}} \sigma^B.$$

Finally, the last graph shows that terms-of-trade for country A is worse in the liquidity case when production is fully specialized. This is consistent with its higher leverage, which leads to higher output and undermines the whole country's terms-of-trade – the pecuniary externality in BruSan15.

Bond liquidity also affects equilibrium drift and diffusion of country A 's wealth share, the state variable. As the top left graph of Figure 1.2 shows, for state variable values such that each country specializes in producing goods it has advantage of, bond liquidity lowers the drift of country A 's wealth share for two reasons. First, bond liquidity fuels higher equilibrium leverage, which increases output of goods a , undermining country A 's terms-of-trade. Second, bond liquidity benefits the creditors by allowing them to more efficiently utilize capital investment opportunities (and thus increasing their wealth growth

³⁰In addition to incentives from production technology differences due to different prices of the two intermediate goods.

³¹The result that higher liquidity lowers asset prices is seemingly in contradiction to a large literature showing how liquidity injections support asset prices. However, they essentially share the same mechanism. In those models, when there is scarce liquidity in the market, capital price is too low relative to where it should be, and an increase in the supply of liquidity helps restore the price of capital to its fundamental value (partially or fully). In my model, due to the assumption of no technological adjustment cost, capital price should be 1 without liquidity constraints. Price of capital is higher than 1 in equilibrium only because of a binding liquidity constraint related to financing capital investment. When relaxing this constraint, capital price again moves towards its fundamental value ($q_t = 1$).

³²Bond liquidity may also introduce a version of sunspot risks to the economy as described in BruSan15. Details are in the Appendix.

Figure 1.2: Tradeoff of growth and stability

rate). When the state variable is close to the boundaries (η_t close to 0 or 1), bond liquidity increases the drift of country A 's wealth share because the terms-of-trade hedge is switched off.

The top right graph of Figure 1.2 shows that bond liquidity almost always increases volatility (diffusion) of the state variable. Again, this is due to two reasons. First, higher leverage increases country A 's risk exposure, and hence makes its net worth share more volatile. Second, lower price of capital magnifies the endogenous risk coming from prices (see Proposition 1), namely σ_t^{qA} and σ_t^{qB} , which has a higher impact on the leveraged country.

Naturally, economic growth is higher when bond can serve as internal liquidity. Growth rate differences between no-liquidity case and with-liquidity case mostly reflect equilibrium debt level, which first increases and then decreases as country A gets richer. Higher economic growth comes at the cost of a less stable stationary distribution. While in the no-liquidity case, the economy spends most of its time near the stochastic steady state ($\eta = 0.5$), with bond liquidity the stationary distribution is considerably flatter – that is, an economy with bond liquidity is more volatile, and spends a higher fraction of time in states where one country is moderately undercapitalized. The bottom two graphs of Figure 1.2 illustrate this tradeoff.

The left graph of Figure 1.3 plots economic welfare, which given symmetry is defined as the sum of welfare of each country's representative agent, as a function of the state variable η_t . We see that with bond liquidity, global welfare is higher for all of the state space. This is not surprising given the fact that bond liquidity relaxes an economy-wide financing friction, despite the fact that the benefit mostly accrues to creditors, and often

Figure 1.3: Alternative measures of welfare

at the expense of borrowers. However, this measure of welfare could be misleading, as the two regimes (with and without liquidity) have very different stationary distributions as shown in Figure 1.2. The no-liquidity regime, despite having a lower welfare measure for all of the state space, spends more time where welfare is high (η near 0.5) when compared with the no-liquidity regime. Thus, a better measure of (ex-ante) welfare should take this into account.

The middle graph from Figure 1.3 plots economic welfare against a monotone transformation of η , namely, its cumulative distribution function according to the stationary distribution of the economy calculated earlier (see Figure 1.2). This is particularly convenient as the area under the curve is the ex-ante global welfare, i.e., the expectation of global welfare $V^A(\eta_t) + V^B(\eta_t)$ with respect to η_t , which is distributed according to its stationary distribution. As argued in BruSan15, Chen and Phelan (2017), and Mendo (2018), this is the right way to evaluate welfare for the following reason: while the HJB equation accounts for the future behavior of the state variable, it only computes welfare for a par-

ticular initial value of η (the left graph). Since there is no clear prior on what the initial wealth distribution should be, the stationary distribution is a natural candidate, as it describes the long-run behavior of the economy. On the other hand, welfare results from the left graph of Figure 1.3 could be misleading because implicitly we are assuming a uniform distribution of the state space.

After considering the effect of bond liquidity on the economy's stationary distribution, we see that welfare implications are reversed: bond liquidity in this example lowers ex-ante welfare. Again, the benefit of bond liquidity is manifested in higher economic growth rates due to relaxing frictions in financing capital investment opportunities, while the cost of bond liquidity is in terms of higher economic instability, in the form of higher volatilities of prices (higher endogenous risk), and higher volatilities of the state variable which results in a less stable stationary distribution. For the set of parameters I considered above, the cost outweighs the benefit, and the ex-ante welfare is higher if neither of the two country's debt can serve as financial liquidity³³.

Finally, the right graph of Figure 1.3 plots country A 's welfare against the CDF of η . We see clearly that when country A is a natural borrower, i.e., when $\eta \in (0, 0.5)$, it is worse off when bond liquidity is introduced, because lower equilibrium interest rates induce them to borrow more and hence lower the price of their output. Symmetrically, bond liquidity increases its welfare when country A is a natural lender, which is due to both a relatively better terms-of-trade (compared to the case without bond liquidity) and more efficient financing of investment opportunities.

³³Given the symmetry of parameters, economic welfare is also higher if only one country's debt stops providing liquidity. However, the gain is unevenly distributed to this country, at the expense of the other country whose debt still serves as liquidity.

1.3 Comparative Statics

In this section, I look at how fundamental volatility, i.e., σ^I , as well as bond market transaction cost, i.e., γ^I , affect the welfare implications of bond liquidity, for $I \in \{A, B\}$. Broadly speaking, lower fundamental volatility can be interpreted as improvements in production technology, while lower bond market transaction cost can be interpreted as lower credit market frictions resulting from financial developments³⁴. Both are features typically associated with developed countries.

1.3.1 Fundamental Volatility

The left graph of Figure 1.4 contrasts between equilibrium dynamics and welfare results as we gradually increase bond resaleability, between the case where (symmetric) fundamental volatility is 0.05 as in Section 1.3, and the case where fundamental volatility is 0.1.

We see here that when $\sigma^A = \sigma^B = 0.05$, bond liquidity lowers (ex-ante) economic welfare, but with a higher fundamental volatility at $\sigma^A = \sigma^B = 0.1$ bond liquidity improves welfare. In particular, increasing bond liquidity from 0 to 5% brings about a consumption-equivalent loss³⁵ of roughly 3.5% when $\sigma^A = \sigma^B = 0.05$, while the same increase in bond liquidity brings a consumption-equivalent gain of roughly 0.6% when $\sigma^A = \sigma^B = 0.1$.

³⁴ Alternatively, it could be interpreted as resulting from deregulation of an (implicitly) intermediating financial sector.

³⁵ Consumption-equivalent ϕ is defined as the percentage change in consumption in current and all future states needed to bring about the change in welfare level. If $V^A = \mathbb{E}[\int_0^\infty e^{-\rho^A t} \log(c_t^A) dt]$, then the consumption-equivalent ϕ for a different value $\tilde{V}^A$ is calculated from $\tilde{V}^A = \mathbb{E}[\int_0^\infty e^{-\rho^A t} \log((1 + \phi)c_t^A) dt] = \int_0^\infty e^{-\rho^A t} \log(1 + \phi) dt + V^A$. Thus, $\phi = \exp((\tilde{V}^A - V^A) \cdot \rho^A) - 1$.

Figure 1.4: Welfare as a function of bond liquidity

While there are many general equilibrium effects behind this pattern, one reason is that with a lower endogenous volatility, if the economic system starts with misallocated capital, it takes longer in expectation for the system to go back to its stochastic steady state where capital is efficiently allocated. Thus, the welfare cost of getting into a bad state of the world is higher. In addition, the increase in equilibrium leverage per unit increase in bond liquidity (in terms of resaleability) is more pronounced for lower levels of fundamental volatility. This is because the convenience yield creditors get from holding bonds, $\chi\phi^I(q_t - 1)$, and thus the equilibrium drop of interest rate from higher bond liquidity, is not sensitive to changes in fundamental volatility. But per unit drop in interest rate, increase in equilibrium leverage, and hence the distortion on stationary distribution, is more pronounced for lower levels of fundamental volatility. Ultimately, this means that in economies with low fundamental volatility, stationary distribution (or the amount of tail risk in particular) is more sensitive to changes in bond liquidity. Indeed, as the other two graphs of Figure 1.4 shows, when $\sigma = 0.1$, after introducing bond liquidity, the fraction of

time the economy spends in $\eta \in [0, 0.4]$ increases by a small fraction (around 20%), but this percentage increase when $\sigma = 0.05$ is much higher (more than 300%).

1.3.2 Transaction Cost

Bond market transaction cost in this model is meant to capture frictions in the primary bond market such as that arising from costly state verification, and reductions in transaction cost may reflect better financial technologies such as lower monitoring costs. Transaction cost has a stabilizing effect because it reduces incentives to lend ³⁶and can help correct the pecuniary externality. As Figure 1.5 shows, transaction cost lowers economic volatility and improves the terms-of-trade hedge for the borrowing country, leading to a more stable stationary distribution, at the expense of a drop in economic growth rate. This is consistent with the empirical growth literature on one hand, which finds positive effect of financial development on growth, and the banking/currency crisis literature on the other, which finds aggregate credit as one of the best predictors of crises and its related economic downturns (See Loayza and Ranciere (2005) for a summary). This tradeoff has received

³⁶As opposed to bond liquidity, which adds to the incentives to lend. The differences between these two are the following: first, for the sake of simplicity I only study constant transaction cost in this paper. In contrast, bond liquidity generates a convenience yield that is state-contingent: recall that the liquidity premium of private bond is effectively $\chi\phi^I(q(\eta_t) - 1)$, so the effect of an increase in liquidity brings about a more significant change in convenience yield (and hence more pronounced real interest rate changes) when the economy is in good states (when price of capital is high). Second, while a reduction in transaction cost affects borrowers in qualitatively the same way as an increase in bond liquidity (cheaper cost of borrowing), variations in liquidity affect growth rate of the current creditors, while variations in transaction cost affect the degree of their consumption smoothing. Finally in terms of asset prices, an increase in liquidity lowers price of capital, while a drop in transaction cost of lending increases the price of capital. The latter does not affect investment, and capital price goes up because (static) capital allocation becomes more efficient. While (static) capital allocation also improves when liquidity increases, its effect on capital price is dominated by that from additional investment which makes (existing) capital less scarce.

renewed attention after the 2007 financial crisis as policy makers debate whether financial innovations can be excessive (Popov 2018).

The reduction in credit is more beneficial when the economy is moderately out of balance – where terms-of-trade hedge is active. At extreme tail states transaction cost lowers the state-contingent welfare since it does not lead to a better terms-of-trade (the rich country is producing both goods, thereby fixing relative prices). However, this is easily overwhelmed by the improvement in stability, and ex-ante welfare is higher for moderate amount of transaction costs.

Figure 1.6 shows how ex-ante welfare changes with liquidity for different levels of bond market transaction cost. Consistent with transaction costs being a stabilizing force, we see that an economy with higher transaction costs has higher welfare, and more importantly, the economy also benefits from increases in bond liquidity, as opposed to the economy with lower transaction costs. The result here speaks to the unintended consequences of a combination of high bond liquidity and low bond market transaction costs, which are often features of advanced economies. Moreover, it speaks to the nuance of welfare implications of financial innovation – lower transaction cost, as one form of financial innovation, may reverse the welfare implications of another form of innovation – increases in bond liquidity.

1.4 Macprudential Policies

While macroprudential tools have been used around the world for many years, they have received renewed interest after the great financial crisis. In this section I discuss two com-

Figure 1.5: Effect of transaction cost on equilibrium dynamics and welfare

Figure 1.6: Interactions between transaction cost and bond liquidity

mon measures aimed at enhancing stability by curbing excessive private credit: leverage constraint and tax on credit. The former directly fixes terms-of-trade by regulating the amount of lending allowed, while the latter indirectly restores terms-of-trade by discouraging lending through prices (interest rate). I show that when bond liquidity is introduced, optimal leverage constraint is significantly looser, while optimal tax on credit also increases, albeit only moderately. This is because leverage constraint fixes terms-of-trade so only its benefit (in terms of more internal liquidity) needs to be considered in designing leverage targets, while credit tax does not fix terms-of-trade so both benefit and cost of bond liquidity needs to be considered together in designing optimal credit tax.

Figure 1.7: An example where bond liquidity affects evaluation of macroprudential policies

1.4.1 Leverage Constraints

A salient feature of BruSan15 as well as Chen and Phelan (2017) is that leverage constraints can be welfare-enhancing. The argument is that while leverage constraint worsens the static inefficiency from capital misallocation, it also reduces the dynamic inefficiency by reducing the fraction of time the economy spends with misallocated capital. Indeed, in BruSan15 it is shown that a complete shut down of international credit market, i.e., a leverage constraint at zero, can still improve welfare.

Bond liquidity may affect welfare implications of such policies non-trivially. Using the same parameters as those in Section 1.4 (with $\gamma^I = 2\%$), Figure 1.7 compares the economy under complete capital control and the market equilibria with (at $\phi^A = \phi^B = 5\%$) and without bond liquidity.³⁷

From the left graph of Figure 1.7, we see that the policy of complete capital control is

³⁷The equilibrium dynamics under complete capital control (as well as its derivation) is identical to Proposition 4 of Brunnermeier and Sannikov (2015).

welfare-improving if one does not take into consideration the value of bond liquidity, but becomes welfare-reducing if one does. Again, the mechanism works through the tradeoff of stability and growth. The middle graph shows that the economy is most stable when capital control is implemented. When credit flow is allowed, economic stability declines, and it declines further when bond provides liquidity service. On the other hand, the complete capital control case has the lowest (global) growth rate, while a laissez-faire economy with bond liquidity features the highest growth rate. This numerical example shows that welfare implications of bond liquidity may be of the same order of magnitude as standard macroprudential policies such as leverage constraint. As a result, optimal policies may be non-trivially different, both qualitatively and quantitatively, once the liquidity value of private debt is considered.

