Politics, Altruism, and the Definition of Poverty

Edward J. Bird

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Edward J. Bird
Associate Professor of Political Science
University of Rochester
109A Harkness Hall, Rochester NY 14627-0158
Ph: 716/275-7840, Fax: 716/271-1616, email: ejbd@troi.cc.rochester.edu

Abstract: The history of poverty lines suggests that they are determined jointly with poverty policy in the same political game. If the definition of poverty is endogenous, however, why do altruistic voters allow poverty to persist indefinitely, as seems to be the case in real life? A simple redistribution model shows that the persistence of poverty imposes fairly strong restrictions on the nature of voter altruism. Specifically, a voter's compassion for the poor must rise as the defined severity of the poverty problem worsens. Given such preferences, political actors face incentives to define poverty as a severe problem and then to use redistribution to reduce it significantly. There is no direct incentive to eliminate poverty, however; indeed voters may prefer a state in which policy always attacks poverty vigorously and yet never defeats it. It follows that social policy should not be judged by its success in eliminating poverty, which may be directly counter to voter interests and therefore practically impossible. Rather, we should ask whether poverty policy provides enough help to people whom voters currently consider to be poor.

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Past work has identified four primary explanations for large-scale redistribution programs: interest group redistribution (Meltzer and Richard, 1981; Kristov et al. 1992; Lambert, 1993); middle class self-insurance against the threat of poverty (Varian, 1980; Moffitt, and Rothschild, 1987; Bird, 1995), the non-poor's interest in keeping the poor from becoming restless and violent (Piven and Cloward, 1971), and the altruistic preferences of the non-poor for the poor (Hochman and Rodgers, 1968). It is not clear which of the four is most important, but altruism is usually the explanation that politicians offer for redistribution policies. There seems to be a reasonable basis for such an argument: poverty is a public "bad" and the collective provision of poverty-reduction corrects for the failure of the charitable-giving market. The purpose of this paper is to extend the redistribution-as-altruism argument by relaxing a key assumption, namely, that the object of altruism, the poor, is a well-defined group. It is fairly clear empirically (as will be shown below) that the definition of poverty is not a fixed constant, and this opens the possibility that the same socio-political process that sets poverty policy also defines the poverty problem. The paper explores whether this interplay creates a problem for the standard models of altruism-based redistribution (it does), and then modifies the standard model to find a form of altruistic preference that solves the problem, i.e. that produces empirically sensible patterns of poverty definition. The main result is that the preferences which work are quite restrictive, and have interesting features involving the egoistic altruism (Andreoni, 1990) of the average citizen.

The paper begins with a brief literature review and overview of the model. Section II then discusses several stylized facts that can be taken from the past experience of developed countries in dealing with poverty. Section III presents a simple model structured around these

facts and derives some basic propositions from it, including a paradox in that pure altruists would prefer to see poverty defined away. Section IV discusses ways to modify altruism so as to allow government and media actors to define poverty as a real problem; it then presents results from some examples which meet the empirical experience of poverty accurately. Section V considers some implications of the results.

I. Introduction: Altruism and redistribution

Hochman and Rodgers (1968) were the first to state formally that redistribution is pareto-efficient for altruistic voters. There have since been many extensions and empirical tests of the idea (Zeckhauser, 1971; Orr, 1976; Feldstein and Clotfelter, 1976; Kingma, 1989; Andreoni, 1990). The literature has been mostly interested in whether government redistribution crowds out private giving dollar-for-dollar; it does not appear to do so, a result that requires voters to have an impure form of altruism (Kingma, 1989; Andreoni, 1990). Impurely altruistic voters receive satisfaction not just from a reduction of poverty in general but also from their own contributions to the poor.

One issue that the literature has not addressed, however, is how perceptions of the poverty problem are produced in the first place. All of the results produced so far implicitly assume that poverty is an unambiguous and fixed social problem, whose level and response to policy can be directly and uncontroversially observed by everyone. In Hochman and Rodgers (1968), for example, each agent has the utility of lower-income agents as a parameterized argument of his or her own utility, as if one person could directly perceive the utility of all others. In Zeckhauser (1971), the poverty line is a fixed constant. In Andreoni (1990) and other papers

in the charitable contributions literature, the actual “public bads” which charitable contributions are supposed to reduce are not defined; instead it is assumed that every dollar contributed has a known, fixed, direct, and positive impact on utility. These approaches overlook the apparent and long-understood “mischievous ambiguity of the word ‘poor’” (Poor Law Report of 1834, p. 395). Poverty is defined differently by different people; they get most of their information about “the poverty problem” from the government or another third party; constructing that information is as much a political and social matter as a scientific, objective one; and given any definition the impact of spending increases is probably subject to diminishing marginal returns.

Overlooking these characteristics of poverty is important. Section III presents a simple model of redistribution that allows the same political process that chooses the redistribution level to define also the severity of poverty, on a scale from 0 to 1. In effect, government and social actors are allowed to set a poverty line; the poverty problem is defined by the poverty rate, the percentage of families with incomes below the line. A solution function then translates pre-transfer poverty into post-transfer poverty in a way that can depend on the amount of redistribution that has been chosen. Voter utility is defined over income and post-transfer poverty. The main result of this model is that all voters will prefer parties that minimize the problem of poverty and define it away if possible. The intuition is simple: voters do not like poverty and they do not like the taxes needed to fight it. Thus no voter will support a government, social, or media actor who reports that poverty is worse than that voter’s own observation suggests. The model predicts that there would never be a book or government report that attempts to raise public consciousness about the plight of the poor, an outcome clearly contradicted by common experience.

Section IV resolves this paradox by constructing a model that replicates the fact that government and media actors do attempt to raise public consciousness about the plight of the poor. The principle result is that, within the context of altruism-based models, *impure and relative* altruism is a necessary condition for this behavior. Under impure relative altruism, utility rises with an agent's own contributions to ending poverty, and the utility received from such contributions rises with poverty's severity. Together these features produce a world in which voters appreciate the fact that the government and media make them aware of the conditions of the poor, because that awareness endows their tax payments with altruistic utility. It is somewhat too strong to say that society pays attention to poverty only because citizens enjoy having a poverty problem to fight, but that is the flavor of the result.

II. Poverty definition: Stylized facts from the historical record

This section draws three empirical lessons from past experience with poverty in developed countries. First, poverty tends to be perceived as a relative concept, and different societies have defined it differently. Second, political and social actors seem able to exercise some influence, formal or informal, over these definitions. It follows from these two that the definition of poverty should be endogenous in models of redistribution. Third, poverty seems to be a perpetual problem in that it does not seem to disappear as a public issue as countries become wealthier. It is significant that this statement is true for both pre- and post-transfer poverty, because it implies both that poverty definitions do produce a poverty problem, and that this problem has never been considered solved by transfer policies. To be empirically accurate, a theory of redistribution policy should produce both outcomes.

A. In practice, poverty seems to be a relative concept

If the poverty problem could be defined absolutely and objectively, there could emerge a simple and unchanging consensus view as to what it is and how it should be defined. In general that has not been the case, and instead the historical record suggests that poverty definitions eventually lose their political meaning and are replaced or updated. There are many examples. In 1901, Seebohm Rowntree used an absolute poverty line and found rates of severe poverty in York of about 10 percent, a figure that fit in well with a consensus at that time that poverty was a serious problem. In response to this and other figures, the U.K. welfare state expanded considerably over the next decade. Repeating the study in 1951, Rowntree found a much lower number in poverty, only 1.7 percent (Himmelfarb 1991, pp. 384-85). Taking this number into account, the social consensus in the U.K. could have been that poverty was basically solved, and that no further expansions of the welfare state were needed. Indeed, it might have been concluded that some reductions were appropriate. Instead, however, U.K. social expenditures continued to rise throughout the next two or three decades. This seems to imply that poverty was not considered solved, from which it follows that Rowntree's standard had become obsolete. As a matter of social consensus and policy determination, countries usually adopt higher poverty standards as national wealth increases.

An example for the U.S. can be found in Smolensky and Plotnick (1993), who present figures showing that the Orshansky poverty line, which is the absolute standard used by the U.S. government to measure poverty, would have produced a 1900 poverty rate of about 80 percent. The calculation assumes no greater inequality in 1900 than in 1947; with the greater inequality earlier in the century, the 1900 Orshansky poverty rate would have been even higher. If the

Orshansky line were to represent some natural, objective, and constant definition of poverty, the poverty issue in 1900 and earlier would have to have commanded an extraordinary amount of attention. However, while poverty was an important issue at the turn of the century, it was not as important as a near 100 percent poverty rate would have required. The Orshansky line therefore seems to represent a poverty concept that would have had no social or political meaning in 1900. The operative definition of poverty in U.S. society must have risen as average incomes rose between 1900 and 1965.

More direct evidence for the relativity of poverty is presented by Kilpatrick (1973). Using subjective poverty opinion data ("about how much money is needed to get along in this community?"), Kilpatrick finds an income elasticity of poverty lines of about 0.75. On the basis of extensive historical data from several countries, Fisher (1995) finds that official poverty lines often move. Citro and Michael (1995, p. 35) present an intriguing figure comparing an explicitly relative poverty concept (one-half of median income) to answers to subjective poverty questions like Kilpatrick's. The resulting pair of poverty lines remain remarkably close over time. Moreover, they both equal the Orshansky absolute poverty line in the early 1960s, when the Orshansky line was designed. The figures suggest that the Orshansky line translates the subjective poverty concepts relative to U.S. society in the 1960s into an objective number.

The inadequacy of objective poverty statistics seems to be generally accepted in the poverty research community. When such figures fail to capture the problems faced by low-income people, researchers use indices of their own construction (e.g. Gottschalk and Danziger, 1993, p. 167). Some of the more influential research on US poverty in recent years has argued not that poverty has risen as measured in absolute terms, but rather that its character has changed.

For example, Mead (1992) argues that the causes of US poverty have changed significantly in the past 30 years and that these changes have forced a sea change in the nature of poverty politics. In other words, poverty can become more severe in a social and political sense even though its prevalence, as measured by an absolute standard, changes very little.

Those researchers who work with absolute poverty standards concede that quantifying poverty is difficult (e.g. Smolensky and Plotnick, 1993, p. 190). When defining it explicitly, these authors often use admittedly relative and subjective terms, such as "less than a decent standard of living" - decency being left unquantified (e.g. Galbraith, 1958, p. 324; Harrington, 1962, p. 2). In practice, quantitative research relies almost exclusively on relative concepts, and many have recommended that official poverty lines be explicitly relative, to be updated from time to time so that they remain socially and politically relevant (e.g. Smeeding, 1991; Rainwater, 1974; Ruggles 1990; Citro and Michael, 1995; Haveman, 1995).

Overall, the evidence supports the view that the severity of poverty seems to be defined with respect to the time and place in which the poverty occurs.

B. Poverty definitions seem to be part of a political equilibrium

If poverty concepts change, they are probably subject to manipulation by political and social actors. Although many people have individual experiences with the poor, anecdotal evidence is not the same as "generally accepted social fact," and only such social facts will change policies. Social facts about poverty are shared and accepted by virtually everyone, and they refer to the aggregate nature of the problem, something that no single individual can perceive without the reports of third parties. In making their reports, these third parties (in

journalism, government, think-tanks, and academia) have to define poverty in some way, formally or informally. On the face of it they have considerable freedom to adopt different standards, and from this one might conclude that poverty's definition should depend entirely on their interests incentives. However these actors also must compete with one another for power and profits. Thus the salience of poverty as a public problem will be the result of the response of the defining third parties to those citizens whose choices (to vote, watch TV, purchase newspapers, books, and magazines, and make donations) support them.

