Curriculum-Context Knowledge: Teacher Learning From Successive Enactments of a Standards-Based Mathematics Curriculum

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ABSTRACT
This study characterizes the teacher learning that stems from successive enactments of innovative curriculum materials. This study conceptualizes and documents the formation of curriculum-context knowledge (CCK) in three experienced users of a Standards-based mathematics curriculum. I define CCK as the knowledge of how a particular set of curriculum materials functions to engage students in a particular context. The notion of CCK provides insight into the development of curricular knowledge and how it relates to other forms of knowledge that are relevant to the practice of teaching, such as content knowledge and pedagogical content knowledge. I used a combination of video-stimulated and semistructured interviews to examine the ways the teachers adapted the task representations in the units over time and what these adaptations signaled in terms of teacher learning. Each teacher made noticeable adaptations over the course of three or four enactments that demonstrated learning. Each of the teachers developed a greater understanding of the resources in the respective units as a result of repeated enactments, although there was some important variation between the teachers. The learning evidenced by the teachers in relation to the units demonstrated their intricate knowledge of the curriculum and the way it engaged their students. Furthermore, this learning informed their instructional practices and was intertwined with their discussion of content and how best to teach it. The results point to the larger need to account for the knowledge necessary to use Standards-based curricula and to relate the development and existence of well-elaborated knowledge components to evaluations of curricula.

INTRODUCTION
This study characterizes the teacher learning that stems from successive enactments of instructional units from a Standards-based mathematics curriculum, which refers to National Science Foundation (NSF)—funded curriculum materials designed with the National Council of Teachers of
Mathematics Standards (NCTM, 1989, 1991) as guiding documents. The knowledge that is relevant to teaching and how teachers develop that knowledge are issues currently under study in the mathematics education community (Hill & Ball, 2004; Hill, Rowan, & Ball, 2005; Ma, 1999). Much of the focus has been on content knowledge and pedagogical content knowledge (PCK), with explicit connections made between the design of curricula and the development of those forms of knowledge (Ball & Cohen, 1996; Davis & Krajcik, 2005; Remillard, 2000). A lesser emphasis has been made on curricular knowledge and how that connects to other forms of teacher knowledge. Given the potentially “educative” nature of Standards-based mathematics curricula (Ball & Cohen, 1996; Remillard, 2000), greater attention needs to be paid to how teachers develop knowledge of how these materials function in particular contexts and how that knowledge is related to the development of content and pedagogical content knowledge. Cases involving Standards-based mathematics curricula are particularly relevant given the large investments in the United States at the local and federal levels in developing and implementing those materials. The curricula were designed to be comprehensive and coherent, develop mathematical ideas in depth, promote sense-making, and engage students through the use of meaningful contexts and applications (Trafton, Reys, & Wasman, 2001), and thus represent an innovation from conventional mathematics curricula.

This study conceptualizes and documents the formation of curriculum-context knowledge (CCK) in relation to a middle school Standards-based mathematics curriculum. I define CCK as the knowledge of how a particular set of curriculum materials functions to engage students in a particular context. The notion of CCK has the potential to be a tool to explore how teachers develop and connect multiple forms of knowledge—content knowledge, pedagogical content knowledge, and curricular knowledge—in practice. This study investigates how CCK develops over time as teachers enact a particular set of instructional materials repeatedly within a given context.

Although the situated nature of teacher learning has been elaborated elsewhere (Peressini, Borko, Romagnano, Knuth, & Willis, 2004), little attention has been paid to the impact of the use of specific curriculum materials as a part of the context that affects teacher learning. Herbel-Eisenmann, Lubienski, and Id-Deen (2006), for example, documented the impact of curricular context on a teacher’s practices by showing how a teacher engaged in different instructional practices according to the materials she was using, with implications for what that teacher might learn as a result of using the curriculum materials.

This study focuses on the development of CCK of three teachers who were veteran users of a Standards-based middle school mathematics curriculum. The study investigates the teachers’ adaptations of instructional units over several enactments and what these adaptations signaled in terms
of teacher learning. The focus on successive adaptations provides insight into the process of how teachers develop knowledge of how the instructional units function to engage students with particular mathematical ideas in particular contexts, with implications for how teachers connect this knowledge to content knowledge and pedagogical content knowledge.