To further illustrate this point, Figure 1.8 plots ex-ante welfare against a symmetric leverage constraint policy ($L^A = L^B = L$) for different values of fundamental volatility³⁸. We see here that in both cases, a small amount of bond liquidity changes the optimal leverage targets significantly. In particular, optimal leverage constraints are looser irrespective of whether private liquidity is beneficial (when $\sigma^A = \sigma^B = 0.1$ as Section 4 shows) or not (when $\sigma^A = \sigma^B = 0.05$ as Section 3 shows). This is because a binding leverage constraint means that increases in bond liquidity generate more internal liquidity without generating higher amount of credit, and hence only the benefit of bond liquidity needs to be considered in adjusting leverage constraint targets.

Also, optimal leverage constraint is tighter when fundamental volatility is lower, consistent with the previous analysis. This is because the dynamic inefficiencies are greater if

³⁸See Chen and Phelan (2017) for details of numerical procedure.

Figure 1.8: Effect of bond liquidity on optimal leverage constraint

the economy ever hits a state with moderately misallocated capital. However, lower fundamental volatility also means that equilibrium leverage is higher and so is total debt outstanding. When bond has liquidity, a tighter leverage constraint in low volatility economies would cause more welfare loss in the sense of a more pronounced drop in the amount of internal liquidity available. The second force seems to be relatively stronger in the sense that when fundamental volatilities are low, the increase of optimal leverage target as bond becomes more liquid is higher in terms of percentages – a 3% increase of bond liquidity changes optimal leverage constraint from 0.14 to 0.18 when $\sigma = 0.1$, compared with that from 0.07 to 0.12 when $\sigma = 0.05$.

1.4.2 Tax on Credit

Taxes on credit have been proposed to limit excessive private credit creation by acting as Pigouvian levies (Jeanne and Korinek 2010). While the idea dates back to at least Keynes

Figure 1.9: Effect of bond liquidity on optimal credit tax

(1936)³⁹, it has received renewed attention after the financial crisis. In this model, tax on credit acts similarly as the credit market transaction cost, with the distinction that proceeds are earned by the government and redistributed to its own agents instead of globally⁴⁰. While the intention is also regulating credit flows as leverage constraints do, tax on credit achieves it indirectly through prices (of lending). As governments increase such taxes, welfare initially improves but ultimately declines as the economy moves closer to financial autarky, where credit flows are inefficiently low.

Figure 1.9 contrasts the optimal level of credit tax for different levels of bond liquidity and fundamental volatility. In contrast with leverage constraint policies, optimal taxes

³⁹“The introduction of a substantial government transfer tax on all transactions might prove the most serviceable reform available, with a view to mitigating the predominance of speculation over enterprises in the United States (Keynes, 1936, p.159-60).”

⁴⁰The main difference between credit tax and bond liquidity is that an increase in bond liquidity directly changes how much capital growth can be financed, while credit tax only indirectly affects capital growth rate through its general equilibrium impact on capital prices (see also Footnote 36). Fundamentally, liquidity innovations directly relax an economy-wide investment financing constraint, while a lower credit tax does not. Thus, while a reduction in credit tax and an increase in bond liquidity have similar qualitative effects on aggregate dynamics, these two policies are not isomorphic to each other.

on credit do not vary much after bond liquidity is introduced. This is because terms-of-trade is not fixed with credit tax, so both the benefit and cost of bond liquidity need to be considered. In addition, it is worth pointing out that the optimal tax on credit is smaller for low levels of fundamental volatility. This is because a high tax on credit, applied uniformly across the state space, makes credit near the stochastic steady state inefficiently low (the financial autarky region inefficiently large)⁴¹. That is, the social planner balances inefficiently low credit near the stochastic steady state with inefficiently high credit away from it. With low fundamental volatility, the economy spends most of its time close to its stochastic steady state, and hence the optimal tax on credit is smaller to avoid too much inefficiency near the steady state. By contrast, leverage constraints do not bind near the stochastic steady state when the natural incentives to borrow and lend are already low in a *lassiez-faire* economy. Thus the tradeoff with leverage constraint is between inefficiently high credit near the stochastic steady state with inefficiently low credit away from it. Thus, while both a tight leverage constraint and a high tax on credit can improve stability by lowering leverage when the economy is moderately out of balance, the tradeoffs of these two policies are different.

In addition, we see that optimal tax on credit increases when bond liquidity is introduced, irrespective of whether bond liquidity increases or lowers welfare in the *lassiez-faire* economy. This is because tax on credit, unlike leverage constraints, actively facilitates the recapitalization process of the debtor country. Higher bond liquidity induces higher equilibrium credit, thereby increasing the gain from imposing a credit tax. Thus, the social planner optimal adjusts credit tax level upwards to realign marginal benefits and

⁴¹Indeed, we can show that it is generally welfare-improving for credit tax to decrease when the economy is very close to its stochastic steady state.

costs.

Finally, while the fully state-contingent optimal credit tax is numerically difficult to compute, we could show that optimal credit tax should roughly follow a hump-shaped pattern⁴² – taxes should be relatively low when the economy is severely out of balance because terms-of-trade hedge is switched off – a lower tax there encourages more credit and hence allow the debtor country to capture more risk premia. Taxes should be relatively high (but at moderate levels) when the economy is moderately out of balance so that debtor country can benefit more from its terms-of-trade hedge. Finally, taxes should be relatively low again when the economy is close to its stochastic steady state where impact of pecuniary externality is small (because “natural” leverage is already low).

1.5 Asymmetric Environments

So far, results on the effects of bond liquidity are all derived in symmetric environments. In this section I consider two sources of asymmetries between countries: fundamental volatility and size. Fundamental volatility refers to the risk parameters in production technology σ^I , $I \in \{A, B\}$, while size is captured by the parameter $\gamma \in (0, 1)$ in the final goods production function⁴³ $Y_t = (Y_t^a)^\gamma (Y_t^b)^{1-\gamma}$. This section shows that in asymmetric environments, the intuition that the distortionary effect of bond liquidity on stationary distribution is larger for low-volatility countries remains true. On the other hand, it is shown that the effect of bond liquidity on stationary distribution does not vary much for

⁴²Welfare improves as we lower taxes close to the economy’s stochastic steady state, and further improves if in addition we lower taxes when the economy is in extremely imbalanced states ($\eta \approx 0$ and $\eta \approx 1$).

⁴³ γ captures size in the sense that this is the steady state wealth share. In addition, with complete markets this parameter also captures the (constant) steady state consumption share.

economies of different sizes. One implication, as discussed below, is that the macroprudential policy negotiation process between low-volatility and high-volatility countries may be very different in nature compared to that between large-sized and small-sized countries.

1.5.1 Asymmetric Volatility

This section considers an asymmetric environment in which $\sigma^A = 0.05$ and $\sigma^B = 0.1$, while all other parameters remain the same as in Section 3. This environment is meant to approximate the interactions between an advanced economy and an emerging economy⁴⁴. In this environment, agents in country A are willing to hold more capital for a given level of wealth relative to agents in country B, which worsens the natural terms-of-trade hedge for country A (more so relative to country B). The stationary distribution of this economy without bond liquidity is plotted in the left graph of Figure 1.10. The left part of this graph is flatter than the right part – as a result of country A’s stronger incentive to take on leverage, the economy will spend more fraction of time with country A being relatively poor ($\eta_t < 0.5$). The middle graph of Figure 1.10 shows the effect of bond liquidity on the stationary distribution of this economy. The blue line is the CDF of the stationary distribution without bond liquidity, and it shows that country A is relatively poor ($\eta_t \leq 0.5$) for about 60% of the time. The red and magenta lines are the CDFs when (only) country B’s and country A’s debt can serve as liquidity, respectively. We see that the distortion on stationary distribution is more pronounced when country A’s debt is liquid. Again,

⁴⁴In Appendix. E I show that qualitative results in this section remain in an asymmetric environment in which $\sigma^A = 0.035$ and $\sigma^B = 0.05$, which is meant to approximate the interactions between (asymmetrically) advanced countries, such as the U.S. and Germany. However, the effects are quantitatively much smaller.

the intuition is that due to lower risks, equilibrium leverage choice of country A is more sensitive to changes in interest rate (resulting from changes in debt liquidity).

The right graph of Figure 1.10 plots ex-ante welfare of each country as well as the social welfare, which is again defined by assigning a Pareto weight $(0.5, 0.5)$ to the two countries. The magenta line is social welfare and it mildly goes down as debt (issued by both country A and country B) becomes more liquid. However, this is the net result of significant changes of ex-ante welfare of the two individual countries in opposite directions. Country B tends to benefit a lot from higher bond liquidity, while country A is hurt by a lot from it (the red line and the blue line, respectively). While country A benefits from an increase in liquidity of debt issued by country B (in terms of both more efficient financing of growth opportunities as well as a favorable shift in the stationary distribution), it is hurt much more from an increase of the same magnitude (in terms of resaleability) in its own debt liquidity (a much worse adverse shift in the stationary distribution).⁴⁵

A potentially interesting dimension of bond liquidity is how it may affect the negotiation process of international macroprudential standards between advanced and emerging economies. As pointed out in Chen and Phelan (2018), advanced economies hold substantial influence in writing international financial regulatory standards, and this they argue may be due to the fact that advanced economies provide safe assets to emerging economies by issuing debt liabilities. In particular, when markets are incomplete, systemic risks (e.g., pecuniary externalities) may not be correctly compensated by returns, and should be addressed through macroprudential regulation. The gains from interna-

⁴⁵Note that given the asymmetric environment, other Pareto weights can also be reasonable. In general, for high enough weights on country B, there would be a net gain in social welfare due to an inherent redistributive motive of the social planner.

Figure 1.10: Welfare implications of bond liquidity in economies of different volatilities

tional coordination of macroprudential policies, however, may be unequally distributed. Advanced economies (economies with low production volatilities) tend to lose under coordination, while emerging economies (economies with high production volatilities) tend to gain under coordination relative to the Nash equilibrium outcome. The intuition is that high-risk economies tend to benefit from credit flows which allow for better international risk sharing, while the low-risk economies tend to lose from credit flows by acting as a global intermediary (absent of regulations) and as a result being exposed to “excess” instability. The conclusion is that low-risk economies hold strong bargaining power when negotiating macroprudential policies as they have a credible threat of reverting to Nash equilibrium and imposing excessively tight (from a social perspective) policies.

Bond liquidity will give low-volatility countries even more bargaining power, because coordination essentially requires them to take more systemic risk (looser capital inflow constraints) so that the benefit of higher growth from bond liquidity can be realized. Indeed, when there is no bond liquidity ($\phi^A = \phi^B = 0$), the Nash equilib-

rium is financial autarky (both countries imposing the tightest leverage constraints, i.e., $L^A = L^B = 0$) with $\mathbb{E}[V^A(\eta_t)] = 8.2903$ and $\mathbb{E}[V^B(\eta_t)] = 7.8770$. The socially optimal leverage constraints are $L^A = 0.13$, $L^B = 0.08$ and ex-ante welfare of country A decreases to $\mathbb{E}[V^A(\eta_t)] = 7.9042$, while that of country B increases to $\mathbb{E}[V^B(\eta_t)] = 8.4161$. With higher bond liquidity ($\phi^A = \phi^B = 3\%$), socially optimal leverages become $L^A = 0.18$, $L^B = 0.115$, and ex-ante welfare of country A drops further to $\mathbb{E}[V^A(\eta_t)] = 7.6928$, while that of country B further improves to $\mathbb{E}[V^B(\eta_t)] = 8.7756$ (Nash equilibrium is still $L^A = L^B = 0$). Thus, when there is bond liquidity, the low-volatility country loses more from coordination, while the high-volatility country benefit more from it. As a result, bond liquidity in general will give low-volatility countries even more bargaining power in negotiating international macroprudential policies.

1.5.2 Asymmetric Size

Next, suppose country A's relative size becomes higher, i.e., suppose γ increases from 0.5 to 0.6⁴⁶, while all other parameters are the same as in Section 3. The effect of bond liquidity in this environment is summarized in Figure 1.11. The left graph shows that the stationary distribution shifts to the right with $\eta = 0.6$ being the new steady state, while the symmetric shape of the distribution is relatively unchanged. In addition, the effect of an increase in bond liquidity issued by either country on the stationary distribution is quantitatively similar, as shown in the middle graph of Figure 1.11. Finally, whether bond liquidity increases social welfare or not depends on the Pareto weights of the social planner. In the right graph of Figure 1.11, we increase both country's debt liquidity and calculate ex-ante

⁴⁶To preserve tractability, production function is still constant return to scale.

Figure 1.11: Welfare implications of bond liquidity in economies of different sizes

global welfare under two sets of Pareto weights. We see that when we continue to use symmetric Pareto weights $(0.5, 0.5)$ (the blue line), global welfare increases with bond liquidity, while if we use Pareto weights $(\gamma, 1 - \gamma)$ (the red line), global welfare decreases with bond liquidity. Roughly speaking, because country A's goods are more important for final goods production, agents in country A are more willing to take on higher leverage when bond liquidity increases relative to agents in country B. Thus if we increase the liquidity of both country's debt, the stationary distribution (slightly) shifts to the left – without liquidity, the economy spends roughly 50% of the time with $\eta_t \leq \gamma$, but with liquidity at $\phi^A = \phi^B = 5\%$, that probability increases to about 54%. This redistribution of ex-ante welfare from country A to country B is viewed more favorably under the Pareto weight of $(0.5, 0.5)$ ⁴⁷.

As for macroprudential policy coordination, the two economies with asymmetric sizes

⁴⁷Ex-ante welfare of country A mildly decreases as bond liquidity increases, while that of country B mildly increases.

would actually both gain from coordination relative to the Nash equilibrium, and higher bond liquidity will increase both of those gains mildly. In particular, without bond liquidity, the Nash equilibrium is again financial autarky (both countries imposing the tightest leverage constraints, i.e., $L^A = L^B = 0$) with $\mathbb{E}[V^A(\eta_t)] = 13.7119$ and $\mathbb{E}[V^B(\eta_t)] = 5.7106$. The socially optimal leverage constraints are $L^A = 0.055$, $L^B = 0.08$ and ex-ante welfare of the two countries increases to $\mathbb{E}[V^A(\eta_t)] = 13.7338$ and $\mathbb{E}[V^B(\eta_t)] = 5.7408$. With higher bond liquidity ($\phi^A = \phi^B = 3\%$), socially optimal leverage becomes $L^A = 0.09$, $L^B = 0.135$, and ex-ante welfare of both countries improves further: $\mathbb{E}[V^A(\eta_t)] = 13.7742$ and $\mathbb{E}[V^B(\eta_t)] = 5.7946$. Thus, the conflict between two countries of different sizes when it comes to stability policy coordination is much less significant, compared with that between two countries with different production volatilities.