While concrete evidence of these interactions is not easy to find, it does seem to be true that many higher-level actors believe they have some influence over public perceptions of poverty's severity; they take part energetically in debates about the poverty problem and whether or not it demands public attention. Official bodies charged with the task of defining poverty usually face politically-inspired criticisms (e.g. Citro and Michael, 1995). The assessment of War on Poverty policies in the U.S. seems to have involved a complex interaction of politics and social perceptions of poverty (Aaron, 1978). Politicians are accused of "sophisticated and 'scientific' attempt[s] to define [the poor] out of existence" (Harrington, 1984, p. 7). Think tanks and special interest groups present and interpret statistics that make poverty look like more or less of a problem than official statistics claim (e.g. Bartlett, 1996, pp. 58-59). These examples suggest that poverty perceptions are considered part of the ongoing political and social debate. The process that determines poverty perceptions seems little different from the process that determines poverty policy.

C. All societies seem to have some poverty

Debates about poverty seem rarely to produce the outcome that poverty is not a social problem. The consensus view seems to be that some people deserve help, and their poverty should not be ignored whatever the current state of poverty policy. This near-universal persistence of poverty can be observed in several ways. The persistence of poverty policy in all developed countries (Pierson, 1994; Burtless, 1994, p. 52) is indirect evidence that most people in most countries believe that at least a pre-transfer poverty problem exists.

More interesting is the evidence suggesting that post-transfer poverty has also persisted as a social problem. Official poverty rates, which are usually based on post-transfer income, have apparently never approached zero in any society in which they have been constructed (Fisher, 1995). The implication is that no government has declared a poverty standard so low that its own anti-poverty efforts have been successful in eliminating the poverty problem. One would think that the incentive would be quite strong to define the problem at some minimal level and then defeat it with an appropriate policy; failure to do so makes governments seem ineffective. Despite this incentive, official post-transfer poverty remains stubbornly positive wherever it is measured.

Even in countries without official post-transfer poverty indexes, there is persistent public interest in the poverty problem that remains after government has conducted its policy. Aaron (1978, p. 38) noted the phenomenon for the U.S., describing it as a difference between the official poverty problem and "poverty," the poverty problem as seen by the public. The U.S. War on Poverty in the 1960s created spending and growth initiatives that seem to have cut the official poverty rate by one-half. Yet it was perceived as a failure; the effect of the transfers on official poverty did not translate into an equivalent effect on "poverty." The implication is that the post-

transfer poverty problem remained severe despite a great increase in transfers; the public did not accept the view that the transfers had succeeded in defeating poverty. On the other hand, the public did not conclude that the failure to eradicate poverty meant that the effort should be abandoned. On the contrary, per capita means-tested spending on the poor *rose* by over 400 percent between 1968 and 1994 (source: Committee on Ways and Means, 1996 Green Book, p. 1317). By this standard, the War on Poverty and the principles on which it was founded were immensely popular. One can only conclude that the public's view of what made anti-poverty spending successful was different from the view of the academic world; perhaps the object was not the elimination of poverty but rather an increasing (but not overwhelming) effort against it.

The same point can be made using studies of OECD countries that reveal ongoing public concern about poverty in even the richest, most equal, and most aggressively redistributing countries in the world (Atkinson, Rainwater, and Smeeding, 1995; OECD, 1990; Hauser, 1989).¹ In literally no country is there a general consensus that transfers have been successful in defeating poverty. As poverty lines trend upward, transfer spending continues to increase, reducing poverty as a social problem but never eliminating it. In all countries, at all times, poverty both exists and is addressed seriously by public policy.

The apparently universal persistence of the poverty problem even in the face of growing transfers seems to suggest two things. First, it may be that the transfers affect how poverty is defined, so that rising transfers produce rising standards, which in turn make the transfers seem inadequate. Second, even given a fixed definition, there may be diminishing returns to anti-poverty spending.

These conclusions are not trivial; they go directly against the opinion voiced by many

authors that it ought to be comparatively simple to eradicate poverty permanently (Levine, 1970, p.6; see Lampman, 1974, p. 68 for a discussion). Galbraith (1958, p. 329), for example, says that "an affluent society, that is also both compassionate and rational, would, no doubt, secure to all who needed it the minimum income essential for decency and comfort." Similarly, Harrington (1962, p. 179) identifies poverty simply with the absence of goods that "our present state of scientific knowledge specifies as necessary for life as it is now lived in the United States." The language such authors employ suggests that fighting poverty is a technical problem, akin to the problem of building a space station or a highway.

Yet it seems apparent from the historical record that scientific knowledge is not the ingredient that defines poverty; rather it is socio-political action that defines poverty, and it seems to do so in a way that pays close attention to average living standards. If that is so, then defeating poverty may be much more difficult than these authors suggest. It is not a matter of setting an objective, scientific standard of living and then simply making sure everyone has the goods necessary to put that standard of living into practice. Even if one could do this in any one year, history suggests that society would soon raise its standards and discover that in fact poverty had not disappeared. Defining poverty is actually a social process rather than a scientific one, and this makes it hard to assess anti-poverty policy. If defeating poverty is impossible, under what conditions should one conclude that anti-poverty spending has succeeded?

D. Summary

This discussion yields several facts about poverty that will turn out to constrain significantly the set of allowable models. These facts are: 1) Poverty is relative and its definition

is changeable. 2) The definition is subject to political manipulation. 3) As a social problem, poverty persists through time, and both before and after transfers.

Together these facts present a fairly serious problem for the existing literature on altruism-based redistribution, most especially in requiring that the definition of poverty be endogenous. In the existing literature, the object of the voter's altruism is always exogenous rather than endogenous. In interdependent utility models (Hochman and Rogers, 1968), for example, each voter directly perceives the utility of the other voters, overlooking the fact that with the exception of close friends and family members, perception of the happiness of others is unquestionably indirect. This is even more true of poverty perceptions. Very few middle- and upper-class voters have direct perceptions of the utility of individual poor people; rather, some third party writes an article or a report about their well-being, which the wealthier agent then observes. But this only returns us to the central issues of the paper: how do these third parties define the group of people on whose well-being they report? How do they judge that well-being? Similar objections can be raised against the other models in the literature: in leaving the definition of the public "bad" of poverty exogenous, they are overlooking an aspect of reality. The next section explores whether ignoring the endogeneity of poverty makes any difference in what these models predict.

III. A Pure Altruism Model of Redistribution With Poverty Definition

A. Setup

To fix ideas, consider a model similar to the standard pure-altruism models in the literature. Agents in the model are middle- and upper-class voters. To account for the idea that

the poor have very little political power (Heclo, 1994, p. 397), assume that the voters in the model do not receive any of the spending that is redistributed; all cash assistance goes to the poor, who do not vote.² There are n voters (n odd), indexed $i = 1, \dots, n$, with incomes y_i . Voters are subject to a lump-sum tax t .³ Voter i 's consumption is $c_i = y_i - t$.

All tax monies, in an amount $T = nt$, are devoted to reducing poverty. Spending to reduce poverty may mean many things: giving cash to the poor, spending money on goods (education, housing, etc., as well as one's own labor in volunteering) and giving these to the poor. For simplicity, this spending will be referred to as "transfers" throughout.

The effect of transfers on the poverty problem depends on the way poverty is perceived. Let the severity of the pre- and post-transfer poverty problems in society be indicated by real numbers r and p respectively, bounded by 0 and 1, with larger numbers indicating greater severity. The severity of pre-transfer poverty is partially individual-specific and partially public. Specifically, agent i has her own observation of the severity of poverty, denoted o_i . As reported by government (and/or media and other social actors; the distinction does not matter for the results⁴), however, the severity of poverty is given as g . Voters assign a weight α , $0 \leq \alpha \leq 1$, to their own observations and $(1-\alpha)$ to government reports. Pre-transfer poverty is thus $r_i = \alpha o_i + (1-\alpha)g$. To solidify intuition one might think of a mental process in which each voter sets her own poverty line and has some sense of what fraction of the local population falls below it; this number is o_i . Government actors, however, have different poverty lines and a broader perspective, which result in different reported poverty rates. Distilling the various reports into a single number, the voter considers g the average reported poverty rate in the nation. Her overall perception of poverty is a weighted average of her own view of poverty and the country's.

Post-transfer poverty depends on pre-transfer poverty and on the total amount of spending, according to the function $p = p(r, T)$. Conceptually, some of the properties of this function are fairly obvious. It should rise with r and fall with T . Moreover it should be convex in transfers: diminishing returns dictate that as transfers increase each dollar should have less of an impact on poverty.⁵ Furthermore it seems sensible to assume $p(r, 0) = r$ and $p(0, T) = 0$. However there seems little reason to assert that the function is either convex or concave in r , and some comparative statics later in the paper are greatly simplified if $p(r, T)$ is linear in r . Hence assume $p_r > 0$, $p_{rr} = 0$, $p_T < 0$, $p_{TT} > 0$, and $p_{rT} \leq 0$.⁶ Recalling that $r_i = \alpha o_i + (1 - \alpha)g$, and that $T = nt$, $p_i = p(r_i, T)$ is convex in g and t , therefore $1 - p(r_i, T)$ is concave in both.

Voter ideal points maximize the utility function $u(c_i, 1 - p_i)$, separable and continuously twice-differentiable with $u_1 > 0$, $u_{11} < 0$, $u_2 < 0$, $u_{22} > 0$, where numerical subscripts $j = 1, 2$ indicate partial derivatives of utility with respect to its j th argument. The function implies pure altruism: voters receive utility from their transfers only because they reduce poverty, and transfers have the same effect regardless of who gives them. Inserting the above definitions, utility is a concave function of tax levels and the government definition of pre-transfer poverty:

$$u_i = u(y_i - t, 1 - p(\alpha o_i + (1 - \alpha)g, nt)) \quad (1)$$

B. Implications

In this setup each voter has some interest in how poverty is defined by the government, since its reporting activity influences her perception of the severity of poverty. Indeed with concavity the voter has a unique ideal point (g_i^*, t_i^*) . Given a distribution of ideal points over

(g,t) space, one can imagine that electoral competition will produce some outcome (g_0, t_0) . To be concrete, assume that there are two parties and that the party winning a majority-rule election has the power to set an official poverty line, which then generates an official poverty rate g between zero and one.⁷ Further, let the winning party also select the amount of transfer t . Thus both g and t are simply chosen by the dominant party in government. Since the issue space is two-dimensional the median voter theorem does not hold, of course. However some important results can be obtained without predicting which point will win under majority rule.

Omitting i-subscripts, for any voter the ideal point solves the following first-order conditions:

$$u_t = -u_l - u_2 n p_t = 0 \quad (2)$$

and

$$u_g = -(1 - \alpha) u_2 p_r = 0 \quad (3)$$

If one holds g fixed, the voter maximizes utility only with respect to the tax t . In that case equation 2 will yield a positive ideal point in t for at least some voters, which raises the possibility that redistributive spending to combat the public bad of poverty can be efficient. This is the basis of the altruism argument for redistribution.

If g is not fixed, however, but is itself part of the competition, both first-order conditions must be met. This yields the following paradoxical result:

Proposition 1: For all voters i , $g_i^ = 0$.*

Proof: In equation (3), both u_2 and p_r are positive and therefore (3) is negative for all g .

With utility monotonically negative in g , $g_i^* = 0$ for all i . ■

Thus all voters prefer that the government's official poverty rate be zero. (Any government can easily comply with these wishes by setting the official poverty line at a sufficiently low number, for example \$1.) With unanimous preferences over g , electoral competition is reduced to a one-dimensional battle over taxes. In equation 2, setting $g = 0$ yields a single-peaked distribution of ideal points in t ; by the median voter theorem the median of this distribution becomes the political outcome. Redistribution exists, but it is affected only by individual observations of poverty, o_i , while all political platforms contain a plank promising that "We in Party X believe there is no poverty in this country and we will see to it that official statistics reflect that belief."