BACKGROUND

The mathematics education research community has shown great interest in defining teacher knowledge necessary to teach using Standards-based curricula. For example, Confrey et al. (2008) state that

Research must elaborate and delve more deeply into the knowledge components that teachers need to teach NSF-supported mathematics curricula effectively and how these components develop during course work and teaching. We must refine and verify our descriptions of the knowledge components, investigate connections between the components, and investigate how the components vary by types of curricula being taught. (p. 108)

Much of the research on teacher knowledge has focused on conceptions of content knowledge and pedagogical content knowledge, with an emphasis on how those components develop while teaching. In their review of the literature on teacher knowledge in mathematics education, Ball, Lubienski, and Mewborn (2001) focused on the knowledge that teachers develop and use in practice, which they term “knowledge of mathematics in and for teaching” (p. 449). This knowledge encompasses pedagogical content knowledge, which “bundles mathematical knowledge with knowledge of learners, learning, and pedagogy” (p. 453). PCK includes “the most useful forms of representation of [topics], the most powerful analogies, illustrations, examples, explanations, and demonstrations—in a word, the ways of representing and formulating the subject that make it comprehensible to others” (Shulman, 1986, p. 9).

Researchers have focused on the role of Standards-based curriculum materials in developing teachers’ PCK, describing these materials as “educative” (Ball & Cohen, 1996). Educative curricula articulate the design rationale (Davis & Krajcik, 2005), which reflects the “developers’ pedagogical judgments” (p. 5), so that teachers can make informed decision about how best to adapt the materials to suit their purposes. The design rationale elaborates the disciplinary concepts, the use of representations, the learning trajectories, and the anticipated student strategies in the curriculum materials. Davis and Krajcik claim that the inclusion of the design rationale may help to develop teachers’ PCK in regards to the use of specific materials and more generally to “develop knowledge they can apply flexibly in new situations” (p. 3).
Hill and Ball (Hill & Ball, 2004; Hill, Rowan, & Ball, 2005) have explored the content knowledge that teachers develop and use in practice and that impacts student learning. Past measures of teacher knowledge have focused on course descriptors, such as courses taken in college, but Hill and Ball have developed a set of items that measure teachers’ content knowledge that is relevant to the mathematics that teachers encounter while teaching. They found a positive association between teachers’ performance on their measures and student achievement; however, they did not elaborate the process by which teachers develop such knowledge.

Ma’s (1999) study of Chinese teachers sheds light on how teachers come to develop content knowledge and PCK. Ma describes the kind of knowledge developed by the teachers as “knowledge packages.” Ma describes a knowledge package as a densely connected set of understandings of the concepts and procedures related to a particular topic, such as subtraction of fractions. Many of the Chinese teachers lacked extensive formal teacher preparation in relation to mathematics teaching. Instead, the teachers developed these knowledge packages by reading the student text materials intensively to learn how tasks functioned to engage students with mathematical ideas, how tasks were ordered, and how to teach particular lessons. Over many years, the Chinese teachers developed knowledge of how to adapt the task representations in the textbooks to better engage students with mathematical concepts and in the process developed well elaborated content knowledge and PCK.

Ma’s (1999) work illustrates the link between content knowledge, PCK, and the third component of Shulman’s (1986) tripartite characterization of teacher knowledge, curricular knowledge. Shulman described curricular knowledge along several dimensions, the primary one of which involves understanding the characteristics of the various curricular options in one’s content domain. The notion of CCK emphasizes the challenge of coming to understand and learn the characteristics of a particular set of curriculum materials, whereas Shulman emphasized more general understandings across a variety of curriculum materials. Given the difficulty many teachers have had implementing Standards-based curricula (Collopy, 2003; Davenport, 2000; Keiser & Lambdin, 1996; Manouchehri & Goodman, 1998), developing such general understandings may be more challenging than Shulman indicated, especially if the goal is to flexibly select and apply the resources in those curricula to develop a comprehensive and connected set of learning experiences.

**RESEARCH DESIGN**

The study is guided by the following research questions:
1. What is the nature of teacher learning that results from successive enactments of curriculum materials, and how is that learning affected by context?

2. What is the evidence for this learning and how does it vary across teachers?

The study investigates the existence of CCK and how it develops over several enactments of the same curriculum materials. The research design is based on the proposition that CCK is evidenced in the ways that teachers adapt curriculum over time. In particular, the research design focuses on the ways that teachers relate their reflections of the enactments of tasks to subsequent adaptations.