1.6 Public Bonds

So far, only the implications of private bond liquidity, that is, liquidity of debt issued by non-government sectors are considered. Given that government bonds, especially the U.S. Treasuries, are a significant source of financial liquidity, it is natural to consider the potential role of public liquidity supply in this model. Indeed, Pozsar (2011) argues that it is precisely the “insufficient” supply of Treasuries that fueled the shadow banking boom which involved “excessive” private liquidity creation. More importantly, the fiscal policy of a government affects real interest rate when markets are incomplete, and to the extent that private borrowing is excessive due to pecuniary externalities, it is natural to consider whether fiscal policy can be used to correct this externality. In this section I discuss three

channels via which a government can potentially use its own debt to address excessive private debt creation. One is the crowding-out channel as in Greenwood, Hanson, and Stein (2015) where government competes with the private sector in creating liquidity-providing debt, with the intention of reducing equilibrium level of (excessive) private leverage. They show that such policy improves welfare if the government has a comparative advantage in managing excess volatility – i.e., if the social cost of more volatile future taxes is smaller than that from private sector fire sales. However as I show below, such crowding-out policies can be ineffective if the government is careless about how it manages its debt position over time. This is because the way government balance sheet is managed has important implications on how much taxes are to be collected. Forward looking agents will adjust their leverage position to hedge their tax exposure, in addition to reacting to changes in interest rates. With a simple policy where the government targets public debt issuance to be a constant fraction of private debt level, these two forces almost exactly cancel each other out, and equilibrium leverage barely changes for much of the state space.

I then discuss a seigniorage-redistribution channel where the government issues public debt to buy private debt, resembling an asset purchase program. A government can earn seigniorage profits from such activities if its debt has lower yield, and redistributing such profits increases welfare in two ways. First, it pumps more liquidity into the economy without generating more private debt, thus boosting growth without sacrificing stability; second, it helps the poor country recapitalize faster thus further improves stability⁴⁸. Public bond via this redistributive channel always increases welfare because it generates more internal liquidity without worsening debtor country's terms-of-trade hedge.

⁴⁸While both countries earn seigniorage profits and redistribute to their own agents, in the symmetric case the poor country's wealth share nonetheless improves due to its lower base.

Finally, I discuss the risk-premia channel of government bond, in which public bonds are used to self-insure against exogenous funding shocks. By controlling their supply, governments could potentially affect the cost of such insurance and hence the equilibrium level of leverage. In a two-country setting, however, lack of international coordination can be inefficient as the two countries compete to issue government bond at low prices to earn more seigniorage profits, while the socially optimal allocation requires both governments to control their public bond supply in order to drive up the cost of credit which enhances global stability.

1.6.1 Crowding-out Channel

In this section I show how a simple crowding-out policy can be completely ineffective as agents' incentive to hedge their tax exposure cancels out the effect on leverage from changes in interest rates. Suppose the debtor country's government decides to implement a crowding-out policy and issues public debt of amount equal to a constant fraction $\theta^I \geq 0$ of (equilibrium level) private debt. The rationale is that by increasing demand for credit, interest rate should go up and thus discourage agents from leveraging up. As in Greenwood et al (2015), I assume that the government keeps no assets and maintains its debt level solely via taxes – government targets its debt level and pays interests and finances its net changes in debt level via pro rata taxes on its agents. In contrast to Greenwood et al (2015), I assume the government has no expenditure to finance so that fiscal policy is conducted solely for the purpose of crowding out private debt. Formally, denote government I 's net worth as N_t^{IG} for $I \in \{A, B\}$. This will always be equal to the negative of the government

debt level as it keeps no assets⁴⁹. I denote its law of motion as⁵⁰

$$\frac{dN_t^{IG}}{q_t K_t} = \mu_t^{N^{IG}} dt + \sigma_t^{N^{IG}A} dZ_t^A + \sigma_t^{N^{IG}B} dZ_t^B, I \in \{A, B\}$$

The balance sheet of the government at any time $t \geq 0$ is illustrated below. Formally, I summarize the equilibrium dynamics in the proposition 2.

Asset	Liability
0	Gov debt: $-N_t^{IG}$
	Net worth: N_t^{IG}

Proposition 2. The state space is divided into three regions. For $\eta \in (0, \underline{\eta})$, country A issues debt of amount $\theta^A(\psi_t^{Aa} + \psi_t^{Ab} - \eta_t)q_t K_t$. The state variable follows an Ito process

$\frac{d\eta_t}{\eta_t} = \mu_t^\eta dt + \sigma_t^{\eta A} dZ_t^A + \sigma_t^{\eta B} dZ_t^B$ where:

$$\sigma_t^{\eta A} = \frac{1}{\eta_t} [\psi_t^{Aa}(1 - \eta_t)\sigma^A + (\psi_t^{Aa} - \eta_t)\sigma_t^{qA} - \sigma_t^{N^{AG}A}],$$

$$\sigma_t^{\eta B} = \frac{1}{\eta_t} [\eta_t(\psi_t^{Aa} - 1)\sigma^B + (\psi_t^{Aa} - \eta_t)\sigma_t^{qB} - \sigma_t^{N^{AG}B}],$$

$$\begin{aligned} \mu_t^\eta = & \frac{1}{1 - \theta^A[\psi^{Aa}(\eta_t) - 1]} \left\{ \frac{1 - \eta_t}{\eta_t} [1 + \theta^A(\psi_t^{Aa} - \eta_t)] + \theta^A(\psi_t^{Aa} - \eta_t) \right\} (\mu_t^{qK} - \mu_t^{NB}) \\ & + \frac{1}{2} \frac{\theta^A}{\eta_t} \psi^{Aa}(\eta_t) (\eta_t)^2 [(\sigma_t^{\eta A})^2 + (\sigma_t^{\eta B})^2] + \frac{\theta^A}{\eta_t} \sigma_t^{\psi^{Aa} - \eta, A} \sigma_t^{qKA} \\ & + \frac{\theta^A}{\eta_t} \sigma_t^{\psi^{Aa} - \eta, B} \sigma_t^{qKB} + (\sigma_t^{N^A A})^2 + (\sigma_t^{N^A B})^2 - \sigma_t^{N^A A} \sigma_t^{qKA} - \sigma_t^{N^A B} \sigma_t^{qKB} \end{aligned}$$

⁴⁹We could interpret the authority to levy taxes as a government's asset. Empirically, the U.S. government net worth has consistently been negative and have had a precipitous decline after the 1980s. Current level stands at less than -14 trillion dollars.

⁵⁰Since equilibrium private debt is an Ito process, government net worth is an Ito process as well. I scale it by global net worth as $\frac{dN_t^{IG}}{N_t^{IG}}$ blows up when the state variable approaches the autarky levels.

Figure 1.12: Welfare implication of the crowding-out policy

In terms of asset pricing, when country B produces both goods we have

$$\begin{aligned}
 & \frac{\bar{a}(P_t^a - P_t^b)}{q_t} + \sigma^A \sigma_t^{qA} - \sigma^B \sigma_t^{qB} + \chi \phi^A (q_t - 1) - \gamma^B \\
 &= \left[\frac{\psi_t^{Aa}}{\eta_t} (\sigma^A + \sigma_t^{qA}) - \frac{\sigma_t^{N^A G A}}{\eta_t} \right] (\sigma^A + \sigma_t^{qA}) + \left(\frac{\psi_t^{Aa}}{\eta_t} \sigma_t^{qB} - \frac{\sigma_t^{N^A G B}}{\eta_t} \right) \sigma_t^{qB} \\
 & \quad - \frac{\psi_t^{Ba} + \psi_t^{Bb}}{1 - \eta_t + \theta^A (\psi_t^{Aa} - \eta_t)} [(\sigma^B + \sigma_t^{qB})^2 + (\sigma_t^{qA})^2]
 \end{aligned}$$

and

$$\underline{a}P_t^a = \bar{a}P_t^b.$$

when countries specialize in production only the first equation holds.

In terms of consumption market clearing, we have:

$$\begin{aligned}
 \{ \eta_t \rho^A + [1 - \eta_t + \theta^A (\psi_t^{Aa} - \eta_t)] \rho^B \} q_t &= [(\bar{a} \psi_t^{Aa} + \underline{a} \psi_t^{Ba}) \bar{a} \psi_t^{Bb}]^{\frac{1}{2}} - \frac{q_t - 1}{\kappa} \\
 &\quad - (\phi^A + \phi^{AG} \theta^A) \chi q_t (\psi_t^{Aa} - \eta_t).
 \end{aligned}$$

capital market clearing conditions remains to be:

$$\psi_t^{Aa} + \psi_t^{Ab} + \psi_t^{Ba} + \psi_t^{Bb} = 1.$$

The left graph of Figure 1.12 plots ex-ante welfare of country A against different values of θ^A , the targeted ratio of public debt and private debt (set by government A). All other parameters are set as in Section 3. The surprising result is that the crowding-out policy actually lowers ex-ante welfare of the representative agent in the debtor country. The middle graph compares equilibrium (private) leverage for different levels of government debt issuance. Here we see that such policies only work when wealth share is low – for much of the state space (and recall that the economy spends most of the time closer to the stochastic steady state at $\eta = 0.5$ than $\eta = 0$) the crowding-out policy has virtually no effect on leverage – as a matter of fact, while it is hard to see on the graph, leverage slightly increases as the government becomes more active in enacting such crowding-out policies.

The rationale behind this has to do with the forward looking behavior of private agents. In particular, the right graph of Figure 1.12 plots the equilibrium government debt level as a function of the state variable η_t . Suppose we are at a point η_t that is on the downward sloping portion of the curve, say $\eta_t \in (0.2, 0.5)$. Positive shocks will lead to a higher wealth share of country A , meaning that the government will shrink its balance sheet, which means higher taxes to its agents. Thus, agents' taxes have a positive exposure to the fundamental shocks, and the (effectively) risk averse agents want to hedge such exposure. They do so precisely by taking on more leverage to earn more risk premia. In the end, while equilibrium interest rate does rise due to additional borrowings from the government, its effect on private leverage is completely canceled out by agents' motive to hedge this Brownian tax exposure – agents choose a slightly higher leverage despite a higher interest rate.

Similarly, crowding-out is pretty effective when η_t is on the upward sloping portion of the government debt curve, as in this region agents are already forced to take some exposure to the fundamental shocks via taxes. However, this is also the region where crowding-out is unnecessary, as the terms-of-trade hedge is switched off due to country B producing goods a . Thus it comes as no surprise that the overall tradeoff of such crowding-out policy results in a reduction in country A 's ex-ante welfare.

While this ineffectiveness of crowding out private leverage shares some ostensible similarities with the concept of Ricardian equivalence, they are fundamentally distinct in important ways. The classical Ricardian equivalence results from complete markets (and no friction in borrowing and lending) so that there is one effective intertemporal budget constraint for the representative agent as well as the government, and thus all equilibrium quantities will remain the same under alternative ways of financing government expenditures of the same discounted value. Here, markets are incomplete and net worth dynamics matter for equilibrium prices and allocations. Different fiscal policy rules, by generating different laws of motion of real interest rate and differentially state-contingent hedging motive discussed above, will also affect equilibrium variables through their impact on net worth dynamics. It just so happens that with log agents and this particular fiscal policy of targeting a constant fraction of private debt level, in some part of the state space the two effects from fiscal policy (cost of capital and tax exposure hedging) roughly cancel each other out, which leaves private leverage virtually unchanged.

It is worth pointing out that the effectiveness of a general crowding-out policy will likely to improve if we allow the government to change the targeted ratio of government debt to private debt as the state variable changes over time. We can indeed show that a pol-

icy for which government debt level goes “in the opposite direction” of private debt level can improve welfare. Thus, the analysis here points to a more nuanced picture than that in Greenwood et al (2015): it is not only the amount of government debt, but also the way government (dynamically) manages such debt, that matter crucially for the effectiveness of crowding-out policies on the basis of correcting externalities.

1.6.2 Redistributive role of government debt

Government bond can be used to increase welfare via a seigniorage-redistribution channel, as long as it has higher liquidity relative to private bond. It entails the government running a balanced budget and issuing public debt to buy a fraction $\theta^I \geq 0$ of the economy’s private debt. Due to liquidity differentials, private creditors demand a lower return from public bonds, leading to seigniorage profit to the government, which can be redistributed back to its agents. For the debtor country, such policies facilitate the recapitalization process of its agents, and hence improve the stability of the global economy. Thus, to the extent that government bonds have higher liquidity than private sector bonds, there is a stability aspect of issuing public debt via this redistributive channel, which echos the result in Greenwood et al (2015) that the prerequisite for a government to optimally issue public debt in reaction to (excessive) private debt creation is that it has a comparative advantage in doing so. In addition, more liquidity is injected into the economy this way (which means more efficient financing of investment opportunities) without introducing more private indebtedness – that is, public liquidity provided this way works through the intensive margin (how much liquidity is generated per dollar of private debt) rather than

the extensive margin (how much private debt is generated)⁵¹.

Formally, suppose the government of country I can issue public debt up to a fraction $\theta^I \geq 0$ of the (equilibrium) level of private debt in the economy. Also, suppose that government bond is more liquid than its country's private bond, as measured by a higher resaleability parameter ϕ^{IG} , for $I \in \{A, B\}$. I restrict attention to the empirically relevant case where $\theta^A + \theta^B < 1$, so that there is at least some private debt held directly by private agents⁵². Denote the return of public bond of country I as dr_t^{IG} , which is priced from equilibrium real interest rate (interest rate at which agents lend privately to each other) through the no-arbitrage relationship⁵³:

$$\frac{dr_t^{IG}}{dt} + \chi\phi^{IG}(q_t - 1) = \frac{dr_t^F}{dt} + \chi\phi^I(q_t - 1)$$

This condition makes creditors indifferent between holding private bond or exchanging it for public bond, which is necessary for an equilibrium where private and public debt coexist. As long as $\phi^{IG} > \phi^I$, $\frac{dr_t^{IG}}{dt} - \frac{dr_t^F}{dt} < 0$, i.e., government bond can be issued at

⁵¹It is possible for public liquidity to work through extensive margin as well. Essentially it requires public bond to be priced so low so that agents lend privately solely for the purpose of swapping private debt with public debt to capture its higher returns. That is, instead of private bond return pinning down public bond return, it is the other way around that public bond return determines equilibrium level of private credit and its terms. Governments in general have no incentive to go to such extremes as it lowers their seigniorage profit while further fueling instability.

⁵²As opposed to $\theta^A + \theta^B = 1$, which is both empirically unrealistic and unlikely to be optimal. See the previous footnote.