If only for the latter outcome, this model is obviously in conflict with reality. The consequences for poverty policy are serious, however: manipulation of equation 2 indicates that the optimal tax rate of a given voter rises as g rises. [Editor: see mathematical addendum.] The reduction of g to zero, and the implied effort of political and media actors to minimize the seriousness of poverty, therefore reduces the amount of assistance going to the poor, *even if* all voters have fairly high individual observations of poverty's severity. The problem is that all voters lend some credence (measured by the parameter α) to what the social and political elite are saying about the poor. If the elites play down the problem of poverty, the voters will call for less redistribution and lower taxes. This hardly reflects the nature of real-world interactions between citizens and government about poverty; some if not most of these interactions involve efforts to raise social consciousness about poverty, not reduce it. Making poverty endogenous thus presents a serious problem for the standard altruism-based redistribution models in the literature.

This problem would not be quite as serious if one could make the case that poverty, as a social problem, is one that is directly perceived by voters. If α were quite close to 1, most of the government's efforts to hush up poverty would be ignored and transfer policy would be little affected. But is α close to 1 for the poverty problem? It depends on whether voters obtain some kind of direct experience of it. For example, road quality is an issue with a relatively high value of α , since most voters drive. National defense is an issue with a low value of α because typically very few voters have directly experienced an invasion of the country. Given the low levels of contact between poor and non-poor, it would be difficult to argue that poverty is closer to the road quality problem than it is to the national defense problem. Therefore α is probably low. As a result it is probably safe to conclude that the simple model not only overpredicts official down-playing of the poverty problem, it also seriously underpredicts the level of transfers.

IV. A more realistic altruism-based model of poverty

This is not the first paradox in the altruism model of redistribution; treated as an n-player game (i.e. without formal political competition or policy choice as here), the process of making transfers to the poor results in very small transfers under pure altruism when the number of voters is large. This paradox is resolved by Andreoni (1990), who shows that transfers can be large under an impure form of altruism in which the agent receives utility not only from the total contributions of all agents but also from her own contribution. The strategy here will be to explore similar modifications in order to sustain redistribution when the definition of poverty is endogenous.

A. Competing explanations for the paradox.

Before pursuing these alterations of altruism theory it is useful to consider, and argue against, a reasonable, common-sense resolution to the paradox posed by proposition 1. A very simple reason why government and media actors might not dismiss the poverty problem as predicted above is that they pay some price, in terms of consumers or votes, for announcing poverty rates greatly at odds with the average citizen's perceptions. Imagine the reaction if the prestigious Brookings Institution were to announce that, based on its scientifically-designed poverty line of \$1, no families in America were poor. Simply to maintain a reputation for expertise, the argument goes, it would behoove any such institution to keep its poverty lines somewhere near the common sense perceptions of the populace.

One could incorporate this in the model by introducing a third term in utility, $-\lambda(o_i - g)^2$, that measures the consistency between the agent's perceptions and social reports on poverty. With sufficiently high values of λ , this consistency incentive would swamp the incentive to down-play poverty that was derived in the preceding section. Government and media reports would then match the observations of the median voter with about the same frequency that tax levels match those desired by the median voter (i.e. fairly often).

Although this does offer a solution, it has a number of problems. First, it predicts that government and media actors will report the world in a way basically similar to the way the voters/consumers perceive it. But if these actors provide no new information, why are voters and consumers interested in them at all? Of course a market in self-confirming books and reports could exist and be quite large, but in such a market no book could appear that would change national consciousness on the issue of poverty. Those who experienced the work of Henry

Mayhew, Charles Booth, Michael Harrington, or Charles Murray would have difficulty accepting a model that produces such a prediction.

Second, the consistency argument requires voters to be more concerned about a party's perception of reality than it is about the party's policies. Note that the impact of party perception on redistribution policy has already been captured in the model in the preceeding section; the consistency argument maintains that voters have an *additional* desire, independent of policy, simply for consistency of views. While it seems beyond question that voters care somewhat about a party's views, their interest must be limited relative to their interest in what the party actually will do. The λ parameter is therefore probably not very large.

Third, this is in the end a brute-force solution: "Government agencies, think-tanks, and newspapers would not report odd poverty rates because they would be odd. And voters and consumers do not like oddness." It begs the question of why low poverty rates would be considered odd. In a world like that depicted in the preceding section, government and media actors would continually minimize the poverty problem. Assuming that citizens do lend these reports some credence and therefore consider them informative, presumably they would eventually update their perceptions of poverty downward. Their subjective poverty lines would gradually fall and their subjectively-perceived poverty rates (the o_i) would fall with them. Given enough time the model depicted above would produce perfect consistency between reported poverty and individually-perceived poverty, namely that both are zero. In other words, with these simple preferences the market for bad news would be empty, containing neither sellers nor buyers; and the consistency argument simply states that when there are buyers there must be sellers. This is true, but the fundamental question here is why there are buyers: Why do citizens

consume media and government reports about poverty and other bad news? What structure of preferences allows informative reports about poverty to raise individual utility?

Thus for short run reputational reasons one can easily accept that a simple desire for consistency would keep media and government actors from putting out reports that are greatly at odds with common perceptions of poverty. Yet this explanation seems weak from a long-run perspective. A long-run explanation for the persistence of poverty should rest on more fundamental arguments, involving specifically the very real compassion of average citizens for the poor and their very real interest in tax policy and the size of the welfare state.

B. Impure relative altruism.

To this end I will advance a model here that modifies impure altruism by making the utility received from the agent's own contribution depend on how severe the initial public problem is defined to be. This version of altruism will be called *impure relative altruism*, denoted q_i , and it enters as a third argument in the utility function. This argument of utility depends on taxes/contributions to fighting poverty, t , as well as the perceived severity of poverty, r_i : $q_i = q(r_i, t)$. Let $q(r_i, t)$ be concave; assume $q_r > 0$, $q_t > 0$, $q_{rr} < 0$, $q_{tt} < 0$, $q_{rt} > 0$, and $q(r, 0) = q(0, t) = 0$. Utility now takes the form

$$u_i = u(y_i - t, 1 - p(\alpha o_i + (1 - \alpha)g, nt), q(\alpha o_i + (1 - \alpha)g, t)) \quad (4)$$

Utility remains concave in g, t and each voter has a unique ideal point (g_i^*, t_i^*) derived from the following first-order conditions (i subscripts omitted):

$$u_t = -u_l - u_2 np_T + u_3 q_t = 0 \quad (5)$$

and

$$u_g = -u_2 p_r + u_3 q_r = 0 \quad (6)$$

Again because the issue space is two-dimensional it is not possible to predict without further assumptions which voter ideal point will be chosen by the electoral system. However even without a predicted political outcome certain results can be derived.

Proposition 2: $q_r > 0$ is necessary for $g_i^ > 0$, all i .*

Proof: Recall g (a poverty rate) is assumed bounded below by zero; if it is nonzero it must be positive (and below 1). Claim: $q_r \leq 0$ implies $g_i^* = 0$. (1) Assume $q_r = 0$. Then equation (6) reduces to equation (3) and the proof of proposition 1 applies; $g_i^* = 0$, all i . (2) Assume $q_r < 0$. Then by $p_r > 0$ and $u_3 > 0$, $u_g < 0$ for all i . Hence $g_i^* = 0$, all i . ■

Thus a necessary condition for government and media actors to report any poverty at all is that the own-utility aspect $q(r, t)$ of the citizens' anti-poverty efforts must become more intense when the poverty problem is perceived as more severe. The 'warm-glow' effect of fighting poverty must produce more utility when the poverty being fought is worse. If it does, then the reporting activity of government and media produces a utility benefit because it makes people aware of a problem that they would like to help solve. In essence, people pay money for bad news because bad news reminds us that our help for others *whom we do not see every day* is truly beneficial and therefore meaningful.

C. Examples that replicate historical poverty patterns

From the above we can conclude that impure relative altruism at least allows some positive reporting of poverty. Determining whether this model allows patterns of g and t that are realistic requires specific assumptions about the shape of the u , p , and q functions. This section presents some examples that produce “realistic” patterns, with the assessments of “realism” being made relative to the historical stylized facts presented in Section II. Principally, we want to know how economic growth affects the model. Can we replicate the historical patterns of the poverty problem by manipulating y ?

In order to come to concrete results it will be assumed that the political outcome corresponds to the ideal point of the median voter. The assumption can be justified on the grounds that these preferences are “close” to being Order Restricted (Rothstein, 1991) in income; the outcome of majority rule competition is unlikely to yield a point very different the ideal point of the voter with median income.⁸ Also the implications of border solutions will be fairly obvious in each example, so they will not be discussed explicitly for the most part.

C.1. Linear utility, p linear in r , q linear in t , all functions separable.

We begin with a simple formulation that nonetheless yields solutions for optimal g and t . Omitting i subscripts, let utility take the form $u = \tau c + \phi(1-p) + \theta q$ and let $p = r - \gamma \ln T$, $q = \beta \ln r + t$. Taking the first-order conditions and solving results in:

$$r^* = \alpha o + (1 - \alpha)g^* = \frac{\phi}{\theta\beta} \quad (7)$$

and

$$t^* = \frac{\phi\gamma}{n(\tau - \theta)} \quad (8)$$

(Note that with interior solutions, solving for optimal g is equivalent to solving for optimal r ; the expressions have been written with r on the left-hand side only to make them cleaner.) Together these conditions imply the following outcome for post-transfer poverty:

$$p^* = \frac{\phi}{\theta\beta} - \gamma \ln \frac{\phi\gamma}{\tau - \theta} \quad (9)$$

Pre-transfer poverty is a positive constant; the median voter desires the government to choose g^* to make (7) hold, given her poverty observation o . The optimal tax is also a constant that is, however, positive only under certain values of the parameters. Similarly post-transfer poverty is a constant but not necessarily a positive constant.

This example delivers a realistic pattern in that, for certain parameters values, it allows post-transfer poverty to be positive and independent of income. This accords well with the fact that post-transfer poverty seems to have remained a constant aspect of developed societies despite their phenomenal increase in wealth over the past 100 years. However, this historical constancy has not been produced by a roughly constant level of pre-transfer poverty being met by a roughly constant level of taxation, as in the model. Rather, in history poverty standards have risen and tax levels have risen with them. On that score this version of the model fares poorly, which is not surprising of course given that linear utility eliminates any effect of the changing marginal utility of income.

C.2. Cobb-douglas utility, linear p, linear q.

To allow some scope for marginal utility of income, let utility take the following form:

$$u = \tau \ln c + \phi \ln(1 - p) + \theta \ln q \quad (10)$$

with $0 < \tau, \theta, \phi < 1$. Let $p(r, T) = r - \gamma T$ and let $q(r, t) = r + \beta t$. The first-order conditions are:

$$F_l \equiv u_t = -\frac{\tau}{y-t} + \frac{\phi n \gamma}{1-r+\gamma n t} + \frac{\theta \beta}{r+\beta t} = 0 \quad (11)$$

and

$$u_g = -\frac{\phi}{1-r+\gamma n t} + \frac{\theta}{r+\beta t} = 0 \quad (12)$$

Solving (12) for r yields

$$r = \alpha o + (1-\alpha)g = \frac{\theta}{\phi+\theta} + \frac{\theta \gamma n - \phi \beta}{\phi+\theta} t \quad (13)$$

Thus r and hence g can be expressed as a linear function of g, in which the sign depends on the parameters. Post-transfer poverty can also be expressed as a function of t:

$$p = \frac{\theta}{\phi+\theta} - \frac{\phi}{\phi+\theta}(\beta + \gamma n)t \quad (14)$$

Inserting these formulas in (11) and taking the appropriate derivatives shows that $F_{lt} < 0$ and $F_{ly} >$

0. Hence (11) has a unique solution in t and it will be the case that $\partial t^*/\partial y > 0$. Thus rising

incomes are associated with rising tax and transfer levels; through (14) we know that this will imply falling post-transfer poverty levels. Its effect on pre-transfer poverty in (13) depends on the parameters. But even if higher incomes and transfer levels cause increases in pre-transfer poverty in (13), those increases will be more than offset by the effect of the transfers on post-transfer poverty in (14). In short, rising income is associated with falling poverty.