**METHODS**

This study focuses on the development of CCK of three experienced users of the Connected Mathematics Project (CMP) middle school curriculum (Lappan, Fey, Fitzgerald, Friel, & Phillips, 1998, 2006), one of the 13 curriculum projects funded by the NSF in response to the original NCTM Standards (NCTM, 1989). The study is a collective case study, a set of instrumental cases intended to provide insight into an issue (Stake, 2000). The study focuses on the teachers' initial enactments of versions of CMP units that were new to the teachers; therefore, although the teachers were new to the particular instructional units, they were not new to the approach and format of CMP. I used a combination of video-stimulated and semi-structured interviews to examine the ways the teachers adapted the task representations in the units over time. The teachers were a “convenience sample” in that they had participated in a professional development project prior to the study and had indicated a willingness to be videotaped and interviewed. Each teacher was interviewed about successive enactments of an instructional unit and about their general beliefs and practices. These data allowed me to explore how teachers adapted the units, why teachers adapted the units, and how the adaptations changed over time.

**The Teachers**

Given the challenges in using Standards-based curricula and the learning required to use them effectively, I felt it was important to select teachers who reported philosophical and pedagogical alignment with the approach implied in the CMP materials. The teachers were chosen in part because of their willingness to participate and in part because of their support of the approach articulated in the CMP materials. The three teachers selected as
subjects of the study were veteran users of the CMP curriculum, having
taught with the materials for at least 5 years prior to the study, although
they were new to the versions of the instructional units that form the focus
of the data collection and analysis. They professed their support of CMP
and stated beliefs about the teaching and learning of mathematics that are
aligned with the NCTM Standards (1989, 2000). Each of the teachers
attended professional development specifically geared toward the use of
CMP materials and taught in districts in which CMP was the primary
curriculum resource for grades 6–8. Two of the teachers, Ms. Cartwright1
and Ms. Pless, taught in Brookline2 and had used CMP for 5 years, and the
other, Ms. Karp, taught in Fillmore and had used CMP for 6 years.
Brookline and Fillmore are both inner-ring suburbs of a moderately sized
rust belt city. Furthermore, both Cartwright and Pless had taught using only
CMP materials and had spent their entire Brookline careers at the same
grade level.

Teaching Contexts
Brookline, the district in which Cartwright and Pless taught, was one of the
highest-performing districts in the county and the one with the most robust
and longest CMP implementation of the area districts that had adopted
CMP (Choppin, 2008). Fillmore, the district in which Karp taught, typically
scored in the middle or lower half of the county districts in terms of state
test scores and thus was more impacted by state testing concerns; further-
more, the CMP implementation in Fillmore had not had the same level of
consistent support as Brookline. Consequently, Cartwright and Pless had
attended considerably more curriculum-specific training than Karp, includ-
ing workshops in Michigan from the curriculum developers, had more
regular interactions and collaboration with grade-level colleagues, and
were less affected by district-level curriculum modifications related to the
state test.

The CMP Curriculum
CMP is a 3-year curriculum for grades 6–8, designed to provide students
with multiple opportunities to explore and formalize their understanding
of key mathematical ideas within five major “strands” (numbers and opera-
tions, geometry, measurement, data analysis and probability, and algebra).
The curriculum is organized into units, each comprised of three to five
“investigations” where students explore a key mathematical concept or
process. Each investigation begins with the presentation of a problem
situation that embodies the mathematical concept/process under study
(“launch”), followed by a combination of whole-class and small-group
guided explorations (“explore”), and concludes with a discussion in which the mathematical concept/process at the core of the investigation is explicitly identified and its understanding reinforced (“summary”). Each investigation is followed by a series of problems divided into sections entitled Applications, Connections, and Extensions (ACE).

CMP promotes the use of diverse approaches to problems and suggests that students “should pose conjectures, question each other, offer alternatives, provide reasons, refine their strategies and conjectures, and make connections” (Lappan et al., 2004, p. 17). Furthermore, the expectation is that students will not achieve mastery of a skill or concept at the end of each investigation, but instead, students will gain formal and abstract understanding after a progression of activities that allow time for exploration and incremental development of concepts, usually 4–6 weeks for each unit. Initially, there is usually little emphasis in the student text or teacher materials about developing fluency with particular representations or procedures, but over time, each unit develops more formal uses of representations, procedures, and concepts.

The CMP teacher resource materials include the design rationale of the curriculum, such as descriptions and analyses of a variety of student responses to a particular problem, mappings of student learning over time, and consideration of various representations of and connections between mathematical concepts. The teacher resource materials provide a detailed description of the mathematical goals for each investigation and include suggested questions and anticipated student responses for many questions.