⁵³Here I assume that there is a competitive market for buying government bond (and selling private bond), so that all surplus accrues to the government in the form of a flow seigniorage profit, which is redistributed to agents. Nonetheless, aggregate dynamics (such as that of interest rate and asset price) will change under this policy due to general equilibrium effects. For example, price of capital goes down as it becomes less scarce, and there is also some minor changes to equilibrium real interest rate. More importantly however, if the government has less than 100% of bargaining power, then real interest rate will decrease more significantly. Essentially, agents anticipate that some of their bond holdings may be sold to the government at favorable terms, which makes agents more willing to extend credit ex-ante. Thus in general, the model predicts there will be an increase in leverage, a drop in real interest rate, a drop in (real) asset prices, and an increase in investment (growth) once the government implements policies of this kind.

a discount relative to private bond because of its higher liquidity, thereby allowing the governments to earn seigniorage profits. I assume governments redistribute their seigniorage profits to their own agents on a pro rata basis. Again, there is no expenditures to be financed by the government, and its balance sheet is illustrated below.

Asset	Liability
Private bond: $\theta^{AG}(\psi_t^{Aa} - \eta_t)q_t K_t$	Government debt: $\theta^{AG}(\psi_t^{Aa} - \eta_t)q_t K_t$

Figure 1.13 shows the welfare implications of public bond issuance, using parameters from Section 3. Here we see that in contrast to private liquidity, public liquidity always increases welfare. This welfare gain comes from the redistributive role of government bonds which enhances stability, and the increased internal liquidity available to be used to finance investment opportunities. It is worth pointing out that while the debtor country's government bond issuance increases stability, the creditor country's government debt issuance does not, because its seigniorage is redistributed to the creditor country's agents (making the rich richer). However, it does pump more liquidity into the economy and is by itself a (global) welfare enhancing policy – lowering creditor country's government debt issuance will further improve stability (by making the rich less rich), but will also lower global welfare due to less internal liquidity available. That is, it is possible that private liquidity lowers welfare while public liquidity improves it, because the former acts via both the intensive margin (higher liquidity per dollar of private debt) and extensive margin (more dollars of private debt), while the latter does not alter equilibrium private leverage, thereby working only through the intensive margin without adversely affecting the debtor

Figure 1.13: Welfare implications of seignorage-redistribution channel

country's terms-of-trade hedge.

1.6.3 Risk-premia channel of government debt

The previous section attributes the spread between public and private bond to be solely coming from liquidity differentials in financing investment opportunities, i.e., in normal times. In reality, government bond is also valuable due to the unconditionality of its liquidity i.e., liquidity in crisis times, as evidenced by various flight-to-liquidity episodes. Indeed, Longstaff (2002) terms this the “flight-to-liquidity premium”, and shows that it can be quite large in magnitude. To the extent that debt issuers anticipate the occurrences of such flight-to-liquidity episodes, they may decide to hold certain amount of government bond ex-ante as insurance against inefficient fire-sales when lenders refuse to roll over their debt. This insurance comes at a (opportunity) cost which is positively correlated with the government bond yield spread – a high premium in government bond yields increases the opportunity cost of insurance, and as a result there will be lower leverage in equilibrium.

Governments then, by managing the supply of government bond, can potentially regulate private sector leverage – an ample supply increases its return, and thus lowers the opportunity cost of holding government bond. This is what I call the risk-premia channel of government debt. As will be illustrated below, in an international setting the ability of the debtor country's government to control private leverage via public debt issuance management is hampered by actions of the other government. In particular, if one government reduces its bond issuance with the intention of driving down government bond return and hence driving up the cost of private leverage, the other government has an incentive to offer a slightly higher return to its debt and capture all private demand for government bond, thereby increasing its seigniorage profit. This lack of coordination may lead to a Nash equilibrium with inefficiently high levels of government bond and inefficiently low levels of risk premia, and hence socially excessive leverage.

To illustrate this channel more formally, I extend the model by adding funding shocks which capture the rollover risks borrowers face when they issue short-term debt. When a funding shock hits, borrowers must immediately redeem a fraction of their debt. Capital can only be sold at low fire-sale prices at those times, but borrowers could self-insure by holding government bond which is immune to fire-sales. Suppose funding shocks arrive according to a Poisson process with exogenous intensity ω . These shocks are systematic in nature and cannot be diversified away within the debtor country. When a shock hits, a borrower must immediately redeem a fraction $\frac{\varphi}{1+\varphi}$ of her debt, where $\varphi > 0$. Due to exogenous frictions, capital can only be sold for a fraction $1 - d \in (0, 1)$ of its value, but there are no fire sale discounts for government bonds issued by either country⁵⁴. As

⁵⁴They do not have to be equally effective in insuring against funding shocks. The results remain if bonds issued by the two governments have different fire-sale discounts, as long as the discounts are smaller than

in Drechsler et al (2018) I look at parameters such that agents have incentive to hold government bond to fully self-insure against such funding shocks.

Suppose government I can issue public debt of quantity at most $\theta^I = \frac{\varphi}{1+\varphi}$ fraction of the private debt level in equilibrium, which is then used to finance the purchase of private bond⁵⁵, for $I \in \{A, B\}$ ⁵⁶. That is, either country alone has the ability to issue enough public debt to satisfy the demand from debtors as insurance against funding shocks. As before, seigniorage profits are redistributed to their own country's agents proportional to their net worth. When the two governments compete with each other, the only Nash equilibrium would be where each exhausts its debt issuance capacity of θ^I fraction of private debt, at a return that makes the creditors indifferent between holding government bond or private bond. This is because otherwise a government always has the incentive to offer a slightly higher return and attract all demand for government bond to itself. Given our parameter choices, if both countries issue the maximum amount of debt allowed, then in equilibrium creditors will be the marginal pricers of government debt, so the return that government bond has to offer is pinned down by the indifference condition as in Section 6.2⁵⁷. When both governments exhaust their debt capacities at this rate, further price undercutting will not lead to additional seigniorage gains and hence the economy is indeed in a Nash equilibrium. However, if the two governments coordinate, potentially each could

that of physical capital. Here for simplicity we look at the symmetric case where there is no discount for either country's public debt.

⁵⁵Thus, the structure of government balance sheet is identical to that in section 7.2.

⁵⁶A quantity limit is used for simplicity. Obviously if $\theta^A + \theta^B < \frac{\varphi}{1+\varphi}$ then the only equilibrium would be financial autarky. As long as $\theta^A + \theta^B > \frac{\varphi}{1+\varphi}$, the welfare loss from lack of coordination illustrated below will exist. I choose the particular case of $\theta^A = \theta^B = \frac{\varphi}{1+\varphi}$ for simplicity.

⁵⁷Borrowers will only demand government bond up to $\frac{\varphi}{1+\varphi}$ fraction of their debt level. So if each government issues government bond at level $\frac{\varphi}{1+\varphi}$ fraction of private debt, the additional supply must be absorbed by private creditors.

reduce its public bond supply so that together they only issue in total government debt of quantity $\frac{\varphi}{1+\varphi}$ fraction of private debt. Now the marginal pricers will be the two governments, and debtor country agents adjust their leverage based on the return of government bonds set (jointly) by both governments.

Lack of coordination in this setting means that government of the debtor country loses the ability to manage its country's leverage via the impact of its debt supply on risk premia. In the Nash equilibrium, the opportunity cost of holding government bond is fixed implicitly by the foreign private creditors and the debtor country's government is unable to alter it. Thus, in cases where the pecuniary externality is strong and private credit is sufficiently excessive, such inability will lower global economic welfare. To see this, using the same parameters in section 3, and setting ω at 0.3, Figure 1.14 plots the ex-ante welfare under coordination for different choices of government bond spread (the opportunity cost of holding government bond), as well as the level of ex-ante welfare in Nash equilibrium (without coordination). When governments coordinate their debt issuances but choose very low levels of opportunity cost, private credit is barely contained (even less so compared with the Nash equilibrium). But as higher opportunity cost of leverage is coordinated between the two governments, welfare goes up and eventually surpasses that in the Nash equilibrium case. Of course, as governments continue to increase this opportunity cost, eventually the economy will resemble that in financial autarky, which could feature lower welfare compared to the non-coordination case. But for all parameters considered in this paper, an optimally chosen government bond spread with coordination always further increases welfare relative to the non-coordination case.

International coordination has become increasingly important in a financially inte-

Figure 1.14: Welfare comparison of coordination vs. Nash via the risk-premia channel

grated world economy. While most work so far focus on monetary policy coordination (Mishra and Rajan 2016; Shin 2015) and macroprudential policy coordination (Chen and Phelan 2017; Agenor and Pereira da Silva 2019), this paper points to a channel where international fiscal policy coordination can be beneficial.

1.7 Conclusion

I theoretically illustrate the tradeoff of growth and stability concerning liquidity-providing private bond in an international macro framework. I show that private bond liquidity increases economic growth at the expense of less economic stability. The benefit accrues to the creditor country in the form of more efficient financing of investment opportunities, while the cost is largely borne by the debtor country due to the worsening of pecuniary externality that results in a worse terms-of-trade hedge. Numerical experiments, based on a parameterized version of the model, show that bond liquidity can lower global welfare, especially for economies with low fundamental volatilities and low credit market frictions. In

terms of macroprudential regulation, bond liquidity is shown to significantly relax the optimal tightness of leverage constraints, and moderately increase the optimal tax on credit. International coordination of macroprudential policies benefits high-volatility countries at the expense of low-volatility countries, while countries of different sizes (but similar volatilities) both benefit from coordination. In terms of the potential role of government bonds, this paper shows that government bond generally improves welfare via a redistributive channel, but the effectiveness of its crowding-out channel depends crucially on how government manages its balance sheet over time. Finally, effectiveness of the risk-premia channel of government bonds depends on the degree of international coordination.

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Appendix A

Proofs of Propositions in Chapter 1

A.1 Proof of Proposition 1

I prove the proposition for η 's such that country A is borrowing from country B, and there is full specialization of production. All other regions are proved almost identically. I also assume a common discount factor $\rho^A = \rho^B = \rho$ to save some space.

First, assume the debt dynamics are as stated in proposition 1, we write the problem of a representative agent in country A as:

$$\rho V^A(n_t^A, \eta_t) = \max_{\{\zeta_t^A, x_t^{Aa}, x_t^{Ab}\}} \log(\underbrace{\zeta_t^A n_t^A}_{c_t^A}) + \mathbb{E}[dV_t^A]$$

$$s.t. \frac{dn_t^A}{n_t^A} = x_t^{Aa} dR_t^{Aa} + x_t^{Ab} dR_t^{Ab} + (1 - x_t^{Aa} - x_t^{Ab}) dr_t^{AF} - \zeta_t^A dt + \tau_t dt; n_t^A, x_t^{Aa}, x_t^{Ab} \geq 0$$

It is a standard result that there exists an equilibrium where $V^A(n_t^A, \eta_t)$ takes the form of:

$$V^A(n_t^A, \eta_t) = \frac{\log(n_t^A)}{\rho} + h^A(\eta_t),$$

where $h^A(\cdot)$ is twice differentiable and represents investor's stochastic investment opportunities. Plugging in this form of value function, and applying Ito's lemma, we have:

$$\begin{aligned} \rho V^A(n_t^A, \eta_t) &= \max_{\{\zeta_t^A, x_t^{Aa}, x_t^{Ab}\}} \log(n_t^A) + \log(\zeta_t^A) + \\ &\frac{1}{\rho} \left\{ \frac{dr_t^{AF}}{dt} + (x_t^{Aa} + x_t^{Ab}) \left(\frac{aP_t^a - \iota_t^e}{q_t} + \Phi(\iota_t^e) + \mu_t^q - \delta + \sigma^A \sigma_t^{qA} - \frac{dr_t^{AF}}{dt} \right) \right. \\ &\quad \left. - \zeta_t^A - \frac{1}{2} (x_t^{Aa} + x_t^{Ab})^2 [(\sigma^A + \sigma_t^{qA})^2 + (\sigma_t^{qB})^2] \right\}. \end{aligned}$$

Taking first order conditions, we have:

$$\zeta_t^A = \rho,$$

$$\frac{\bar{a}P_t^a - \iota_t^e}{q_t} + \Phi(\iota_t^e) + \mu_t^q - \delta + \sigma^A \sigma_t^{qA} - \frac{dr_t^{AF}}{dt} = (x_t^{Aa} + x_t^{Ab}) [(\sigma^A + \sigma_t^{qA})^2 + (\sigma_t^{qB})^2],$$

$$\bar{a}P_t^a \geq \underline{a}P_t^b, \text{ with equality iff } x_t^{Ab} > 0.$$

The LOM of aggregate wealth of country A is:

$$\begin{aligned} \frac{dN_t^A}{N_t^A} &= \left[\frac{dr_t^F}{dt} + x_t^{Aa} \left(dR_t^{Aa} - \frac{dr_t^{AF}}{dt} \right) + x_t^{Ab} \left(dR_t^{Ab} - \frac{dr_t^{AF}}{dt} \right) - \zeta_t^A + \tau_t \right] dt \\ &\quad + (x_t^{Aa} + x_t^{Ab}) (\sigma^A + \sigma_t^{qA}) dZ_t^A + (x_t^{Aa} + x_t^{Ab}) \sigma_t^{qB} dZ_t^B. \end{aligned}$$

Plugging in FOCs, we get:

$$\begin{aligned} \frac{dN_t^A}{N_t^A} &= \underbrace{\left[\frac{dr_t^{AF}}{dt} + (x_t^{Aa} + x_t^{Ab})^2 [(\sigma^A + \sigma_t^{qA})^2 + (\sigma_t^{qB})^2] - \rho + \tau_t \right]}_{\mu_t^{NA}} dt + \\ &\quad \underbrace{(x_t^{Aa} + x_t^{Ab}) (\sigma^A + \sigma_t^{qA})}_{\sigma_t^{NA A}} dZ_t^A + \underbrace{(x_t^{Aa} + x_t^{Ab}) \sigma_t^{qB}}_{\sigma_t^{NA B}} dZ_t^B. \end{aligned}$$

For agents in country B, we write their HJB as:

$$\rho V(n_t^B, \eta_t) = \max_{\{\zeta_t^B, x_t^{Ba}, x_t^{Bb}\}} \log(c_t^B) + \mathbb{E}[dV_t^B],$$

where now the expectation is taken over the bond-buyer matching shock as well as shocks to efficiency of physical capital, dZ_t^A and dZ_t^B . In particular:

$$\rho V(n_t^B, \eta_t) = \max_{\{\zeta_t^B, x_t^{Ba}, x_t^{Bb}\}} \log(c_t^B) + (1 - \chi)\mathbb{E}[dV_t^{Bn}] + \chi\mathbb{E}[dV_t^{Bi}]$$

$$s.t. \frac{dn_t^{Bn}}{n_t^B} = (1 - x_t^{Ba} - x_t^{Bb})d\tilde{r}_t^{BF} + x_t^{Ba}dR_t^{Ba} + x_t^{Bb}dR_t^{Bb} - \zeta_t^B dt + \tau_t dt,$$

$$\frac{dn_t^{Bi}}{n_t^{Bi}} = (1 - x_t^{Ba} - x_t^{Bb})([d\tilde{r}_t^{BF} + \phi^b(q_t - 1)dt] + x_t^{Ba}dR_t^{Ba} + x_t^{Bb}dR_t^{Bb} - \zeta_t^B dt + \tau_t dt,$$

$$x_t^{Ba}, x_t^{Bb}, n_t^B \geq 0.$$

where:

$$d\tilde{r}_t^{BF} = dr_t^F - \gamma^B \mathbf{1}_{B \text{ is lending}}$$

Again, we conjecture and verify the value function takes the form:

$$V^B(n_t^B, \eta_t) = \frac{\log(n_t^B)}{\rho} + h^B(\eta_t),$$

Plugging in and using Ito's lemma, we have:

$$\begin{aligned} \rho V(n_t^B, \eta_t) = & \max_{\{\zeta_t^B, x_t^{Ba}, x_t^{Bb}\}} \log(\zeta_t^B n_t^B) + \frac{1}{\rho} \left[\frac{d\tilde{r}_t^{BF}}{dt} + \chi \phi^A(q_t - 1) \right] + \\ & \frac{1}{\rho} \{ (x_t^{Ba} + x_t^{Bb}) \left[\frac{aP_t^a - \iota_t^e}{q_t} + \Phi(\iota_t^e) + \mu_t^q - \delta + \sigma^B \sigma_t^{qB} - \frac{d\tilde{r}_t^{BF}}{dt} - \chi \phi^A(q_t - 1) \right] \right. \\ & \left. - \zeta_t^A - \frac{1}{2} (x_t^{Ba} + x_t^{Bb})^2 [(\sigma^B + \sigma_t^{qB})^2 + (\sigma_t^{qA})^2] \}. \end{aligned}$$

From here, we see that the problem of country B is equivalent to a problem where they face a fictitious interest rate of $\frac{dr_t^{BF, fictitious}}{dt} = \frac{d\tilde{r}_t^{BF}}{dt} + \chi\phi^A(q_t - 1)$, which is their expected return to holding bond. country B agents are risk-neutral towards bond-buyer matching risk, despite them being risk averse, due to the nature of the shock: for any time interval of positive measure, an agent is matched with a bond buyer χ fraction of the time almost surely.