This example improves on the previous one in allowing rising incomes to have some effect on the definition of poverty. In particular, with the right parameters the model would allow r to rise with t ; since t rises with y , it would follow that richer societies have higher pre-transfer poverty standards, an outcome which accords well with the historical record. However this example has an obvious flaw in that post-transfer poverty always trends downward with income, implying that the post-transfer poverty problem today should be less important socially than it was 100 years ago. It predicts that eventually every society becomes rich enough that it becomes satisfied with its transfer levels and declares total victory in the War on Poverty. This seems especially at odds with actual experience.

C.3. Cobb-Douglas utility; p linear in r ; q linear in t .

I now construct an example in an effort to replicate as closely as possible the historical pattern of poverty (rising pre-transfer poverty, rising transfers, but constant post-transfer poverty). After testing a number of different functional form assumptions, the following ones performed best at producing the historical pattern. It is instructive that these functional forms are unique and unusual, because this implies that an altruism-based model can only replicate observed reality under quite restrictive assumptions. Thus if altruism is indeed an important

cause of anti-poverty spending, it apparently can only have its effect in the quite special and restricted way described in this section. Conversely, if these restrictions seem implausible, then that must imply that altruism is not a particularly important cause of the spending patterns one observes in reality; rather, the purely-self-interested bases of anti-poverty spending must be more important.

The functional assumptions that allow the altruism model to replicate reality most closely are as follows: Let utility take the Cobb-Douglas form given above, and let $p(r, T) = re^{-\gamma T}$ and $q(r, t) = tr^\beta$. The first-order conditions become

$$F_2 \equiv u_t = -\frac{\tau}{y-t} + \frac{\phi n \gamma r e^{-\gamma t}}{1 - r e^{-\gamma t}} + \frac{\theta}{t} = 0 \quad (15)$$

$$u_g = -\frac{\phi e^{-\gamma t}}{1 - r e^{-\gamma t}} + \frac{\theta \beta}{r} = 0 \quad (16)$$

where it should be recalled that $r = \alpha o + (1-\alpha)g$. Re-arranging (16) yields

$$r = \alpha o + (1-\alpha)g = \frac{\theta \beta}{\phi + \theta \beta} e^{\gamma t}$$

Inserting this into the definition of p gives

$$p = \frac{\theta \beta}{\phi + \theta \beta} \quad (18)$$

a positive constant. Inserting these formulas into (15) and taking derivatives reveals that $F_{2t} < 0$ and $F_{2y} > 0$. Hence $\partial t^*/\partial y > 0$. From this and (17) it follows that $\partial r^*/\partial y > 0$. Economic growth is

accompanied by higher pre-transfer poverty standards and higher transfers, but post-transfer poverty remains positive and constant. These patterns are a close match to the historical patterns.

V. Summary and Implications

It has proved possible to construct a model of endogenous poverty definitions that replicates fairly well the historical record of the poverty problem. With the right functional assumptions, rising wealth in society increases poverty standards. The perceived pre-transfer severity of the poverty problem rises, and society responds with higher levels of redistribution. Yet the rising standards prevent the elimination of poverty as a social problem; redistribution reduces poverty but fails to defeat it.

Two aspects of this outcome are worth highlighting. First, replicating the historical record has required very strong assumptions about the shape of the utility function, the impact of transfers on poverty, and the translation of anti-poverty efforts into individual warm-glow utility. Under general functional forms the trajectory of the poverty problem can be in any direction: poverty may remain constant regardless of income, or it may be totally defeated and eliminated as a social problem. It follows that the universal real-world persistence of poverty in developed countries constrains the set of acceptable theories quite severely.

How should one interpret the severity of the assumptions necessary to make altruism a historically accurate view of poverty and poverty policy? Certainly, the necessity of these assumptions is useful in that it tells us quite clearly how altruism must operate if it operates at all. We learn, for example, that the voter's altruism must rise when poverty becomes more severe, and that the marginal impact of poverty spending must fall as spending increases. These nuances

of voter sentiment shed considerable light on important puzzles, such as the surprising persistence of welfare spending despite apparent widespread dissatisfaction with it.

On the other hand, one can also interpret the uniqueness of the model as an argument against altruism as an explanation of the Welfare State. If altruism explains poverty spending only by taking an unusual form, then perhaps we should conclude that it simply does not explain poverty spending very well. In comparison, the pure self-interest explanations (self-insurance, social order, interest-group redistribution) are the more likely causes of social spending.

Yet if altruism matters as much as most commentators seem to think, the model has important implications for poverty policy. In that version, poverty will not be defeated by increasing wealth or rising transfers. The citizens raise their standards of pre-transfer poverty to keep post-transfer poverty constant. In such an environment a War on Poverty will certainly fail - if the object is to eliminate poverty as a social problem. Regardless of what the War on Poverty does for the poor, the public will remain concerned about their well-being and the problem of poverty will not vanish. To the extent that this version of the model reflects actual preferences, it would seem then that total victory is out of the question: should not evaluate an anti-poverty policy by asking whether it has the chance to eliminate poverty. Instead the standard should be whether the policy helps the poor to the degree that the public desires them to be helped. By that standard most Wars on Poverty are unqualified successes. They are a response to public pressure to increase or change aid to the poor; they match transfer levels to the public's level of compassion and concern for the poor, as elicited by government and media reports on their plight. In short, poverty is always and everywhere a moral problem. With moral problems the right standard of success is not total victory but maintained effort with continual improvement.

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Endnotes

1. If this is true then the poverty problem must persist despite a high level of overall well-being and equality. It seems to do so through the existence of inequality, in that severe poverty can be concentrated in pockets of the population - pockets that may be small but nonetheless evoke policy concern. It may have been this kind of argument that led to the War on Poverty in the U.S. (Galbraith, 1958; Harrington, 1962; Friedman, 1977). Examples of such concerns can be found in other countries. A recent study of Sweden (Vogel, 1990), shows 2 percent poverty among "management" but 24 percent poverty among the marginal group of "social welfare clients" (p.53). The author finds these numbers to be troubling evidence of stagnation in the general trend toward greater equality (p.70). Thus even if official poverty rates seem low, it is always conceivable that poverty rates in a sub-group may be high enough to warrant continued public attention, with the implication that the poverty problem persists even after transfers.
2. Note that this takes the model out of the context of interest-group redistribution (Meltzer and Richard, 1981; Kristov, et al., 1992). In that literature, interest groups muster support for transfers from other people to their own members; such behavior is undoubtedly an important aspect of real-world redistribution and the literature's explanation of it is sensible. This paper, however, is concerned with a different phenomenon, the mustering of political support among the middle- and upper-classes for transfers *from themselves* to the poor. I focus on altruism entirely because this is the most common explanation for anti-poverty spending that one hears about in public discourse. In response to this view, social scientists have done two things. First, they have pointed out that many voters may have entirely non-altruistic reasons for supporting social spending (income insurance, social order, and interest-group redistribution). Second, they have constructed formal arguments supporting the idea that voter altruism can lead to social spending. It is worth stressing that the purpose of the paper is not to dispute the real importance of the directly self-interested motives. It is rather to take the altruism argument at face value and to reassess it under somewhat more realistic assumptions about how poverty is defined. If in the process the altruism argument becomes less convincing, that outcome should be interpreted as an argument that the non-altruism bases of anti-poverty spending are relatively more important than altruism.
3. One could make redistribution costly by adding a leaky-bucket deadweight loss parameter. The addition changes the results in one way only: it allows one to show that the higher the deadweight losses of redistribution, the less redistribution will voters prefer. Otherwise the efficiency costs of redistribution do not affect any of the propositions presented. [Editor: see mathematical appendix with the parameter δ .]
4. The idea that political and social actors maneuver problems onto the political agenda is of course not new (see Riker, 1993a,b and references therein; Johnson, 1991; Gusfield, 1981). It must be the case that the actors who move issues onto the public agenda are themselves subject to competition. In the end the competition of politicians, bureaucrats, special interest groups, journalists, and academics for dollars and/or votes ensures that the problems which successfully appear on the public agenda do so because middle-of-the-road citizens desire them to be there. In

the model, rather than construct theoretical issue competitions among all of these actors (politicians, bureaucrats, media, etc.), the core idea is captured in a tractable way: let the governing political party have the power to define poverty; then two-party electoral competition determines the severity of the poverty problem. If representative voters (Rothstein, 1991) feel that poverty is a great social problem, the political parties will define it as such. A more complex model with several arenas of competition (in academia, media markets, etc.) would almost certainly produce the same basic outcome, so the more tractable set-up has been used here. Nonetheless the text will refer to the incentives and behavior of “parties” and “political, social, and media actors” interchangeably.

5. This is in contrast to the charitable goods literature, in which each dollar contributed to the public good has the same effect as any other.

6. Single subscripts on function symbols indicate the first partial derivative of the function with respect to the variable in the subscript. Double subscripts indicate second derivatives.

7. Suppose for example that the party who wins an election immediately sets up a board to do a scientific re-assessment of the official poverty line. When the board offers its recommendations, the leaders of dominant party direct the members of committees and agencies it controls to select from among the recommendations to produce a poverty line in accord with the leadership’s wishes. Of course in choosing a poverty line the leadership is implicitly choosing a poverty rate, that is, an official statement about the severity of the poverty problem.

8. The justification for examining the median-income voter is that this preference structure is “very nearly” order-restricted in a specific and well-defined sense. According to Rothstein (1991), order-restricted preferences generally allow the existence of a representative voter, a single voter whose preferences are also the ordering dictated by majority rule, regardless of the nature of the alternatives (e.g. the dimensionality of the issue space). Rothstein’s proofs do not apply here directly because his alternatives are drawn from a finite set while the issue space here is infinite. Nonetheless the intuition of Rothstein’s reasoning does apply. Suppose we order voters by income and present them with a pair of alternatives $A = (g_a, t_a)$ and $B = (g_b, t_b)$, and there is one voter indifferent between them. Let this voter have income y_j . Then if it is the case that all the voters richer than y_j prefer A to B and all the voters poorer than y_j prefer B to A (or vice versa), and that this is true regardless of the alternatives, then the preferences of the voter with median income are equivalent to the ordering provided by majority rule.

This reasoning “very nearly” applies here. [Editor: on this material see mathematical addendum, pp. 2-5.] Indeed if income were the only characteristic that distinguished the voters, it is possible to prove that these preferences are order restricted. However voters are also distinguished by the variety of their opinions about poverty. Returning to the above example, there will be voters richer than y_j who because of a particularly high or low observation of poverty will prefer B to A rather than A to B. Thus it will not be the case that the ordering of incomes and the ordering of preferences will be equivalent; order restriction fails.

Nonetheless an argument can be made for concentrating anyway on the preferences of the voter with median income. It seems reasonable to assume that the income distribution is much

wider than the distribution of opinion on poverty. Moreover, in the model the importance of personal observations is regulated by a parameter α , which is probably small. Under these circumstances the preference ordering by income will be nearly decisive, in the sense that the great majority of voters richer than y_j will prefer A to B, and the great majority poorer than y_j will prefer B to A. Thus for this particular application the ordering of incomes is very close to the ordering of preferences; preferences are “very nearly” order restricted. In other words, the outcome of electoral competition can only be a point not very different from the median voter’s ideal point.

The argument can be presented technically and is available from the author on request.