Taking first order conditions, we have:

$$\zeta_t^B = \rho,$$

$$\frac{aP_t^b}{q_t} + \mu_t^q - \delta + \sigma^B \sigma_t^{qB} - \frac{dr_t^{BF, fictitious}}{dt} = (x_t^{Ba} + x_t^{Bb})[(\sigma^B + \sigma_t^{qB})^2 + (\sigma_t^{qA})^2],$$

$$\underline{a}P_t^a \leq \bar{a}P_t^b, \text{ with equality iff } x_t^{Ba} > 0.$$

We write the LOM of aggregate wealth of country B as:

$$\begin{aligned} \frac{dN_t^B}{N_t^B} &= (1 - \chi) \frac{dN_t^{Bn}}{N_t^B} + \chi \frac{dN_t^{Bi}}{N_t^B} \\ &= d\tilde{r}_t^{BF} + \chi\phi^A(q_t - 1)dt \\ &\quad + [x_t^{Ba}(dR_t^{Ba} - [\frac{d\tilde{r}_t^{BF}}{dt} + \chi\phi^b(q_t - 1)]) + x_t^{Bb}(dR_t^{Bb} - [\frac{d\tilde{r}_t^{BF}}{dt} + \chi\phi^b(q_t - 1)])]dt \\ &\quad - \zeta_t^B dt + \tau_t dt + \\ &\quad (x_t^{Ba} + x_t^{Bb})[\sigma_t^{qA} dZ_t^A + (\sigma + \sigma_t^{qB})dZ_t^B]. \end{aligned}$$

Plugging in FOCs, we have:

$$\frac{dN_t^B}{N_t^B} = \underbrace{[\frac{d\tilde{r}_t^{BF}}{dt} + \chi\phi^A(q_t - 1) + (x_t^{Ba} + x_t^{Bb})^2[(\sigma^B + \sigma_t^{qB})^2 + (\sigma_t^{qA})^2] - \rho + \tau_t]}_{\mu_t^{NB}} dt +$$

$$\underbrace{(x_t^{Ba} + x_t^{Bb})\sigma_t^{qA}}_{\sigma_t^{N^BA}} dZ_t^A + \underbrace{(x_t^{Ba} + x_t^{Bb})(\sigma + \sigma_t^{qB})}_{\sigma_t^{N^BB}} dZ_t^B.$$

The LOM for aggregate wealth of the economy, $q_t K_t$, can then be derived from:

$$\frac{d(q_t K_t)}{q_t K_t} = \frac{d(N_t^A + N_t^B)}{q_t K_t} = \frac{dN_t^A}{q_t K_t} + \frac{dN_t^B}{q_t K_t} = \eta_t \cdot \frac{dN_t^A}{N_t^A} + (1-\eta_t) \cdot \frac{dN_t^B}{N_t^B}$$

from which we pin down $\mu_t^{qK}, \sigma_t^{qK,A}, \sigma_t^{qK,B}$ as weighted sum of $\mu_t^{N^A}$ and $\mu_t^{N^B}$, $\sigma_t^{N^AA}$ and $\sigma_t^{N^AB}$ and $\sigma_t^{N^BA}$ and $\sigma_t^{N^BB}$, respectively.

Since $\eta_t \equiv \frac{N_t^A}{q_t K_t}$, using Ito's lemma we have:

$$\begin{aligned} \frac{d\eta_t}{\eta_t} = & \underbrace{[\mu_t^{N^A} - \mu_t^{qK} + (\sigma_t^{qK,A})^2 + (\sigma_t^{qK,B})^2 - \sigma_t^{N^AA} \sigma_t^{qK,A} - \sigma_t^{N^AB} \sigma_t^{qK,B}]}_{\mu_t^\eta} dt + \\ & \underbrace{(\sigma_t^{N^AA} - \sigma_t^{qK,A})}_{\sigma_t^{\eta A}} dZ_t^A + \underbrace{(\sigma_t^{N^AB} - \sigma_t^{qK,B})}_{\sigma_t^{\eta B}} dZ_t^B. \end{aligned}$$

We still need to derive σ_t^{qA} and σ_t^{qB} . Again, they are derived from Ito's lemma:

$$\begin{aligned} \sigma_t^{qA} = \frac{q'_t(\eta_t)\eta_t}{q_t(\eta_t)}\sigma_t^{\eta A} & \implies \sigma_t^{qA} = \frac{q'_t(\eta_t)\psi_t^{Aa}(1-\eta_t)}{q(\eta_t) - [\psi_t^{Aa} - \eta_t]q'_t(\eta_t)}\sigma^A, \\ \sigma_t^{qB} = \frac{q'_t(\eta_t)\eta_t}{q_t(\eta_t)}\sigma_t^{\eta B} & \implies \sigma_t^{qB} = -\frac{q'_t(\eta_t)\eta_t(1-\psi_t^{Aa})}{q(\eta_t) - [\psi_t^{Aa} - \eta_t]q'_t(\eta_t)}\sigma^B. \end{aligned}$$

The capital market clearing condition is standard; consumption market clearing follows from the fact that the aggregate amount of debt in the economy is $q_t K_t(\psi_t^{Aa} - \eta_t)$, so aggregate amount of internal liquidity is $\chi\phi_t^A q_t K_t(\psi_t^{Aa} - \eta_t)$.

Finally, we need to prove our assumption that when $\eta_t \in (0, 0.5)$, the only active debt market is for credit to flow from country B to country A. First we check for local deviations. Since we have interior optimality in the equilibrium defined in the text, the

shadow value of a marginal dollar for a lender is $1 + [\frac{dr_t^F}{dt} - \gamma + \chi\phi^A(q_t - 1)]dt$. Similarly, the shadow value of a marginal dollar for a borrower (country A agent) in equilibrium is $1 + \frac{dr_t^F}{dt}dt$. If $\gamma = 0$, the latter is strictly less than the shadow value of a dollar for a country B agent, thus, absent frictions an individual country A agent (agent A henceforce) and an individual country B agent (agent B henceforce) always have an incentive to work out a contract such that agent A lends a marginal dollar to agent B, at any interest rate $\frac{dr_t^{F, fictitious}}{dt} \in (\frac{dr_t^F}{dt}, \frac{dr_t^F}{dt} + \chi\phi^A(q_t - 1))$. Once we introduce a cost of lending γ_t for agent A, however, there is no gains from trade as long as $\gamma_t \geq \frac{1}{2} \cdot \chi\phi^A(q_t - 1)$, which is Assumption 1.

Now we check for deviations such that an agent deviates completely from the conjectured credit decision, that is, if country A agent does not borrow at all and only lend, and/or country B agent does not lend at all and only borrow. First we consider a deviating agent in country B who only borrows. Her HJB is written as:

$$\begin{aligned} \rho V^{Borrow}(n_t^B) = & \max_{\{\zeta_t^B, x_t^{Bb, borrow}\}} \log(n_t^B) + \log(\zeta_t^B) + \frac{\frac{dr_t^{F, fictitious}}{dt}}{\rho} + \\ & \frac{1}{\rho} \{ x_t^{Bb, borrow} \left(\frac{aP_t^b - \iota_t}{q_t} + \Phi(\iota_t) + \mu_t^q - \delta + \sigma^B \sigma_t^{qB} - \frac{dr_t^{F, fictitious}}{dt} \right) \\ & - \zeta_t^B - \frac{1}{2} (x_t^{Bb, borrow})^2 [(\sigma^B + \sigma_t^{qB})^2 + (\sigma_t^{qA})^2] \}, \end{aligned}$$

where $\frac{dr_t^{F, fictitious}}{dt}$ is a fictitious interest rate contracted by a deviating agent in country B with a deviating agent in country A. We aim to show that there is no such interest rate that achieves a Pareto improvement for both deviating agents, therefore there is no incentive to deviate, and our conjectured debt market dynamics is validated.

To do this, we first write the deviating B-agent's FOC:

$$\frac{aP_t^b - \iota_t}{q_t} + \Phi(\iota_t) + \mu_t^q - \delta + \sigma^B \sigma_t^{qB} - \frac{dr_t^{F, \text{fictitious}}}{dt} = (x_t^{Bb, \text{Borrow}})[(\sigma^B + \sigma_t^{qB})^2 + (\sigma_t^{qA})^2].$$

Plugging this back to her HJB, we get:

$$\begin{aligned} \rho V^{\text{Borrow}}(n_t^B) = & \log(n_t^B) + \log(\rho) + \frac{\frac{dr_t^{F, \text{fictitious}}}{dt}}{\rho} \\ & + \frac{\frac{1}{2}(x_t^{Bb, \text{borrow}})^2[(\sigma^B + \sigma_t^{qB})^2 + (\sigma_t^{qA})^2] - \rho}{\rho} \end{aligned}$$

Recall the FOC for agent B in the conjectured equilibrium (where $\frac{dr_t^F}{dt}$ is the (equilibrium) interest rate at which country B agents lend to country A agents in the conjectured equilibrium):

$$\begin{aligned} \frac{aP_t^b - \iota_t}{q_t} + \Phi(\iota_t) + \mu_t^q - \delta + \sigma^B \sigma_t^{qB} - \frac{dr_t^F}{dt} + \gamma - \chi \phi^A(q_t - 1) \\ = (x_t^{Bb, \text{Lend}})[(\sigma^B + \sigma_t^{qB})^2 + (\sigma_t^{qA})^2]. \end{aligned}$$

Similarly for the conjectured equilibrium, we have:

$$\begin{aligned} \rho V^{\text{Lend}}(n_t^B) = & \log(n_t^B) + \log(\rho) + \frac{\frac{dr_t^F}{dt} - \gamma + \chi \phi^A(q_t - 1)}{\rho} \\ & + \frac{\frac{1}{2}(x_t^{Bb, \text{Lend}})^2[(\sigma^B + \sigma_t^{qB})^2 + (\sigma_t^{qA})^2] - \rho}{\rho} \end{aligned}$$

Thus, given $\frac{dr_t^{F, \text{fictitious}}}{dt}$, this agent deviates iff:

$$\begin{aligned} \frac{dr_t^F}{dt} - \gamma + \chi \phi^A(q_t - 1) + \frac{1}{2}(x_t^{Bb, \text{lend}})^2[(\sigma^B + \sigma_t^{qB})^2 + (\sigma_t^{qA})^2] \\ \leq \frac{dr_t^{F, \text{fictitious}}}{dt} + \frac{1}{2}(x_t^{Bb, \text{borrow}})^2[(\sigma^B + \sigma_t^{qB})^2 + (\sigma_t^{qA})^2], \end{aligned}$$

$\Longleftrightarrow$

$$\frac{dr_t^F}{dt} - \gamma + \chi\phi^A(q_t - 1) - \frac{dr_t^{F, \text{fictitious}}}{dt} \leq \frac{1}{2}(x_t^{Bb, \text{borrow}} + x_t^{Bb, \text{lend}})(x_t^{Bb, \text{borrow}} - x_t^{Bb, \text{lend}})[(\sigma^B + \sigma_t^{qB})^2 + (\sigma_t^{qA})^2].$$

Subtracting the two FOCs, we have:

$$(x_t^{Bb, \text{borrow}} - x_t^{Bb, \text{lend}})[(\sigma^B + \sigma_t^{qB})^2 + (\sigma_t^{qA})^2] = -\frac{dr_t^{F, \text{fictitious}}}{dt} + \frac{dr_t^F}{dt} - \gamma + \chi\phi^A(q_t - 1),$$

Plugging in:

$$\frac{dr_t^F}{dt} - \gamma + \chi\phi^A(q_t - 1) - \frac{dr_t^{F, \text{fictitious}}}{dt} \leq \frac{1}{2}(x_t^{Bb, \text{borrow}} + x_t^{Bb, \text{lend}})\left[-\frac{dr_t^{F, \text{fictitious}}}{dt} + \frac{dr_t^F}{dt} - \gamma + \chi\phi^A(q_t - 1)\right],$$

$$\Longleftrightarrow$$

$$2 \leq (x_t^{Bb, \text{borrow}} + x_t^{Bb, \text{lend}})$$

$$2[(\sigma^B + \sigma_t^{qB})^2 + (\sigma_t^{qA})^2] \leq 2\left[\frac{aP_t^b - \iota_t}{q_t} + \Phi(\iota_t) + \mu_t^q - \delta + \sigma^B \sigma_t^{qB}\right] - \frac{dr_t^{F, \text{fictitious}}}{dt} - \frac{dr_t^F}{dt} + \gamma - \chi\phi^A(q_t - 1)$$

$$\Longleftrightarrow$$

$$\frac{dr_t^{F, \text{fictitious}}}{dt} + \frac{dr_t^F}{dt} - \gamma + \chi\phi^A(q_t - 1) \leq 2\left\{\left[\frac{aP_t^b - \iota_t}{q_t} + \Phi(\iota_t) + \mu_t^q - \delta + \sigma^B \sigma_t^{qB}\right] - [(\sigma^B + \sigma_t^{qB})^2 + (\sigma_t^{qA})^2]\right\}$$

Using the FOC for the conjectured equilibrium,

$$\begin{aligned} & \frac{aP_t^b - \iota_t}{q_t} + \Phi(\iota_t) + \mu_t^q - \delta + \sigma^B \sigma_t^{qB} - [(\sigma^B + \sigma_t^{qB})^2 + (\sigma_t^{qA})^2] \\ &= \frac{dr_t^F}{dt} - \gamma + \chi\phi^A(q_t - 1) + (x_t^{Bb, \text{lend}} - 1)[(\sigma^B + \sigma_t^{qB})^2 + (\sigma_t^{qA})^2]. \end{aligned}$$

Plugging in,

$$\begin{aligned}
\frac{dr_t^{F,fictitious}}{dt} + \frac{dr_t^F}{dt} - \gamma + \chi\phi^A(q_t - 1) &\leq 2\left\{\frac{dr_t^F}{dt} - \gamma + \chi\phi^A(q_t - 1) \right. \\
&\quad \left. + (x_t^{Bb,Lend} - 1)[(\sigma^B + \sigma_t^{qB})^2 + (\sigma_t^{qA})^2]\right\} \\
\frac{dr_t^{F,fictitious}}{dt} &\leq \frac{dr_t^F}{dt} - \gamma_t + \chi\phi^A(q_t - 1) - \underbrace{2(1 - x_t^{Bb,lend})[(\sigma^B + \sigma_t^{qB})^2 + (\sigma_t^{qA})^2]}_{>0} \\
&\leq \frac{dr_t^F}{dt} + \frac{1}{2}\chi\phi^A(q_t - 1) \\
&\leq \frac{dr_t^F}{dt} + \gamma_t.
\end{aligned}$$

Thus, an agent in country A considering local deviation will not participate in the lending market.

If an agent in country A deviates completely and only lend to this deviating country B agent, we use the same steps as above and find that for her to prefer lending (only) to borrowing (only), her lending rate must satisfy:

$$\begin{aligned}
\frac{dr_t^{F,fictitious}}{dt} &\geq \frac{dr_t^F}{dt} + \gamma_t + 2\left(\frac{\psi_t^{Aa}}{\eta_t} - 1\right)[(\sigma^A + \sigma_t^{qA})^2 + (\sigma_t^{qB})^2] \\
&\geq \frac{dr_t^F}{dt} + \gamma_t.
\end{aligned}$$

Thus, as long as lending cost for country A agents satisfy Assumption 1, there is no market interest rate for country A agents to lend to country B agents that achieves a Pareto improvement over the conjectured equilibrium. Therefore, this debt market will stay inactive.

A.2 Proof of Proposition 2

Again, I derive the equilibrium dynamics for $\eta \in (0, 0.5)$, where country A is borrowing from country B.

Country A issues public debt of amount a constant fraction θ^A of the equilibrium level of private debt, that is:

$$D_t^{AG} \equiv \theta^A(\psi_t^{Aa} - \eta_t)q_t K_t$$

First, using Ito's lemma,

$$d(\psi_t^{Aa} - \eta_t) = [\psi^{Aa'}(\eta_t) - 1]d\eta_t + \frac{1}{2}\psi^{Aa''}(\eta_t)(d\eta_t)^2$$

Continue using A's wealth share as the state variable:

$$\eta_t \equiv \frac{N_t^A}{q_t K_t}$$

where:

$$\frac{d\eta_t}{\eta_t} = \mu_t^\eta dt + \sigma_t^{\eta A} dZ_t^A + \sigma_t^{\eta B} dZ_t^B$$

Also, denote

$$\frac{dq_t}{q_t} = \mu_t^q dt + \sigma_t^{qA} dZ_t^A + \sigma_t^{qB} dZ_t^B$$

where using the fact that q_t is a function of the state variable:

$$\frac{dq_t}{q_t} = \frac{q'(\eta_t)}{q(\eta_t)}\eta_t\mu_t^\eta dt + \frac{1}{2}\frac{q''(\eta_t)}{q(\eta_t)}\eta_t^2[(\sigma_t^{\eta A})^2 + (\sigma_t^{\eta B})^2]dt + q'(\eta_t)\eta_t\sigma_t^{\eta A}dZ_t^A + q'(\eta_t)\eta_t\sigma_t^{\eta B}dZ_t^B$$

This will be used later to derive interest rates.

Plugging in the dynamics of η_t , we have:

$$\begin{aligned} d(\psi_t^{Aa} - \eta_t) = & \underbrace{\{[\psi^{Aa'}(\eta_t) - 1]\eta_t\mu_t^\eta + \frac{1}{2}\psi^{Aa''}(\eta_t)\eta_t^2[(\sigma_t^{\eta A})^2 + (\sigma_t^{\eta B})^2]\}}_{\mu_t^{\psi^{Aa}-\eta}} dt \\ & + \underbrace{[\psi^{Aa'}(\eta_t) - 1]\eta_t\sigma_t^{\eta A}}_{\sigma_t^{\psi^{Aa}-\eta,A}} dZ_t^A + \underbrace{[\psi^{Aa'}(\eta_t) - 1]\eta_t\sigma_t^{\eta B}}_{\sigma_t^{\psi^{Aa}-\eta,B}} dZ_t^B \end{aligned}$$

At t , Gov-A borrows D_t^{AG} units of numeraire. At $t+dt$, Gov-A owes $D_t^{AG} + D_t^{AG}r_t^{AG}dt$, plus the $D_t^{AG}T_tdt$ “rebates” from global intermediaries (since gov net worth is negative it means government has to pay). Gov-A finances part of the repayment with new borrowing D_{t+dt}^{AG} , and the remaining is:

$$D_t^{AG} + D_t^{AG}r_t^{AG}dt + D_t^{AG}T_tdt - D_{t+dt}^{AG} = -dD_t^{AG} + D_t^{AG}(r_t^{AG} + T_t)dt$$

so the tax “rate” Gov-A charges to its agents would be

$$\frac{D_t^{AG}(r_t^{AG} + T_t)dt}{N_t^A} - \frac{dD_t^{AG}}{N_t^A} = \frac{\theta^A(\psi_t^{Aa} - \eta_t)q_tK_t(r_t^{AG} + T_t)dt}{q_tK_t} \cdot \frac{q_tK_t}{N_t^A} - \frac{dD_t^{AG}}{q_tK_t} \cdot \frac{q_tK_t}{N_t^A}$$

Thus,

$$d(Cumtax) = \frac{1}{\eta_t}\theta^A(\psi_t^{Aa} - \eta_t)(r_t^{AG} + T_t)dt - \frac{1}{\eta_t} \frac{dD_t^{AG}}{q_tK_t}$$

since

$$N_t^{AG} = -D_t^{AG}$$

$$d(Cumtax) = \frac{1}{\eta_t}\theta^A(\psi_t^{Aa} - \eta_t)(r_t^{AG} + T_t)dt + \frac{1}{\eta_t} \frac{dN_t^{AG}}{q_tK_t}$$

Denote¹

$$dCumtax \equiv \tau_t^A dt + \tau_t^{AA} dZ_t^A + \tau_t^{AB} dZ_t^B$$

$$\frac{dN_t^{AG}}{q_tK_t} \equiv \mu_t^{N^{AG}} dt + \sigma_t^{N^{AG}A} dZ_t^A + \sigma_t^{N^{AG}B} dZ_t^B$$

Plugging in,

$$dCumtax = \underbrace{\frac{\theta^A(\psi_t^{Aa} - \eta_t)(r_t^{AG} + T_t) + \mu_t^{N^{AG}}}{\eta_t}}_{-\tau_t^A} dt + \underbrace{\frac{\sigma_t^{N^{AG}A}}{\eta_t}}_{-\tau_t^{AA}} dZ_t^A + \underbrace{\frac{\sigma_t^{N^{AG}B}}{\eta_t}}_{-\tau_t^{AB}} dZ_t^B$$

¹I define $\frac{dN_t^{AG}}{q_tK_t}$ this way instead of $\frac{dN_t^{AG}}{N_t^{AG}}$ because the latter “blows up” towards $\eta = 0$.

$$\begin{aligned}\frac{d(q_t K_t)}{q_t K_t} &= \frac{d(N_t^A + N_t^{AG} + N_t^B)}{q_t K_t} \\ &= \frac{dN_t^A}{N_t^A} \cdot \frac{N_t^A}{q_t K_t} + \frac{dN_t^{AG}}{q_t K_t} + \frac{dN_t^B}{N_t^B} \cdot \frac{N_t^B}{q_t K_t}\end{aligned}$$

recall:

$$N_t^A = \eta_t q_t K_t$$

$$N_t^{AG} = -\theta^A(\psi_t^{Aa} - \eta_t) q_t K_t$$

$$N_t^B = [1 - \eta_t + \theta^A(\psi_t^{Aa} - \eta_t)] q_t K_t$$

plugging in,

$$\begin{aligned}\frac{d(q_t K_t)}{q_t K_t} &= \underbrace{\{\eta_t \mu_t^{N^A} + \mu_t^{N^{AG}} + [1 - \eta_t + \theta^A(\psi_t^{Aa} - \eta_t)] \mu_t^{N^B}\}}_{\mu_t^{qK}} dt \\ &\quad + \underbrace{\{\eta_t \sigma_t^{N^A A} + \sigma_t^{N^{AG} A} + [1 - \eta_t + \theta^A(\psi_t^{Aa} - \eta_t)] \sigma_t^{N^B A}\}}_{\sigma_t^{qKA}} dZ_t^A \\ &\quad + \underbrace{\{\eta_t \sigma_t^{N^A B} + \sigma_t^{N^{AG} B} + [1 - \eta_t + \theta^A(\psi_t^{Aa} - \eta_t)] \sigma_t^{N^B B}\}}_{\sigma_t^{qKB}} dZ_t^B\end{aligned}$$

recall:

$$N_t^{AG} \equiv -\theta^A(\psi_t^{Aa} - \eta_t) q_t K_t$$

$$dN_t^{AG} \equiv -\theta^A d[(\psi_t^{Aa} - \eta_t) q_t K_t]$$

By Ito's lemma,

$$\frac{dN_t^{AG}}{q_t K_t} = -\theta^A [d(\psi_t^{Aa} - \eta_t) + (\psi_t^{Aa} - \eta_t) \frac{d(q_t K_t)}{q_t K_t} + \frac{d(\psi_t^{Aa} - \eta_t) d(q_t K_t)}{q_t K_t}]$$

plugging in,

$$\begin{aligned}
\frac{dN_t^{AG}}{q_t K_t} = & \underbrace{(-\theta^A)[\mu_t^{\psi^{Aa}-\eta} + (\psi_t^{Aa} - \eta_t)\mu_t^{qK} + \sigma_t^{\psi^{Aa}-\eta,A}\sigma_t^{qKA} + \sigma_t^{\psi^{Aa}-\eta,B}\sigma_t^{qKB}]}_{\mu_t^{NAG}} dt \\
& + \underbrace{(-\theta^A)[\sigma_t^{\psi^{Aa}-\eta,A} + (\psi_t^{Aa} - \eta_t)\sigma_t^{qKA}]}_{\sigma_t^{NAGA}} dZ_t^A \\
& + \underbrace{(-\theta^A)[\sigma_t^{\psi^{Aa}-\eta,B} + (\psi_t^{Aa} - \eta_t)\sigma_t^{qKB}]}_{\sigma_t^{NAGB}} dZ_t^B
\end{aligned}$$

First we pin down the σ terms:

$$\sigma_t^{NAGA} = (-\theta^A)\{[\psi^{Aa'}(\eta_t) - 1]\eta_t\sigma_t^{\eta A} + (\psi_t^{Aa} - \eta_t)\sigma_t^{qKA}\}$$

$$\sigma_t^{NAGB} = (-\theta^A)\{[\psi^{Aa'}(\eta_t) - 1]\eta_t\sigma_t^{N^A A} - [\psi^{Aa'}(\eta_t)\eta_t - \psi_t^{Aa}]\sigma_t^{qKA}\}$$

First,

$$\begin{aligned}
\sigma_t^{NAGA} = & (-\theta^A)\{[\psi^{Aa'}(\eta_t) - 1]\eta_t\sigma_t^{N^A A} \\
& - [\psi^{Aa'}(\eta_t)\eta_t - \psi_t^{Aa}]\underbrace{[\eta_t\sigma_t^{N^A A} + \sigma_t^{NAGA} + [1 - \eta_t + \theta^A(\psi_t^{Aa} - \eta_t)]\sigma_t^{qKA}]}_{\sigma_t^{qKA}}\}
\end{aligned}$$

after simplifications, we get:

$$\begin{aligned}
[\psi^{Aa'}(\eta_t)\eta_t - \psi_t^{Aa} - \frac{1}{\theta^A}]\sigma_t^{NAGA} = & \eta_t[\psi^{Aa'}(\eta_t) - 1 - \psi^{Aa'}(\eta_t)\eta_t + \psi_t^{Aa}]\sigma_t^{N^A A} \\
& - [\psi^{Aa'}(\eta_t)\eta_t - \psi_t^{Aa}][1 - \eta_t + \theta^A(\psi_t^{Aa} - \eta_t)]\sigma_t^{N^B A}
\end{aligned}$$

similarly,

$$\begin{aligned}
[\psi^{Aa'}(\eta_t)\eta_t - \psi_t^{Aa} - \frac{1}{\theta^A}]\sigma_t^{NAGB} = & \eta_t[\psi^{Aa'}(\eta_t) - 1 - \psi^{Aa'}(\eta_t)\eta_t + \psi_t^{Aa}]\sigma_t^{N^A B} \\
& - [\psi^{Aa'}(\eta_t)\eta_t - \psi_t^{Aa}][1 - \eta_t + \theta^A(\psi_t^{Aa} - \eta_t)]\sigma_t^{N^B B}
\end{aligned}$$

Recall the first order condition for Country A is written as:

$$\begin{aligned} \frac{aP_t^a - \iota_t}{q_t} + \Phi(\iota_t) + \mu_t^q - \delta + \sigma^A \sigma_t^{qA} - r_t \\ - [x_t^{Aa}(\sigma^A + \sigma_t^{qA}) + \tau_t^{AA}](\sigma^A + \sigma_t^{qA}) - (x_t^{Aa} \sigma_t^{qB} + \tau_t^{AB}) \sigma_t^{qB} = 0 \end{aligned}$$

as for B, there is no tax so the FOC stays the same as in Proposition 1.