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Profit Sharing (with workers) Facilitates Collusion (among firms)

**Dan Bernhardt
Christopher Chambers**

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Profit Sharing (with workers) Facilitates Collusion (among firms)

Dan Bernhardt¹
University of Illinois

Christopher Chambers ²
University of Rochester

June 13, 2001

¹463 Wohlers Hall, University of Illinois, Champaign, Illinois 61820, danber@uiuc.edu

²Department of Economics, University of Rochester, Rochester NY 14627

Profit Sharing (with workers) Facilitates Collusion (among firms)

Abstract

This paper shows how profit sharing by firms with workers always facilitates increased collusion among firms in a dynamic oligopoly environment with uncertain demand. Expected firm profits are increased both if worker wages are tied to market conditions, or if workers instead receive a share of firm profits. We first show that firm profits are always increased by tying wages to market conditions. The optimal profit-sharing agreement generally features only partial profit sharing, because profit sharing raises the expected price-wage differential, but reduces price-wage variability. The attraction of profit-sharing agreements with workers is even greater. Indeed, we show that for any cartel size, there are always market conditions for which expected firm profits are increased simply by transferring some expected profit to workers, through the impact of this transfer on the incentive to cheat on the cartel.

1 Introduction

This paper addresses a basic contracting issue: How does the sharing of profits between a firm and its employees affect the ability of firms to support collusive oligopoly pricing and oligopoly profits? We explore this question in the dynamic oligopoly environment with uncertain demand first characterized by Julio Rotemberg and Garth Saloner (1986).

We obtain a stark answer: Profit sharing between firms and their employees *always* facilitates collusion between firms in the output market, raising expected firm profits. Indeed, we show that expected profits can even be raised by *giving* workers a share of firm profits, *without extracting a corresponding reduction in wages*. However, the optimal profit sharing agreement features only partial profit sharing, so that, along the equilibrium path, net firm profit rises monotonically with the demand realization.

It is important to emphasize that our findings do not revolve around either worker moral hazard or insurance explanations. In our barebones economy there is no problem eliciting efficient worker effort; profit sharing is not required to overcome worker moral hazard. Further, all parties are risk neutral, so there is no role for insurance.

Rather, the driving economic force is that profit sharing between workers and firms favorably impacts on the strategic interaction of firms in the output market. The economics underlying our results can be gleaned from recalling equilibrium outcomes in Rotemberg and Saloner (1986). There, firms repeatedly interact in an economy with independently and identically distributed demand shocks. Because expected future collusive profits do not depend on the period demand shock, but the gains from cheating on the cartel rise with period demand, firms can support monopoly profits only if the demand shock is sufficiently low. If demand is too high, then total industry output is increased beyond the monopoly level just until firms cease to have an incentive to cheat on the cartel.

In this environment, consider the impact of tying worker compensation to market conditions. Incentive compatible profit sharing with workers effectively lowers worker compensation in low demand states, and raises worker compensation in high demand states. But it is in the low demand states where firms do not have an incentive to cheat on the cartel, so that in these low demand states greater firm profits are then realized due to the lower worker

compensation. In contrast, in sufficiently high demand states, firm profit is completely unaffected by increased worker compensation, as profit is already constrained by the heightened incentive to cheat on the cartel.

The optimal profit sharing agreement generally features only imperfect profit sharing because while profit sharing increases the expected price-worker compensation difference, it also reduces the variance of this difference. Since the period profit function is convex, as profit sharing is increased, eventually expected firm profits decline (unless it is the case in the Rotemberg and Saloner environment that monopoly profits cannot be sustained even for the lowest demand realization). However, since the variance has only a second order impact on firm profits, but raising the expected price-worker compensation difference has a first order impact on firm profits, it follows that some profit sharing is always optimal.

We first consider an environment in which there is not explicit profit sharing between firms and workers, but suppose that workers' wages can be tied to market conditions in a linear fashion, so that wages are higher when market demand is higher. This framework highlights the tradeoff of increased profit sharing on (i) the expected price-wage difference and (ii) the variance of the price-wage difference. In this environment, we show that incentive compatible profit sharing between firms and workers always allows firms to capture increased profit. This result has the more general implication that the optimal wage contracts for firms are not independent of demand.

We then explicitly allow for profit sharing, so that employees can receive a share of firm profits. Here another benefit of profit sharing on the incentive of a firm to cheat on the cartel is revealed: If a firm cheats by increasing output and hence profits, it must (i) hire more workers to produce the output, and (ii) pay higher compensation to each worker because its profits are higher. Consequently, an explicit profit sharing agreement with workers reduces a firm's incentive to cheat on a cartel. To highlight this latter feature, we provide sufficient conditions under which firms can increase expected profits simply by *giving* their employees a share of firm profits, without demanding a lower wage in return. We show that for a cartel of any size, there exist market conditions such that giving workers a share of firm profits is optimal; the reduction in the incentive to cheat on the cartel supported by this transfer to workers supports monopoly pricing in enough additional higher demand states to more than

offset the transfer of surplus to employees.

2 The Model

The basic framework is that of Rotemberg and Saloner (1986): n firms producing a homogeneous good repeatedly interact in an infinite horizon economy with stochastic demand. Period market demand at time t is given by

$$D_t(\theta_t, P_t) = \theta_t - P_t,$$

where θ_t is an independently and identically distributed demand shock, with distribution function F on its positive support, $[\underline{\theta}, \bar{\theta}]$.¹ The risk neutral firms share common discount factor $\beta \in (0, 1)$, and compete in prices in the output market.

Because we want to study the impact of profit sharing between workers and firms, we have to model the production function of a firm more explicitly than do Rotemberg and Saloner. We assume that one hour of labor is required to produce one unit of the good. Workers are risk neutral, and have an outside opportunity of $\bar{w}$ per hour, which, without loss of generality, we normalize to zero.

Absent profit sharing, our framework corresponds to Rotemberg and Saloner. Firms support maximal period profits by threatening to revert to marginal cost pricing if any firm ever deviates from the collusive pricing agreement. Let $\Pi^m(\theta) = \left(\frac{\theta - \bar{w}}{2}\right)^2$ denote monopoly profits if the demand shock is θ , and let V represent expected period profit per firm along the equilibrium path in which firms co-operate. Along a co-operative path, firms split total profit equally. Given a demand shock θ , monopoly profits are sustainable if and only if

$$\left(\frac{\Pi^m(\theta)}{n}\right) + \left(\frac{\beta}{1 - \beta}\right) V \geq \Pi^m(\theta).$$

That is, incentive compatibility mandates that the expected profits from continued cooperation (the left hand side), exceed the one-time gains from cheating on the cartel by lowering price slightly to capture the entire market profit (the right hand side). The following proposition formalizes this intuition, where $\Pi^s(\theta)$ is sustainable profit in state θ .

¹None of our results qualitatively depend on the independently and identically distributed nature of the demand shocks. Demand shocks could follow a Markov process, or there could be cycles in demand as in John Haltiwanger and Joseph Harrington (1991).

Proposition 1 (Rotemberg and Saloner) *Suppose that monopoly profits can be sustained in some demand states, but not in others. That is, there exists a $\theta^* \in (\underline{\theta}, \bar{\theta})$ which solves*

$$\left(\frac{\beta}{1-\beta} \right) \left[\int_{\underline{\theta}}^{\theta^*} \Pi^m(\theta) F(d\theta) + (1 - F(\theta^*)) \Pi^m(\theta^*) \right] = (n-1) \Pi^m(\theta^*).$$

Then optimal pricing strategies support monopoly profits in demand states $\theta \geq \theta^$, but only support profits of $\Pi^m(\theta^*)$ in higher demand states $\theta > \theta^*$:*

$$\Pi^s(\theta) = \begin{cases} \Pi^m(\theta), & \theta \leq \theta^* \\ \Pi^m(\theta^*), & \theta > \theta^* \end{cases}.$$

In particular, along the equilibrium path, for low demand states $\theta \leq \theta^*$, firms set prices equal to the monopoly price of $\frac{\theta}{2}$, and each firm produces $\frac{\theta}{2n}$. For higher demand realizations, $\theta > \theta^*$, firms set price at $\frac{\theta^*}{2}$, less than the monopoly level. Higher prices cannot be sustained in these high demand states, as firms would have an incentive to cheat on them to garner the entire market profit.

2.1 Profit sharing: Demand-linked wage contracts

We first explore how outcomes are affected when worker compensation is tied to market conditions. Each period, before any demand shock is realized, firms and workers can sign binding wage agreements, in which the wage depends on the market shock. For simplicity, we focus on linear wage contracts,

$$\omega(\theta_t, a_t^i) = \omega(a_t^i) + a_t^i(\theta_t - E(\theta)).$$

Here, a_t^i is a firm specific variable which weights the difference between the current demand state and the expected demand state, and $\omega(a_t^i)$ is the fixed component of compensation that does not depend on the demand shock. The wage contracts signed at each firm are public information.

Incentive compatibility mandates that workers expect to beat their reservation alternative when signing the wage contract:

$$\int_{\underline{\theta}}^{\bar{\theta}} (\omega(a_t^i) + a_t^i(\theta - E(\theta))) Q(\theta, a_t^i) F(d\theta) \geq \int_{\underline{\theta}}^{\bar{\theta}} \bar{\omega} Q(\theta, a_t^i) F(d\theta).$$

Here, $Q(\theta, a_t^i)$ is a function detailing the total quantity of labor hired when the demand realization is θ and the wage contract parameter is a_t^i . For the moment, take $Q(\theta, a_t^i)$ to be

increasing in θ . If $a_t^i = 0$, then $\omega(\theta_t, a_t^i) = \bar{\omega}$. However, if $a_t^i \in (0, 1)$, then because (i) worker compensation is higher in high demand states, and (ii) firms produce more output in these high demand states (where workers beat their reservation alternative), the firm can lower the fixed component of wages, $\omega(a_t^i)$, and the wage contract will still be incentive compatible. Finally, if $a_t^i = 1$, then the firm is ‘fully-insured’ against the demand shock, so that its output level (in a symmetric equilibrium in which all firms choose $a_t^i = 1$) will not vary with market conditions, so that again $\omega(\theta_t, a_t^i) = \bar{\omega}$.

After the wage contract is signed, the period demand shock is realized. Upon observing the demand shock, given the history of past demand shocks, wage contracts and past price selection, firms simultaneously decide on prices. That is, each firm’s strategies are a sequence of functions $\{(a_t^i, P_t^i)\}$ where a_t^i maps histories of length $t - 1$ into wage contracts, and P_t^i maps from histories of length $t - 1$ together with (a_t, θ_t) into a price selection.

Before proceeding with our analysis, we first note that the assumption that firms and workers can sign binding period wage contracts is made only for simplicity; our findings extend when binding contracts cannot be signed. When binding contracts can be signed, in equilibrium, firms extract all surplus in wage negotiations, so that a worker’s *ex ante* incentive compatibility constraint will hold at equality. Then, *ex post*, if θ_t is sufficiently low, workers will regret signing the binding wage contract. However, to retrieve *ex post* incentive compatibility for workers even in the lowest demand state $\underline{\theta}$, firms only need to give workers a slightly higher fixed wage component, $w(a_t^i)$, and threaten never to hire a worker who rejects a wage offer. Section 2.2 offers a more complete characterization of *ex post* incentive compatible profit sharing.

Our analysis focuses on symmetric equilibrium paths; so, from now on, without loss of generality, we drop the index i from strategies. We look at firm trigger strategies in which if any firm has deviated in the past from the specified symmetric wage contract and pricing choices, then each firm sets price equal to marginal cost in the future. Such strategies support maximal firm profits.

Along a symmetric action path, period market profit, $\Pi_t(\theta_t, P_t)$, is given by

$$\Pi_t = (\theta_t - P_t) (P_t - (\omega(a_t) + a_t(\theta_t - E(\theta)))) .$$

Along the equilibrium path, firms choose the wage contract parameter, a_t , and period price

function $P(a_t, \theta_t)$, to maximize the sum of discounted expected profits subject to the constraint that the wage contract be incentive compatible for workers. Since the optimal wage contract parameter will not vary with time, we denote it simply by a , and let

$$\Pi^m(\theta, a) = \left(\frac{\theta + a(E(\theta) - \theta) - \omega(a)}{2} \right)^2$$

denote monopoly profits as a function of demand shock and wage contract parameter, a . We consider symmetric equilibrium outcomes in which each firm produces an identical amount of output, splitting profits equally.