Write out the LOM for wealth of country A and country B, and plugging in, we get:

$$\sigma_t^{N^A A} = \frac{\psi_t^{Aa}}{\eta_t} (\sigma^A + \sigma_t^{qA}) - \frac{\sigma_t^{N^A G A}}{\eta_t}$$

$$\sigma_t^{N^A B} = \frac{\psi_t^{Aa}}{\eta_t} \sigma_t^{qB} - \frac{\sigma_t^{N^A G B}}{\eta_t}$$

$$\sigma_t^{N^B A} = \frac{\psi_t^{BA} + \psi_t^{Bb}}{1 - \eta_t + \theta^A (\psi_t^{Aa} - \eta_t)} \sigma_t^{qA}$$

$$\sigma_t^{N^B B} = \frac{\psi_t^{BA} + \psi_t^{Bb}}{1 - \eta_t + \theta^A (\psi_t^{Aa} - \eta_t)} (\sigma^B + \sigma_t^{qB})$$

Plugging in, we get:

$$\begin{aligned} [\psi^{Aa'}(\eta_t) - 1 - \frac{1}{\theta^A}] \sigma_t^{N^A G A} = \psi_t^{Aa} [\psi^{Aa'}(\eta_t) - 1 - \psi^{Aa'}(\eta_t) \eta_t + \psi_t^{Aa}] \sigma^A \\ + \psi^{Aa'}(\eta_t) (\psi_t^{Aa} - \eta_t) \sigma_t^{qA} \end{aligned}$$

$$[\psi^{Aa'}(\eta_t) - 1 - \frac{1}{\theta^A}] \sigma_t^{N^A G B} = \psi^{Aa'}(\eta_t) (\psi_t^{Aa} - \eta_t) \sigma_t^{qB} - [\psi^{Aa'}(\eta_t) \eta_t - \psi_t^{Aa}] (1 - \psi_t^{Aa}) \sigma^B$$

From Ito's lemma:

$$\sigma_t^{qA} = \frac{q'(\eta_t) \eta_t}{q(\eta_t)} \sigma_t^{\eta A}$$

$$\sigma_t^{qA} = \frac{q'(\eta_t) \eta_t}{q(\eta_t)} \{ (1 - \eta_t) \sigma_t^{N^A A} - \sigma_t^{N^A G A} - (\psi_t^{BA} + \psi_t^{Bb}) \sigma_t^{qA} \}$$

Plugging in:

$$\sigma_t^{qA} = \frac{q'(\eta_t)}{q(\eta_t)} \{ (1 - \eta_t) \psi_t^{Aa} \sigma^A + (\psi_t^{Aa} - \eta_t) \sigma_t^{qA} - \sigma_t^{N^{AG}A} \}$$

Plugging in $\sigma_t^{N^{AG}A}$ and after simplifications, we get:

$$\sigma_t^{qA} = - \frac{\psi_t^{Aa} [\frac{1}{\theta^A} - \eta_t - \frac{\eta_t}{\theta^A} + \psi_t^{Aa}]}{(1 + \frac{1}{\theta^A})(\psi_t^{Aa} - \eta_t) + \frac{q(\eta_t)}{q'(\eta_t)}(\psi^{Aa'}(\eta_t) - 1 - \frac{1}{\theta^A})} \sigma^A$$

similarly,

$$\sigma_t^{qB} = \frac{(1 - \psi_t^{Aa})[(1 + \frac{1}{\theta^A})\eta_t - \psi_t^{Aa}]}{(\psi_t^{Aa} - \eta_t)(1 + \frac{1}{\theta^A}) + (\psi^{Aa'}(\eta_t) - 1 - \frac{1}{\theta^A}) \frac{q(\eta_t)}{q'(\eta_t)}} \sigma^B$$

Now we can explicitly solve for other diffusion terms.

Now we derive the drift term μ_t^η .

We have 5 equations:

$$\mu_t^{NA} = r_t + x_t^{Aa} \left(\frac{aP_t^a - \iota_t}{q_t} + \Phi(\iota_t) + \mu_t^q - \delta + \sigma^A \sigma_t^{qA} - r_t \right) - \rho^A + T_t + \tau_t^A$$

$$\mu_t^{NB} = \frac{d\tilde{r}_t^F}{dt} + (x_t^{Ba} + x_t^{Bb}) \left(\frac{aP_t^b - \iota_t}{q_t} + \Phi(\iota_t) + \mu_t^q - \delta + \sigma^B \sigma_t^{qB} - \frac{d\tilde{r}_t^F}{dt} \right) - \rho^B + T_t$$

$$\mu_t^{qK} = \eta_t \mu_t^{NA} + \mu_t^{N^{AG}} + [1 - \eta_t + \theta^A (\psi_t^{Aa} - \eta_t)] \mu_t^{NB}$$

$$\mu_t^{\psi^{Aa}-\eta} = [\psi^{Aa'}(\eta_t) - 1] \eta_t \mu_t^\eta + \frac{1}{2} \psi^{Aa''}(\eta_t) (\eta_t)^2 [(\sigma_t^{\eta A})^2 + (\sigma_t^{\eta B})^2]$$

$$\mu_t^{N^{AG}} = (-\theta^A) [\mu_t^{\psi^{Aa}-\eta} + (\psi_t^{Aa} - \eta_t) \mu_t^{qK} + \sigma_t^{\psi^{Aa}-\eta, A} \sigma_t^{qKA} + \sigma_t^{\psi^{Aa}-\eta, B} \sigma_t^{qKB}]$$

first,

$$\begin{aligned}\mu_t^{qK} &= \eta_t \mu_t^{NA} - \theta^A [\mu_t^{\psi^{Aa}-\eta} + (\psi_t^{Aa} - \eta_t) \mu_t^{qK} + \sigma_t^{\psi^{Aa}-\eta,A} \sigma_t^{qKA} + \sigma_t^{\psi^{Aa}-\eta,B} \sigma_t^{qKB}] \\ &\quad + [1 - \eta_t + \theta^A (\psi_t^{Aa} - \eta_t)] \mu_t^{NB}\end{aligned}$$

$$\begin{aligned}\mu_t^{NA} - \mu_t^{qK} &= (1 - \eta_t) \mu_t^{NA} + \theta^A [\mu_t^{\psi^{Aa}-\eta} + (\psi_t^{Aa} - \eta_t) \mu_t^{qK} + \sigma_t^{\psi^{Aa}-\eta,A} \sigma_t^{qKA} + \sigma_t^{\psi^{Aa}-\eta,B} \sigma_t^{qKB}] \\ &\quad - [1 - \eta_t + \theta^A (\psi_t^{Aa} - \eta_t)] \mu_t^{NB}\end{aligned}$$

$$\begin{aligned}\mu_t^{NA} - \mu_t^{qK} &= (1 - \eta_t) (\mu_t^{NA} - \mu_t^{NB}) + \theta^A \mu_t^{\psi^{Aa}-\eta} + \theta^A (\psi_t^{Aa} - \eta_t) (\mu_t^{qK} - \mu_t^{NB}) \\ &\quad + \theta^A \sigma_t^{\psi^{Aa}-\eta,A} \sigma_t^{qKA} + \theta^A \sigma_t^{\psi^{Aa}-\eta,B} \sigma_t^{qKB}\end{aligned}$$

then:

$$\begin{aligned}\mu_t^{NA} - \mu_t^{NB} &= r_t + x_t^{Aa} \left(\frac{aP_t^a - \iota_t}{q_t} + \Phi(\iota_t) + \mu_t^q - \delta + \sigma^A \sigma_t^{qA} - r_t \right) - \rho^A + \tau_t^A - \frac{d\tilde{r}_t^F}{dt} \\ &\quad - (x_t^{Ba} + x_t^{Bb}) \left(\frac{aP_t^b - \iota_t}{q_t} + \Phi(\iota_t) + \mu_t^q - \delta + \sigma^B \sigma_t^{qB} - \frac{d\tilde{r}_t^F}{dt} \right) + \rho^B\end{aligned}$$

plugging in FOCs, we get:

$$\begin{aligned}\mu_t^{NA} - \mu_t^{NB} &= -\chi \phi^A (q_t - 1) + c^B - \frac{\theta^A (\psi_t^{Aa} - \eta_t) (r_t^{AG} + T_t) + \mu_t^{NAG}}{\eta_t} \\ &\quad + \frac{\psi_t^{Aa}}{\eta_t} \left\{ \left[\frac{\psi_t^{Aa}}{\eta_t} (\sigma^A + \sigma_t^{qA}) - \frac{\sigma_t^{NAG A}}{\eta_t} \right] (\sigma^A + \sigma_t^{qA}) + \left(\frac{\psi_t^{Aa}}{\eta_t} \sigma_t^{qB} - \frac{\sigma_t^{NAG B}}{\eta_t} \right) \sigma_t^{qB} \right\} \\ &\quad - \left(\frac{\psi_t^{Ba} + \psi_t^{Bb}}{1 - \eta_t + \theta^A (\psi_t^{Aa} - \eta_t)} \right)^2 [(\sigma^B + \sigma_t^{qB})^2 + (\sigma_t^{qA})^2] + \rho^B - \rho^A\end{aligned}$$

then:

$$\begin{aligned}\mu_t^{qK} - \mu_t^{NB} &= \eta_t \mu_t^{NA} + \mu_t^{NAG} + [1 - \eta_t + \theta^A (\psi_t^{Aa} - \eta_t)] \mu_t^{NB} - \mu_t^{NB} \\ &= \eta_t (\mu_t^{NA} - \mu_t^{NB}) + \mu_t^{NAG} + \theta^A (\psi_t^{Aa} - \eta_t) \mu_t^{NB}\end{aligned}$$

$$\begin{aligned}\iff [1 + \theta^A (\psi_t^{Aa} - \eta_t)] (\mu_t^{qK} - \mu_t^{NB}) \\ = \eta_t (\mu_t^{NA} - \mu_t^{NB}) - \theta^A \mu_t^{\psi^{Aa}-\eta} - \theta^A \sigma_t^{\psi^{Aa}-\eta,A} \sigma_t^{qKA} - \theta^A \sigma_t^{\psi^{Aa}-\eta,B} \sigma_t^{qKB}\end{aligned}$$

recall:

$$\mu_t^{qK} - \mu_t^{NB} = \eta_t(\mu_t^{NA} - \mu_t^{NB}) + \mu_t^{NAG} + \theta^A(\psi_t^{Aa} - \eta_t)\mu_t^{NB}$$

plug in $\mu_t^{NA} - \mu_t^{NB}$, we have:

$$\begin{aligned} \mu_t^{qK} - \mu_t^{NB} = & \\ & -\eta_t\chi\phi^A(q_t - 1) + \eta_t c^B - \theta^A(\psi_t^{Aa} - \eta_t)(r_t^{AG} + T_t) \\ & + \psi_t^{Aa} \left\{ \left[\frac{\psi_t^{Aa}}{\eta_t}(\sigma^A + \sigma_t^{qA}) - \frac{\sigma_t^{NAG A}}{\eta_t} \right] (\sigma^A + \sigma_t^{qA}) + \left(\frac{\psi_t^{Aa}}{\eta_t} \sigma_t^{qB} - \frac{\sigma_t^{NAG B}}{\eta_t} \right) \sigma_t^{qB} \right\} \\ & - \eta_t \left(\frac{\psi_t^{Ba} + \psi_t^{Bb}}{1 - \eta_t + \theta^A(\psi_t^{Aa} - \eta_t)} \right)^2 [(\sigma^B + \sigma_t^{qB})^2 + (\sigma_t^{qA})^2] \\ & + \eta_t(\rho^B - \rho^A) + \theta^A(\psi_t^{Aa} - \eta_t)\mu_t^{NB} \end{aligned}$$

plugging in μ_t^{NB} , we get:

Thus,

$$\begin{aligned} \mu_t^{qK} - \mu_t^{NB} = & \\ & [\theta^A(\psi_t^{Aa} - \eta_t)\phi^{AG} - \eta_t\phi^A]\chi(q_t - 1) + [\eta_t - \theta^A(\psi_t^{Aa} - \eta_t)]c^B \\ & + \psi_t^{Aa} \left\{ \left[\frac{\psi_t^{Aa}}{\eta_t}(\sigma^A + \sigma_t^{qA}) - \frac{\sigma_t^{NAG A}}{\eta_t} \right] (\sigma^A + \sigma_t^{qA}) + \left(\frac{\psi_t^{Aa}}{\eta_t} \sigma_t^{qB} - \frac{\sigma_t^{NAG B}}{\eta_t} \right) \sigma_t^{qB} \right\} \\ & + [\theta^A(\psi_t^{Aa} - \eta_t) - \eta_t] \left(\frac{\psi_t^{Ba} + \psi_t^{Bb}}{1 - \eta_t + \theta^A(\psi_t^{Aa} - \eta_t)} \right)^2 [(\sigma^B + \sigma_t^{qB})^2 + (\sigma_t^{qA})^2] \\ & + \eta_t\Delta\rho - \theta^A(\psi_t^{Aa} - \eta_t)\rho^B \end{aligned}$$

then, recall:

$$\begin{aligned} [1 + \theta^A(\psi_t^{Aa} - \eta_t)](\mu_t^{qK} - \mu_t^{NB}) &= \eta_t(\mu_t^{NA} - \mu_t^{NB}) - \theta^A\mu_t^{\psi^{Aa}-\eta} \\ &\quad - \theta^A\sigma_t^{\psi^{Aa}-\eta,A}\sigma_t^{qKA} - \theta^A\sigma_t^{\psi^{Aa}-\eta,B}\sigma_t^{qKB} \\ \iff [1 + \theta^A(\psi_t^{Aa} - \eta_t)](\mu_t^{qK} - \mu_t^{NB}) &+ \theta^A(\mu_t^{\psi^{Aa}-\eta} + \sigma_t^{\psi^{Aa}-\eta,A}\sigma_t^{qKA} + \sigma_t^{\psi^{Aa}-\eta,B}\sigma_t^{qKB}) \\ &= \eta_t(\mu_t^{NA} - \mu_t^{NB}) \end{aligned}$$

$$\begin{aligned}
&\Longleftrightarrow \mu_t^{N^A} - \mu_t^{N^B} \\
&= \frac{1}{\eta_t} \{ [1 + \theta^A(\psi_t^{Aa} - \eta_t)](\mu_t^{qK} - \mu_t^{N^B}) + \theta^A(\mu_t^{\psi^{Aa}-\eta} + \sigma_t^{\psi^{Aa}-\eta,A} \sigma_t^{qKA} + \sigma_t^{\psi^{Aa}-\eta,B} \sigma_t^{qKB}) \}
\end{aligned}$$