As in Rotemberg and Saloner, the expected profit-maximizing strategy profile along the equilibrium path consists of a pricing schedule such that there exists some $\theta^*(a)$ for which each firm uses monopoly pricing for $\theta \leq \theta^*(a)$, and prices so that period profits are just equal to $\Pi^m(\theta^*(a), a)$ for $\theta > \theta^*(a)$. Off the equilibrium path, firms choose prices that generate zero profit.

Denoting these sustainable profits as a function of demand shock and wage contract by $\Pi^s(\theta, a)$, we have

$$\Pi^s(\theta, a) = \begin{cases} \Pi^m(\theta, a), & \theta \leq \theta^*(a) \\ \Pi^m(\theta^*(a), a), & \theta > \theta^*(a) \end{cases}.$$

Therefore, the expected profits for any given firm under these strategies are:

$$(1/n) \sum_{t=0}^{\infty} \beta^t E[\Pi^s(\theta, a)].$$

The equilibrium level of a^* maximizes these profits (where the worker incentive compatibility constraint is now implicit). Our first proposition details that in equilibrium, as long as firms cannot support monopoly profits in the Rotemberg and Saloner environment in *every* demand state, then it is always beneficial for firms to tie worker compensation to market conditions:

Theorem 1 *If $\theta^*(0) < \bar{\theta}$, so that monopoly profits cannot be supported in all demand states, then firms optimally tie worker compensation to the demand realization, setting $a^* > 0$.*

Since we show that the optimal linear wage contract features tied wages, an immediate corollary is that the optimal unconstrained wage contract also features wages that are non-trivially tied to demand.

Figure 1: Change in Expected Profits when $\theta^*(0) < E[\theta]$.

To understand Theorem 1, consider a version of the model in which firms do not extract lower fixed wage concessions in return for demand sensitive wages, so that the wage is $\bar{w} + a(\theta - E[\theta])$. Since firms produce more when demand is higher, such wage contracts transfer some of the surplus generated by increased collusion to workers, reducing the profitability of tying wages to market conditions. Consider first Figure 1, which contrasts period monopoly profits when firms do not tie wages to market conditions, $\Pi^m(\theta, 0)$, with monopoly profits when they do, $\Pi^m(\theta, a > 0)$. Note that the two profit levels are equal if $\theta = E[\theta]$, so that $\bar{w} + a(\theta - E[\theta]) = \bar{w}$. Figure 1 depicts the case where if wages are not tied to market conditions, then the maximal demand state that supports monopoly profits, $\theta^*(0)$, is less than $E[\theta]$. In particular, this implies that period monopoly profits are not sustainable for demand shocks exceeding $E[\theta]$; for all $\theta \geq \theta^*(0)$, maximal sustainable profits are just $\Pi(\theta^*(0), 0)$. In sharp contrast, using the same continuation payoffs, if $a > 0$, greater collusive profits can be supported for all $\theta < \theta^*(0)$, and there is no offsetting reduction in period profits for $\theta > \theta^*(0)$. The profit increase from tying wages to market conditions corresponds to the shaded area A in the diagram. Further, since expected period profits are increased, continuation payoffs must also be greater, so that more collusive pricing can be supported even if $\theta > \theta^*(0)$. Indeed,

Figure 2: Change in Expected Profits when $\theta^*(0) > E[\theta]$.

the reason that tying wages perfectly to market conditions, *i.e.* setting $a = 1$, is not optimal is that eventually, monopoly profits can be supported for demand realizations $\theta > E[\theta]$, creating a cost to tying wages to market conditions.

Figure 2 illustrates this second case, where $\theta^*(0) > E(\theta)$. Here, by tying wages to demand firms gain profit in low demand states (shaded area A), but lose profit in high demand states (shaded area B). If $B > A$, then it is better not to tie wages to market conditions at all than to tie them to the degree that $B > A$.

What the proof of theorem 1 does is show that when firms extract all surplus in contracting from workers, expected firm profits are always increased by slightly tying wages to market conditions, *i.e.* by choosing an $a > 0$ that is sufficiently small. Since $w(a) < \bar{w}$, the expected price-wage differential is greater when wages are tied to the demand realization. However, the variance of the price-wage differential is reduced. Since the profit function is convex, reducing this variance reduces expected profits. But, since the increase in the mean of the price-wage differential has a first order impact on expected profits, whereas the reduction in its variance has a second order impact, it follows that slightly tying wages to market

conditions is always optimal.

Formally, we show in the appendix that

$$\frac{\partial}{\partial a} E[\Pi^s(\theta, a)]|_{a=0} = \int_{\underline{\theta}}^{\theta^*(0)} \theta \left(\frac{E(\theta) - \theta - \frac{\partial \omega(a)}{\partial a}|_{a=0}}{2} \right) F(d\theta) \quad (1)$$

$$> \int_{\underline{\theta}}^{\bar{\theta}} \theta \left(\frac{E(\theta) - \theta - \frac{\partial \omega(a)}{\partial a}|_{a=0}}{2} \right) F(d\theta) \quad (2)$$

$$> \frac{1}{2} \int_{\underline{\theta}}^{\bar{\theta}} \theta \left(E(\theta) - \theta - \frac{\int_{\underline{\theta}}^{\bar{\theta}} (E(\theta) - \theta) (\theta/2) F(d\theta)}{\int_{\underline{\theta}}^{\bar{\theta}} (\theta/2) F(d\theta)} \right) F(d\theta) = 0, \quad (3)$$

where $\Pi^s(\theta, a)$ in this expression is the sustainable level of period one profit when after period one, all firms use the Rotemberg and Saloner equilibrium ($a = 0$).

In general, significant tying of wages to market conditions may be optimal. Figure 3 illustrates how the optimal degree to which wages should be tied to market conditions varies with $\theta^*(0)$, the highest demand realization for which monopoly profits can be supported. Note that $\theta^*(0)$, which rises with the discount factor β and falls with number of firms, n , is a sufficient statistic for the primitives describing the economy. Figure 3 illustrates the optimal wage-tying rule when the demand shock is uniformly distributed on $[1, 10]$. As collusion becomes sustainable in more demand states, the degree to which wages should be tied to market conditions declines. That is, a cartel that can sustain monopoly profits in very high demand states has more to lose by signing wage tying contracts with workers than a cartel that cannot. Observe that perfectly tying wages to market conditions is optimal only when, in the absence of tying wages to market conditions (*i.e.* in the Rotemberg and Saloner environment), the cartel cannot sustain monopoly profits even in the worst demand state.

2.2 Binding wage contracts

We now return to our earlier discussion regarding the robustness of our results to relaxing the assumption that wage contracts are binding. Consider now an environment in which workers are infinitely lived, with common discount factor $\gamma \in (0, 1)$, and that *after* the demand shock is realized, workers make a decision whether or not to abide by the wage agreement. Firms could then use strategies which threaten to not hire a worker in the future if the worker ever chooses not to abide by the wage contract terms. Firms must now augment the fixed wage

Figure 3: Optimal wage-tying parameter a as a function of $\theta^*(0)$

component to $\omega(a) + \epsilon(a)$, where $\epsilon(a)$ provides workers just enough expected surplus from the future worker-firm relationship to ensure that it is incentive compatible *ex post* for workers to supply their labor. *Ex post* incentive compatibility requires that the workers willingly accept the wage contract even in the worst demand state, $\underline{\theta}$,

$$(\omega(a) + \epsilon(a) + a(\underline{\theta} - E(\theta))) \left(\frac{\underline{\theta}}{2} \right) + \left(\frac{\gamma}{1-\gamma} \right) \int_{\underline{\theta}}^{\bar{\theta}} (\omega(a) + \epsilon(a) + a(\theta - E(\theta))) Q(\theta, a) F(d\theta) \geq 0,$$

where the right-hand side reflects the normalization, $\bar{\omega} = 0$.

For any given $\epsilon(a) > 0$, this relationship is always satisfied for all γ sufficiently close to one. Indeed, if workers are arbitrarily patient, firms can obtain *ex-ante* incentive compatibility of workers by offering a vanishingly small fixed wage increase, $\epsilon(a) \rightarrow 0$. In turn, this implies that our analysis extends immediately. More generally, if workers are more impatient, then firms must offer workers a greater share of the surplus from tying wages to market conditions. In this case, tying wages to market conditions raises expected collusive firm profits only if, in the absence of tying wages, it is sufficiently difficult to support monopoly profits, *i.e.*, if $\theta^*(0)$ is sufficiently small relative to the other parameters characterizing the economy.

2.3 Pure Profit Sharing

It is straightforward to show that if, rather than tying wages to market conditions, firms share profits and extract the lowest incentive compatible fixed wage in return, it is always optimal for firms to share some of their profits with their workers. This leads us to pose a more ambitious question, asking under which conditions can firms raise profits by actually *giving* a share of profits back to workers, *without demanding any wage concessions*? The structure of the economy is the same as before, save that we now assume that one worker produces one unit of good. The period t compensation given to workers is $\bar{w} + a_t \Pi_t(\theta_t)$, where

$$\Pi_t = (\theta_t - P_t)(P_t - \bar{w}),$$

where a_t is the profit sharing rule, and the demand shock is θ_t . Incentive compatibility for workers is not an issue, as period profits are always positive. We again maintain the assumption that $\bar{w}$ is normalized to zero.

We study symmetric equilibria in which for any a , firms use optimal pricing whenever sustainable. To support such pricing, upon deviation by any firm, firms use marginal cost pricing. Letting V denote the expected period profits along the equilibrium path from colluding, we see that optimal profits (denoted here by Π^*) in period zero are sustainable for state θ given profit sharing level a when

$$\frac{(1 - aL(\theta, a)) \Pi^*(\theta)}{n} + \left(\frac{\beta}{1 - \beta} \right) V \geq (1 - anL(\theta, a)) \Pi^*(\theta)$$

Here, $L(\theta, a)$ is the total number of workers needed per firm to produce the profit maximizing output. The left hand side of the above inequality represents expected discounted profits from continued collusion. Profits per firm are $\Pi^*(\theta)/n$, but each firm gives a share of $aL(\theta, a)$ away to workers, according to the profit-sharing agreement. Continued cooperation in the current period implies that the expected discounted future payoffs are then $\frac{\beta}{1-\beta} V$.

A firm that deviates from the cartel agreement optimally does so by marginally undercutting the price set by others. If a firm deviates it can obtain as much of the entire market as it desires. If the deviating firm meets the entire market demand, it can earn a profit of $\Pi^*(\theta)$, but it must return a share $anL(\theta, a)$ back to its employees. Because the firm must hire more workers to produce the entire market output, not only must each worker be paid more if it

cheats on the cartel, the firm must also pay more workers. In turn, this reduces the attraction of cheating on the cartel. It is for this reason that simply signing contracts that give workers a share of firm profits can raise expected cartel profits, a form of ‘addition by subtraction’.

The following result then obtains.