Thus:

$$\begin{aligned}
\mu_t^{N^A} - \mu_t^{qK} &= (1 - \eta_t)(\mu_t^{N^A} - \mu_t^{N^B}) + \theta^A \mu_t^{\psi^{Aa}-\eta} + \theta^A(\psi_t^{Aa} - \eta_t)(\mu_t^{qK} - \mu_t^{N^B}) \\
&\quad + \theta^A \sigma_t^{\psi^{Aa}-\eta,A} \sigma_t^{qKA} + \theta^A \sigma_t^{\psi^{Aa}-\eta,B} \sigma_t^{qKB}
\end{aligned}$$

$$\begin{aligned}
\mu_t^{N^A} - \mu_t^{qK} &= \left\{ \frac{1 - \eta_t}{\eta_t} [1 + \theta^A(\psi_t^{Aa} - \eta_t)] + \theta^A(\psi_t^{Aa} - \eta_t) \right\} (\mu_t^{qK} - \mu_t^{N^B}) \\
&\quad + \frac{\theta^A}{\eta_t} (\mu_t^{\psi^{Aa}-\eta} + \sigma_t^{\psi^{Aa}-\eta,A} \sigma_t^{qKA} + \sigma_t^{\psi^{Aa}-\eta,B} \sigma_t^{qKB})
\end{aligned}$$

Recall:

$$\mu_t^{\psi^{Aa}-\eta} = [\psi^{Aa'}(\eta_t) - 1] \eta_t \mu_t^\eta + \frac{1}{2} \psi^{Aa''}(\eta_t) (\eta_t)^2 [(\sigma_t^{\eta A})^2 + (\sigma_t^{\eta B})^2]$$

$$\mu_t^\eta = \mu_t^{N^A} - \mu_t^{qK} + (\sigma_t^{N^A A})^2 + (\sigma_t^{N^A B})^2 - \sigma_t^{N^A A} \sigma_t^{qKA} - \sigma_t^{N^A B} \sigma_t^{qKB}$$

Plugging in, we get:

$$\begin{aligned}
\{1 - \theta^A[\psi^{Aa'}(\eta_t) - 1]\} \mu_t^\eta &= \left\{ \frac{1 - \eta_t}{\eta_t} [1 + \theta^A(\psi_t^{Aa} - \eta_t)] + \theta^A(\psi_t^{Aa} - \eta_t) \right\} (\mu_t^{qK} - \mu_t^{N^B}) \\
&\quad + \frac{1}{2} \frac{\theta^A}{\eta_t} \psi^{Aa''}(\eta_t) (\eta_t)^2 [(\sigma_t^{\eta A})^2 + (\sigma_t^{\eta B})^2] \\
&\quad + \frac{\theta^A}{\eta_t} \sigma_t^{\psi^{Aa}-\eta,A} \sigma_t^{qKA} + \frac{\theta^A}{\eta_t} \sigma_t^{\psi^{Aa}-\eta,B} \sigma_t^{qKB} \\
&\quad + (\sigma_t^{N^A A})^2 + (\sigma_t^{N^A B})^2 - \sigma_t^{N^A A} \sigma_t^{qKA} - \sigma_t^{N^A B} \sigma_t^{qKB}.
\end{aligned}$$

Appendix B

Sudden Stops

Brunnermeier and Sannikov (2015) define two kinds of sudden stops, those that happen naturally on equilibrium path and those that can be triggered by sunspots. In both cases, a necessary condition for sudden stops to occur is for $q(\eta_t)$ to be sufficiently concave. Bond liquidity modeled in this paper has such effect and thus can cause sudden stops to occur, as the following example shows.

Here I take $\gamma^A = \gamma^B = 0.9\%$ and $\sigma^A = \sigma^B = 3.5\%$ and contrast the case where bond has no liquidity versus the case where $\phi^A = \phi^B = 5\%$.

As the left graph shows, there is procyclical leverage (sudden stop of the first kind) when bond provides liquidity service, but none when it does not. As for the second kind of sudden stop, Figure 1.1 in the main text shows how bond liquidity makes $q(\eta)$ more concave, and thus potentially make the economy subject to sudden stops triggered by sunspots. Sunspot-induced instabilities are much more thoroughly illustrated in Mendo (2018).

Figure B.1: An example where bond liquidity induces sudden stops

Appendix C

Microfoundation for Capital Illiquidity

Consider a time period from time t to $t + dt$, call it period t .

Agents enter into period t with some capital holding k_t and bond holding b_t .

In standard BruSan models, output of order $Ak_t dt$ realizes at t , and agents decide the investment rate, ι_t . Then at $t + dt$ (the end of period t), agents solve consumption-saving problem (choosing consumption rate, and rebalance their portfolios by modifying leverage);

Suppose now output realizes at $t + dt$. Investment opportunities still arise at t^+ . Here I use t^+ to denote the instant right after agents have rebalanced their portfolio and started the production process. So agents have to borrow/trade with others who have resources at t^+ – the “outside bond buyers”, or dealers for short (because they are buying bond at t^+ and selling them back at $t + dt$, same as a repo trade).

In particular, I assume that a continuum of dealers are born at t and die at $t + dt$. They have deep pockets and serve as intra-period liquidity providers, that is, they could

potentially buy capital and bond at t^+ , and sell these back to agents at $t + dt$. Thus, at t^+ agents can potentially sell bond and capital to these dealers.

Assume there is technological constraint on the size of such trades, in the sense that trades can only happen at no higher than some maximum rate. In particular, suppose an agent starts the period with position k_t unit of capital and b_t unit of bond, then assume she can sell capital to dealers at rate no higher than $m_{max}k_t$, and bond at rate no higher than $m_{max}b_t$. That is, agents can transact at most $m_{max}k_t \cdot dt$ units of capital and $m_{max}b_t \cdot dt$ units of bond over this period¹.

For bond, this is what the agents will try to do. In my model I assume agents can sell bond at rate ϕb_t . I restrict attention to cases where $\phi < m_{max}$, and an increase in liquidity is interpreted as an increase in ϕ , while still respecting $\phi < m_{max}$, the technological constraint.

Agents also want to sell capital, but the problem is that capital is subject to asymmetric information as in Kurlat (2013). In particular, when production process starts at t , a fraction δdt of an agent's capital holding depreciates over this period. Which units have depreciated is private information of the capital holder at t^+ , but becomes public knowledge at $t + dt$. Thus, there is no asymmetric information problem at $t + dt$, so capital can trade freely at $t + dt$.

Suppose at t^+ , the market for capital is an anonymous and competitive market, as in the market for capital at $t + dt$. If $m_{max} \leq \delta$, then a dealer knows that the capital any agent sells will be publicly known as worthless at $t + dt$, so dealers will refuse quoting a

¹This constraint is not binding when agents rebalance their portfolios at $t + dt$. The constraint makes sense at t^+ because aggregate output is of order dt , so short-lived dealers will only transact up to $Y_t dt$, the output to be realized at $t + dt$.

positive price. Even if $m_{max} > \delta$, as long as m_{max} is small enough such that the pooling price of capital is sufficiently smaller than q_t , the current price of capital (the opportunity cost of holding the capital instead of selling), agents will want to hold on to her capital.

In particular, suppose there is an active anonymous market of capital trading between agents and dealers at a pooling price of $\hat{q}_t$. We know that worthless ones sold in this market are of quantity $\delta K_t dt$ (every agents try to sell their privately known depreciated ones), and good ones (those that didn't depreciate at all) are of quantity at most $(m_{max} - \delta) K_t dt$. Since 1 unit of good capital is worth q_t , we have:

$$\hat{q}_t \leq \frac{0 \cdot \delta K_t dt + q_t \cdot (m_{max} - \delta) K_t dt}{m_{max} K_t dt} = \frac{m_{max} - \delta}{m_{max}} q_t$$

Suppose an agent contemplates whether to sell good capital at t^+ . 1 unit of such capital is worth q_t to her, but alternatively, she can sell it for $\hat{q}_t$ consumption goods to the dealer, turning this into $\hat{q}_t$ unit of (good) capital, which then she can sell at q_t per unit (everything here is of order dt). So she doesn't want to sell good capital if and only if:

$$\hat{q}_t q_t \leq q_t \iff \hat{q}_t \leq 1$$

Thus, a sufficient (and also necessary) condition for capital to not trade actively is:

$$\frac{m_{max} - \delta}{m_{max}} \leq \frac{1}{q_t}$$

If this inequality is satisfied, then in equilibrium there is no market for capital due to the classic lemon's problem – the price of capital that a dealer is willing to quote is so low that agents only want to sell the bad ones (those that fully depreciate at $t + dt$), but then the price of capital will fall to 0. Equivalently, dealers will not accept capital collateral due to asymmetric information.

Appendix D

Correlated Fundamental Shocks

Instead of assuming that aggregate shocks from the two countries are independent, suppose $Cor[Z_t^A, Z_t^B] = \rho^{corr} \neq 0$. Equilibrium dynamics is summarized in Proposition D1.

PROPOSITION D1: The state space is divided into three regions as before. Law of motion for η_t is written as:

$$\frac{d\eta_t}{\eta_t} = \mu_t^\eta dt + \sigma_t^{\eta A} dZ_t^A + \sigma_t^{\eta B} dZ_t^B,$$

where:

$$\begin{aligned} \mu_t^\eta = & (1 - \eta_t)(\mu_t^{NA} - \mu_t^{NB}) + (\sigma_t^{qK,A})^2 + (\sigma_t^{qK,B})^2 + 2\rho^{corr} \sigma_t^{qKA} \sigma_t^{qKB} \\ & - \sigma_t^{NAA}(\sigma_t^{qK,A} + \rho \sigma_t^{qKB}) - \sigma_t^{NAB}(\sigma_t^{qK,B} + \rho \sigma_t^{qKA}), \end{aligned}$$

$$\mu_t^{NA} = \frac{dr_t^F}{dt} + (x_t^{Aa})^2 [(\sigma^A + \sigma_t^{qA})^2 + (\sigma_t^{qB})^2 + 2\rho^{corr}(\sigma^A + \sigma_t^{qA})\sigma_t^{qB}] - \rho^A + T_t,$$

$$\mu_t^{NB} = \frac{d\tilde{r}_t^F}{dt} + (x_t^{Ba} + x_t^{Bb})^2 [(\sigma^B + \sigma_t^{qB})^2 + (\sigma_t^{qA})^2 + 2\rho^{corr}\sigma_t^{qA}(\sigma^B + \sigma_t^{qB})] - \rho^B + T_t,$$

and all other terms are algebraically the same as in Proposition 1;

in terms of asset pricing, in the left region we have:

$$\begin{aligned} \frac{a(P_t^a - P_t^b)}{q_t} + \sigma^A(\sigma_t^{qA} + \rho^{corr}\sigma_t^{qB}) - \chi\phi^B(q_t - 1) - \sigma^B(\sigma_t^{qB} + \rho^{corr}\sigma_t^{qA}) + c^A = \\ \frac{1 - \psi_t^{Bb}}{1 - \eta_t^B} [(\sigma^A + \sigma_t^{qA})^2 + (\sigma_t^{qB})^2 + 2\rho^{corr}(\sigma^A + \sigma_t^{qA})\sigma_t^{qB}] \\ - \frac{\psi_t^{Bb}}{\eta_t^B} [(\sigma^B + \sigma_t^{qB})^2 + (\sigma_t^{qA})^2 + 2\rho^{corr}(\sigma^B + \sigma_t^{qB})\sigma_t^{qA}], \end{aligned}$$

and a symmetric condition in the right region. All other conditions are same as in Proposition 1.

Numerically, it turns out that non-zero correlation among fundamental shocks does not matter much for most of equilibrium dynamics, e.g., leverage changes very little even when the two shocks are significantly correlated. This is because from an agent's perspective, correlation among country-specific shocks matter only to the extent that it affects the capital price dynamics, that is, the endogenous risk in this economy (σ_t^{qA} and σ_t^{qB}). It turns out that for the parameterizations in this paper, most of the time endogenous risk is a very small portion of the overall risk faced by agents, and hence the small adjustment in equilibrium dynamics. It is worth pointing out that equilibrium leverage slightly increases as the two fundamental shocks become more positively correlated. This is because the levered agents expect that the drop in capital price will be less pronounced after they experience a negative shock in their country, because the other country is also likely to experience a negative shock and hence wealth share is distorted by less (so capital price drops by less). As a result, they choose to slightly increase their leverage because endogenous risk is smaller.

Figure D.1: Effect of bond liquidity when fundamental shocks are correlated

However, non-zero correlation matters a lot for the stationary distribution – after all, when the two shocks are perfectly correlated, the stationary distribution becomes degenerate (mass point at $\eta = 0.5$ in a symmetric world). As a result, correlation also matters in terms of the welfare implications of bond liquidity. For example, in Section 1.4 it is demonstrated that with high fundamental volatilities ($\sigma^A = \sigma^B = 0.1$), higher bond liquidity improves ex-ante global welfare. However, when we set $\rho^{corr} = 0.5$ and keep other parameters unchanged, higher bond liquidity lowers ex-ante global welfare. This is illustrated in Figure D.1. The left two graphs show the effect of an increase in bond liquidity for different values of fundamental correlation. When there is positive fundamental correlation, the stationary distribution is more (adversely) affected by an increase in bond liquidity. The right graph shows that contrary to the results in Section 4 (solid blue line), bond liquidity actually lowers ex-ante global welfare when there is positive fundamental correlation (dashed red line).

Appendix E

Interactions between Advanced Economies

Figure E.1 plots the effect of bond liquidity on stationary distribution and ex-ante welfare when $\sigma^A = 3.5\%$ and $\sigma^B = 5\%$. When there is no bond liquidity ($\phi^A = \phi^B = 0$), the Nash equilibrium is financial autarky (both countries imposing the tightest leverage constraints, i.e., $L^A = L^B = 0$) with $\mathbb{E}[V^A(\eta_t)] = 8.8265$ and $\mathbb{E}[V^B(\eta_t)] = 8.7560$. The socially optimal leverage constraints are $L^A = 0.065$ and $L^B = 0.055$, and ex-ante welfare of country A decreases to $\mathbb{E}[V^A(\eta_t)] = 8.7794$, while that of country B increases to $\mathbb{E}[V^B(\eta_t)] = 8.8420$. With higher bond liquidity ($\phi^A = \phi^B = 3\%$), socially optimal leverages become $L^A = 0.115$, $L^B = 0.105$, and ex-ante welfare of country A drops further to $\mathbb{E}[V^A(\eta_t)] = 8.7378$, while that of country B further improves to $\mathbb{E}[V^B(\eta_t)] = 8.9675$ (Nash equilibrium is still $L^A = L^B = 0$).

Figure E.1: Effects of bond liquidity in an environment with two advanced economies with asymmetric volatilities