Theorem 2 *Suppose that the demand shock, θ , is uniformly distributed on $[\underline{\theta}, \bar{\theta}]$.² Then a sufficient condition for expected cartel profits to be raised by giving workers a share of profits, i.e. setting $a^* > 0$, is*

$$\left[2(n+1)\bar{\theta}\theta^*(0) - (2n+1)\theta^*(0)^2 - \bar{\theta}^2 - \bar{\theta}\sqrt{\bar{\theta}^2 - \theta^*(0)^2} + \theta^*(0)^2 \log \left(\frac{\bar{\theta} + \sqrt{\bar{\theta}^2 - \theta^*(0)^2}}{\theta^*(0)} \right) \right] > \frac{\theta^*(0)^4 - \underline{\theta}^4}{2\theta^*(0)^2},$$

where $\theta^*(0)$ is the maximal demand shock such that monopoly profits can be supported in the absence of profit sharing (i.e. in the Rotemberg and Saloner environment). In particular,

- (i) for any n , given $\underline{\theta}$, if $\theta^*(0) - \underline{\theta}$ and $\bar{\theta} - \underline{\theta}$ are sufficiently small, firms can raise expected cartel profits by giving workers a share of profits,
- (ii) for any $(\underline{\theta}, \bar{\theta}, \beta)$, there exists a $n(\underline{\theta}, \bar{\theta}, \beta)$ such that for all $n \geq n(\underline{\theta}, \bar{\theta}, \beta)$, profit sharing is optimal, and
- (iii) for any $(\underline{\theta}, n)$, there exists a $\bar{\theta}(\underline{\theta}, n)$ such that for any $\bar{\theta} < \bar{\theta}(\underline{\theta}, n)$ there exists a $\beta^*(\underline{\theta}, \bar{\theta}, n) < 1$ such that for any $\beta \leq \beta^*(\underline{\theta}, \bar{\theta}, n)$, profit sharing is optimal.

The proof proceeds by finding conditions under which firms can support greater expected cartel profits simply by giving workers a very small share, a , of those profits, rather than setting $a = 0$. In the neighborhood of $a = 0$, firms optimally set price equal to the monopoly price whenever the monopoly price can be supported, and if a firm decides to deviate from the cartel agreement, it optimally supplies the entire market. For larger profit-sharing rules, firms price in excess of the monopoly price whenever possible, choosing price to maximize *residual* profits retained by firms, $(1 - aL)(\theta - L)L$, with associated optimal price, $\frac{2a\theta - 1 + \sqrt{a^2\theta^2 - a\theta + 1}}{3a}$.

²The uniform assumption is made only to simplify the characterization.

While raising price above the monopoly level reduces total profit, it also reduces the number of workers hired, which, in turn, increases the net profits that cartel members actually retain. For similar reasons, if a is substantially greater than zero, a deviating firm would choose to ration some of the market, rather than meet the entire market by hiring the necessary number of workers. Thus, Theorem 2 provides a sufficient, but not necessary condition, for net cartel profits to be raised by giving workers a share of profits.

The intuition underlying part (ii) of Theorem 2 is that as n rises, $\theta^*(0)$ decreases, so that the gain to sharing profits is larger. Moreover, each firm employs fewer workers so that the profit that each firm returns to its workforce declines, while a deviating firm must return the same entire market profit back to its workforce, independently of n . Figures 4-6 shows how the marginal impact on expected period profits induced by sharing a marginal amount of profits with workers varies with $\theta^*(0)$ for $n \in \{2, 4, 10\}$ when the demand shock is uniformly distributed on the interval $[0, 10]$. For these parameter values, (i) when there are two firms, marginal profit sharing is never optimal; (ii) if there are four firms, marginal profit sharing is only optimal for intermediate values of θ^* ; and (iii) if there are ten or more firms, marginal profit sharing is optimal unless θ^* is very large, in which case period monopoly profits can almost always be supported even in the absence of profit sharing.

3 Conclusion

This paper offers an explanation for the prevalence of profit-sharing agreements between workers and firms. Importantly, our explanation revolves around neither moral hazard, nor risk sharing. Rather, we show that profit-sharing agreements between workers and firms can facilitate collusion among firms in the output market. We highlight two features of profit-sharing agreements: (a) profit sharing ties worker compensation to market conditions, so that worker compensation is higher when demand is higher, and (b) if, to cheat on the cartel, the firm must hire more workers, then the firm must return a greater portion of those profits back to workers, reducing the attraction of cheating on the cartel.

We first show that it is always optimal to tie worker compensation to market conditions. Tying compensation to market conditions raises the expected price-wage differential, but reduces its variance. Since raising the expected price-wage differential has a first order impact

Figure 4: $n = 2$

Figure 5: $n = 4$

Figure 6: $n = 10$

on expected profits, but reducing its variance has only a second order impact on profits, it follows that it is always optimal to tie compensation to market conditions at least to some limited extent. We illustrate that while it is not optimal to perfectly tie worker compensation to market conditions, significant tying remains optimal.

We then focus on how the incentives to cheat on the cartel are affected if, to do so, a firm must hire more workers. A powerful and striking result obtains: For any cartel size, there are always market conditions for which firms can increase expected profits simply by *giving* workers a share of the profits that the firm earns. That is, the increased ability to collude in the output market more than offsets the reduced profit share that the firm receives.

The bottom line is that stock options and other profit sharing agreements may do more than provide workers the incentive to work hard; they may help firms support collusively higher prices in the output market.

4 Appendix

Proof of Theorem 1: We first consider strategies in which after the initial period, firms use strategies which require zero profit sharing, but otherwise, support optimal payoffs. We show that even with these sub-optimal strategies, that wages should be tied to market conditions in period 1. Thus, we let $K = E[\Pi^s(\theta, 0)/n]$, as above. In this appendix, $\Pi^s(\theta, a)$ refers to the sustainable level of profits in period one given these strategies when demand is θ , and a is chosen.

Let $Q(\theta, a)$ denote the associated optimal period one quantity produced when the demand shock is θ , i.e. the output level that supports $\Pi^s(\theta, a)$:

$$Q(\theta, a) = \begin{cases} (\theta/2) & \theta \leq \theta^*(a) \\ \left(\theta + \sqrt{\theta^2 - \theta^{*2}(a)}\right)/2 & \theta > \theta^*(a) \end{cases}$$

where $\theta^*(a)$ is an implicit function of K , solving

$$\Pi^m(\theta^*(a), a) = \left(\frac{n}{n-1}\right) \left(\frac{\beta}{1-\beta}\right) K.$$

The worker's incentive compatibility constraint is

$$\int_{\underline{\theta}}^{\bar{\theta}} (\omega(a) + a(\theta - E(\theta))) Q(\theta, a) F(d\theta) = \int_{\underline{\theta}}^{\bar{\theta}} \bar{\omega} Q(\theta, a) F(d\theta).$$

The left hand side of the equality is what workers obtain by working at the wage structure; the right hand side is what they get if they use their labor to obtain their outside opportunity. In equilibrium, this constraint holds as an equality as firms will never choose $\omega(a)$ any higher than necessary.

We prove the theorem using a sequence of lemmas.

Lemma 1 At $a = 0$,

$$\frac{\partial \omega(a)}{\partial a} = \frac{\int_{\underline{\theta}}^{\bar{\theta}} (E(\theta) - \theta) Q(\theta, 0) F(d\theta)}{\int_{\underline{\theta}}^{\bar{\theta}} Q(\theta, 0) F(d\theta)}.$$

Proof: Recall, as $\bar{\omega} = 0$,

$$\int_{\underline{\theta}}^{\bar{\theta}} (\omega(a) + a(\theta - E(\theta))) Q(\theta, a) F(d\theta) = 0.$$

This expression establishes that $\omega(0) = 0$. Differentiating each side of the equality with respect to a obtains:

$$\int_{\underline{\theta}}^{\bar{\theta}} \left(\frac{\partial \omega(a)}{\partial a} + \theta - E(\theta) \right) Q(\theta, a) + \frac{\partial Q(\theta, a)}{\partial a} (\omega(a) + a(\theta - E(\theta))) F(d\theta) = 0.$$

When evaluated at $a = 0$, the above expression becomes:

$$\frac{\partial \omega(a)}{\partial a} = \frac{\int_{\underline{\theta}}^{\bar{\theta}} (E(\theta) - \theta) Q(\theta, 0) F(d\theta)}{\int_{\underline{\theta}}^{\bar{\theta}} Q(\theta, 0) F(d\theta)}. \quad \blacksquare$$

Lemma 2 At $a = 0$,

$$\frac{\partial}{\partial a} E[\Pi^s(\theta, a)] = \int_{\underline{\theta}}^{\theta^*(0)} \theta \left(\frac{E(\theta) - \theta - \frac{\partial \omega(a)}{\partial a}|_{a=0}}{2} \right) F(d\theta).$$

Proof: Note that

$$E[\Pi^s(\theta, a)] = \int_{\underline{\theta}}^{\theta^*(a)} \left(\frac{\theta + a(E(\theta) - \theta) - \omega(a)}{2} \right)^2 F(d\theta) + \int_{\theta^*(a)}^{\bar{\theta}} \left(\frac{n}{n-1} \right) \left(\frac{\beta}{1-\beta} \right) KF(d\theta).$$

By Leibnitz' rule, we obtain

$$\begin{aligned} \frac{\partial E[\Pi^s(\theta, a)]}{\partial a} &= \int_{\underline{\theta}}^{\theta^*(a)} \frac{\partial}{\partial a} \left(\frac{\theta + a(E(\theta) - \theta) - \omega(a)}{2} \right)^2 F(d\theta). \\ &= \int_{\underline{\theta}}^{\theta^*(a)} (\theta + a(E(\theta) - \theta) - \omega(a)) \left(\frac{E(\theta) - \theta - \frac{\partial \omega(a)}{\partial a}}{2} \right) F(d\theta). \end{aligned}$$

Evaluating at $a = 0$,

$$= \int_{\underline{\theta}}^{\theta^*(0)} \theta \left(\frac{E(\theta) - \theta - \frac{\partial \omega(a)}{\partial a}|_{a=0}}{2} \right) F(d\theta). \quad \blacksquare$$

The above lemma guarantees that $\frac{\partial}{\partial a} E[\Pi^s(\theta, a)]|_{a=0} > 0$ if $\theta^*(0) \leq E(\theta) - \frac{\partial \omega(a)}{\partial a}|_{a=0}$.

The rest of the proof considers the case where $\theta^*(0) > E(\theta) - \frac{\partial \omega(a)}{\partial a}|_{a=0}$.

Lemma 3

$$\int_{\underline{\theta}}^{\bar{\theta}} \theta \left(\frac{E(\theta) - \theta - \frac{\partial \omega(a)}{\partial a}|_{a=0}}{2} \right) F(d\theta) < \int_{\underline{\theta}}^{\theta^*(0)} \theta \left(\frac{E(\theta) - \theta - \frac{\partial \omega(a)}{\partial a}|_{a=0}}{2} \right) F(d\theta).$$

Proof: We note that

$$E(\theta) - \frac{\partial \omega(a)}{\partial a} \Big|_{a=0} = \frac{\int_{\underline{\theta}}^{\bar{\theta}} \theta Q(\theta, 0) F(d\theta)}{\int_{\underline{\theta}}^{\bar{\theta}} Q(\theta, 0) F(d\theta)} > 0.$$

Moreover, note that the expression

$$\theta \left(\frac{E(\theta) - \theta - \frac{\partial \omega(a)}{\partial a} \Big|_{a=0}}{2} \right)$$

is quadratic, taking zeroes at $\theta = 0$ and $\theta = E(\theta) - \frac{\partial \omega(a)}{\partial a} \Big|_{a=0}$. The expression is strictly negative to the right of $E(\theta) - \frac{\partial \omega(a)}{\partial a} \Big|_{a=0}$. Therefore, the conclusion follows. ■

The next lemma guarantees that the increase in the fixed wage component is less than it would be if collusion were sustainable in all states of demand.

Lemma 4

$$\frac{\partial \omega(a)}{\partial a} \Big|_{a=0} = \frac{\int_{\underline{\theta}}^{\bar{\theta}} (E(\theta) - \theta) Q(\theta, 0) F(d\theta)}{\int_{\underline{\theta}}^{\bar{\theta}} Q(\theta, 0) F(d\theta)} \leq \frac{\int_{\underline{\theta}}^{\bar{\theta}} (E(\theta) - \theta) (\theta/2) F(d\theta)}{\int_{\underline{\theta}}^{\bar{\theta}} (\theta/2) F(d\theta)}.$$

Proof: Suppose the wage tying parameter is $a' > 0$. The right hand side of the expression in the statement of the lemma is the change in the fixed component of the wage under perfect monopoly pricing. Denote this fixed component of the wage under perfect monopoly pricing by $\omega_M(a')$. The right hand side of the inequality is $\frac{\partial \omega_M(a)}{\partial a} \Big|_{a=0}$. Note that $\omega(0) = \omega_M(0) = 0$. We will show that

$$\int_{\underline{\theta}}^{\bar{\theta}} (\theta/2) (a'(\theta - E(\theta)) + \omega(a')) F(d\theta) < 0.$$

The left hand side of the inequality is the expected wage workers would receive were monopoly pricing used in all states, and workers were offered contracts with fixed wage component $\omega(a')$.

This will enable us to show $\omega_M(a') > \omega(a')$. As a' is arbitrary, this establishes that at $a = 0$,

$$\frac{\partial \omega(a)}{\partial a} \leq \frac{\partial \omega_M(a)}{\partial a},$$

as required. The incentive compatibility constraint for workers is:

$$\int_{\underline{\theta}}^{\bar{\theta}} (Q(\theta, a')) (a'(\theta - E(\theta)) + \omega(a')) F(d\theta) = 0.$$

Separating:

$$\begin{aligned} & \int_{\underline{\theta}}^{\theta^*(a')} (\theta/2) (a'(\theta - E(\theta)) + \omega(a')) F(d\theta) + \int_{\theta^*(a')}^{\bar{\theta}} (\theta/2) (a'(\theta - E(\theta)) + \omega(a')) F(d\theta) \\ & + \int_{\theta^*(a')}^{\bar{\theta}} \left(\sqrt{\theta^2 - \theta^*(a')^2}/2 \right) (a'(\theta - E(\theta)) + \omega(a')) F(d\theta) = 0. \end{aligned}$$

We claim that

$$\int_{\underline{\theta}}^{\theta^*(a')} (\theta/2) (a'(\theta - E(\theta)) + \omega(a')) F(d\theta) < 0.$$

If not, it is easy to see that

$$\int_{\theta^*(a')}^{\bar{\theta}} (\theta/2) (a'(\theta - E(\theta)) + \omega(a')) F(d\theta) \geq 0,$$

which in turn implies

$$\int_{\theta^*(a')}^{\bar{\theta}} \left(\sqrt{\theta^2 - \theta^*(a')^2}/2 \right) (a'(\theta - E(\theta)) + \omega(a')) F(d\theta) > 0.$$

This follows as the last expression clearly weights higher values of θ at a larger ratio than the first expression through the use of $\left(\sqrt{\theta^2 - \theta^*(a')^2}/2 \right)$ as opposed to $(\theta/2)$. Summing the three inequalities obtains

$$\int_{\underline{\theta}}^{\bar{\theta}} (Q(\theta)) (a'(\theta - E(\theta)) + \omega(a')) F(d\theta) > 0,$$

a contradiction. As

$$\int_{\underline{\theta}}^{\theta^*(a')} (\theta/2) (a'(\theta - E(\theta)) + \omega(a')) F(d\theta) < 0,$$

it must be that

$$\int_{\theta^*(a')}^{\bar{\theta}} (\theta/2) (a'(\theta - E(\theta)) + \omega(a')) F(d\theta) + \int_{\theta^*(a')}^{\bar{\theta}} \left(\sqrt{\theta^2 - \theta^*(a')^2}/2 \right) (a'(\theta - E(\theta)) + \omega(a')) F(d\theta) > 0.$$

However, again, this inequality implies that

$$\int_{\theta^*(a')}^{\bar{\theta}} \left(\sqrt{\theta^2 - \theta^*(a')^2}/2 \right) (a'(\theta - E(\theta)) + \omega(a')) F(d\theta) > 0.$$

The preceding inequality is exactly

$$\int_{\underline{\theta}}^{\bar{\theta}} (Q(\theta) - \theta/2) (a'(\theta - E(\theta)) + \omega(a')) F(d\theta) > 0.$$

Separating, we see

$$\int_{\underline{\theta}}^{\bar{\theta}} (Q(\theta)) (a'(\theta - E(\theta)) + \omega(a')) F(d\theta) > \int_{\underline{\theta}}^{\bar{\theta}} (\theta/2) (a'(\theta - E(\theta)) + \omega(a')) F(d\theta),$$

where the left hand side of the inequality is zero. Hence, we have established that $\omega_M(a') > \omega(a')$ for any $a' > 0$; moreover, then

$$\omega_M(a') - \omega_M(0) > \omega(a') - \omega(0),$$

establishing that at $a = 0$,

$$\frac{\partial \omega_M(a)}{\partial a} \geq \frac{\partial \omega(a)}{\partial a}. \quad \blacksquare$$

Finally, our theorem demonstrates that firms can always achieve higher expected sustainable profits by tying wages to market conditions. We show this by establishing that at $a = 0$,

$$\frac{\partial}{\partial a} E[\Pi^s(\theta, a)] > 0.$$

Proof: Suppose false. Then

$$\frac{\partial}{\partial a} E[\Pi^s(\theta, a)]|_{a=0} \leq 0.$$

The preceding lemmas taken together then imply that

$$\int_{\underline{\theta}}^{\bar{\theta}} \theta \left(\frac{E(\theta) - \theta - \frac{\partial \omega(a)}{\partial a}|_{a=0}}{2} \right) F(d\theta) < 0$$

and

$$\int_{\underline{\theta}}^{\bar{\theta}} \theta \left(\frac{E(\theta) - \theta - \frac{\int_{\underline{\theta}}^{\bar{\theta}} (E(\theta) - \theta)(\theta/2) F(d\theta)}{\int_{\underline{\theta}}^{\bar{\theta}} (\theta/2) F(d\theta)}}{2} \right) F(d\theta) < 0.$$

Thus, we see that

$$\int_{\underline{\theta}}^{\bar{\theta}} -\theta^2 + \frac{\theta \int_{\underline{\theta}}^{\bar{\theta}} \theta^2 F(d\theta)}{\int_{\underline{\theta}}^{\bar{\theta}} \theta F(d\theta)} F(d\theta) < 0$$

and thus

$$-E[\theta^2] + E[\theta] \left(\frac{E[\theta^2]}{E[\theta]} \right) < 0.$$

However, this is an obvious contradiction, as the expression above is identically zero. ■

Thus, we have shown that in period one, it is never optimal to set $a = 0$. This analysis extends to future periods, as the problem for firms is time separable. Further, since period profits are increased by wage tying, it follows that continuation profits would be increased if there is wage tying in each period. In turn, increased continuation profits supports (weakly) more collusive output levels for all demand realizations, further increasing the profitability of tying wages to market conditions.

Proof of Theorem 2: We show that under the conditions detailed in the theorem that setting $a > 0$ is optimal even when (i) firms produce the monopoly level of output, where-ever possible in the first period, and (ii) as in the proof of the theorem 1, there is a reversion after the first period to the strategies that do not feature profit sharing, and correspond to those in Rotemberg and Saloner. Fixing continuation payoffs to correspond to the Rotemberg and Saloner monopoly pricing equilibrium, for any a , we may solve for $\theta^*(a)$, as the θ^* which solves

$$(1 - anL(\theta^*, a)) \Pi^m(\theta^*, a) = \frac{(1 - aL(\theta^*, a)) \Pi^m(\theta^*)}{n} + K,$$

where $K = E[\Pi^s(\theta)]/n$ is the expected period profit for any given firm. To simplify notation, we write θ^* , leaving the dependence on a implicit. Differentiating both sides of the above equality with respect to a and evaluating at $a = 0$ obtains:

$$\frac{\partial \Pi^m(\theta^*)}{\partial \theta^*} \frac{\partial \theta^*}{\partial a} - n \Pi^m(\theta^*) L(\theta^*, a) = \frac{-L(\theta^*, a) \Pi^m(\theta^*) + \frac{\partial \Pi^m(\theta^*, a)}{\partial \theta^*} \frac{\partial \theta^*}{\partial a}}{n}.$$

Rearranging yields

$$\frac{\partial \Pi^m(\theta^*)}{\partial \theta^*} \frac{\partial \theta^*}{\partial a} = (n + 1) \Pi^m(\theta^*) L(\theta^*, a).$$

Recall that expected period profits for the cartel given profit sharing level a are

$$\int_{\underline{\theta}}^{\theta^*} (1 - aL(\theta, a)) \Pi^m(\theta) F(d\theta) + \int_{\theta^*}^{\bar{\theta}} (1 - aL(\theta, a)) \Pi^m(\theta^*) F(d\theta).$$

Differentiating this expression with respect to a , and evaluating at $a = 0$ yields:

$$\int_{\underline{\theta}}^{\theta^*} -L(\theta, a) \Pi^m(\theta) F(d\theta) + \int_{\theta^*}^{\bar{\theta}} -L(\theta, a) \Pi^m(\theta^*) + \frac{\partial \Pi^m(\theta^*)}{\partial \theta^*} \frac{\partial \theta^*}{\partial a} F(d\theta).$$

We establish when this function takes positive values, or (solving) when

$$\int_{\underline{\theta}}^{\theta^*} -L(\theta, a) \Pi^m(\theta) F(d\theta) + \int_{\theta^*}^{\bar{\theta}} \Pi^m(\theta^*) [(n+1)L(\theta^*, a) - L(\theta, a)] F(d\theta)$$

is strictly positive. Substituting for $L(\theta, a)$ and $\Pi^m(\theta)$, as derived in the demand-linked wage contracts case and normalizing, yields the following expression:

$$\frac{\underline{\theta}^4 - \theta^{*4}}{4} + (1/2) \theta^{*2} \left[2(n+1) \bar{\theta} \theta^* - (2n+1) \theta^{*2} - \bar{\theta}^2 - \bar{\theta} \sqrt{\bar{\theta}^2 - \theta^{*2}} + \theta^{*2} \log \left(\frac{\bar{\theta} + \sqrt{\bar{\theta}^2 - \theta^{*2}}}{\theta^*} \right) \right].$$

Rearranging, we see that this expression is positive if:

$$\left[2(n+1) \bar{\theta} \theta^* - (2n+1) \theta^{*2} - \bar{\theta}^2 - \bar{\theta} \sqrt{\bar{\theta}^2 - \theta^{*2}} + \theta^{*2} \log \left(\frac{\bar{\theta} + \sqrt{\bar{\theta}^2 - \theta^{*2}}}{\theta^*} \right) \right] > \frac{\theta^{*4} - \underline{\theta}^4}{2\theta^{*2}}. \quad (4)$$

When this expression is positive, expected period one profits can be raised by signing profit sharing contracts with workers, and using a monopoly pricing scheme. We see that continuation payoffs must also be driven up, as the analysis applies at all periods. Therefore, we conclude that all firms setting $a = 0$ is not optimal.

To see that (i) holds, note that at $\theta^*(0) = \bar{\theta} = \underline{\theta}$, both sides of (4) are equal to zero, but differentiating the left-hand side with respect to $\bar{\theta}$ yields $(2n)\bar{\theta} > 0$, while the derivative of the right-hand side is zero.

To see that (ii) holds, note that the derivative of (4) with respect to $\bar{\theta}$ evaluated at $\bar{\theta}$ when $\theta^* = \underline{\theta}$ is equal to $2(n+1)\theta^* - 2\theta - 2(\theta^2 - \theta^{*2})^{1/2}$. Therefore, there exists n large enough so that this expression is positive, for all $\theta \leq \bar{\theta}$. For such n , there thus exists a neighborhood of $\underline{\theta}$ such that for all θ in this neighborhood, (4) is satisfied. Let $n(\underline{\theta}, \bar{\theta}, \beta)$ be this n . As n increases, we see that (4) is true for any θ^* for which it was previously true (a simple monotonicity argument establishes this). Moreover, as n increases, θ^* decreases to zero. Therefore, the result holds.

Lastly, (iii) is a trivial corollary to part (i), as θ^* is an increasing function of β . ■

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