Data Collection

The sources of data included semistructured interviews about each teacher’s overall perceptions and practices related to CMP, videotaped classroom observations, video-stimulated interviews, and semistructured interviews of subsequent enactments. See Table 1 for an outline of the data collection for each teacher. As noted in the table, there was some variation between the teachers in terms of the number of times they had taught a unit prior to the study. Ms. Karp had never taught any version of the Say It With Symbols unit prior to the study. Ms. Cartwright was teaching the new version of the Growing, Growing, Growing unit for the first time although she had taught the old version several times. Ms. Pless was in her third implementation of the new version of the Bits and Pieces II unit at the beginning of the study and had taught the old version several times as well. Pless had taught the new Bits and Pieces II unit twice the year before, once for her “accelerated” class and later again with her regular class. For each teacher, I conducted one video-stimulated interview and at least one semistructured interview about subsequent enactments. In Pless’s case, I treated the video-stimulated interview as an account of the third enactment and,
TABLE 1
Outline of Data Collection for Each Teacher

<table>
<thead>
<tr>
<th>Teacher (grade)</th>
<th>Unit</th>
<th>Iteration</th>
<th>Type of interview*</th>
<th>Timing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cartwright (8th)</td>
<td>Growing, Growing,</td>
<td>First</td>
<td>Video-stimulated</td>
<td>After</td>
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<tr>
<td></td>
<td>Growing**</td>
<td></td>
<td>interview</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Second</td>
<td>Semistructured</td>
<td>Before +</td>
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<td></td>
<td></td>
<td></td>
<td>interviews</td>
<td>During</td>
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<tr>
<td></td>
<td></td>
<td>Third</td>
<td>Semistructured</td>
<td>Before</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>interview</td>
<td></td>
</tr>
<tr>
<td>Karp (8th)</td>
<td>Say It With Symbols</td>
<td>First</td>
<td>Video-stimulated</td>
<td>Before</td>
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<td></td>
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<td>interview</td>
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<td></td>
<td>Third</td>
<td>Semistructured</td>
<td>After</td>
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<td></td>
<td></td>
<td>interview</td>
<td></td>
</tr>
<tr>
<td>Pless (6th)</td>
<td>Bits and Pieces II***</td>
<td>Third</td>
<td>Video-stimulated</td>
<td>After</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>interview</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Fourth</td>
<td>Semistructured</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>interview</td>
<td></td>
</tr>
</tbody>
</table>

* In addition to the interviews listed, each teacher was interviewed once about her general beliefs and practices in relation to CMP.

** Cartwright had taught Growing, Growing, Growing three times from the previous edition of CMP.

*** Pless had taught Bits and Pieces II twice previously from old edition and twice previously with new edition, although for some of the tasks it was her first time teaching them.

retrospectively, as accounts of her first two enactments of the new version of Bits and Pieces II.

**Interviews About Overall Perceptions and Practices Related to CMP.** I interviewed the participants regarding (1) their beliefs about the learning and teaching of mathematics; (2) the design of CMP in relation to the development of content and the emphasis on processes such as communication, problem solving, and the use of multiple representations; (3) their practices around planning, pedagogy, and assessment; (4) contextual factors such as school demographics, district curriculum guidelines, and the impact of state testing; (5) CMP-specific professional development in which they had participated; and (6) the resources they drew on to plan their mathematics lessons.

**Video Stimulated Interviews.** For each teacher, I videotaped five to eight consecutive classes from the beginning of a particular unit. For each teacher, I selected 10 brief classroom episodes based on the following criteria: (1) they spanned a variety of tasks; (2) they varied in terms of the launch, explore, and summary parts of the investigations; and (3) they
represented various ways that teachers adapted the CMP materials. I used
the video clips to elicit in specific instances of curriculum use (1) the
purposes and processes of how they designed that instance of instruction,
including adaptations and revisions from the base curriculum and
from how they designed the task in previous years, (2) their reflections
on how the task design engaged students, (3) the experiences and
resources they drew on to design instruction, and (4) how they might
revise the instructional design in the future based on how they perceived
student engagement.

Semistructured Interviews of Subsequent Enactments. I interviewed the teach-
ers regarding subsequent implementations of the same instructional unit.
Each teacher was interviewed at least once about the subsequent enact-
ments of the same unit in which they had been videotaped. In the case of
Pless, I interviewed her only about the next enactment, whereas I inter-
viewed Karp and Cartwright about the next two enactments. With Pless,
however, the videotaped enactment, the first one about which she was
interviewed, was the third time she had enacted the instructional unit.

Data Analysis
The data were initially analyzed using both typological and inductive
coding (LeCompte & Preissle, 2003), a process that resulted in a stable
coding scheme which emerged after several passes through the data. The
analysis of the interviews related to overall perceptions and practices was
conducted separately from the other data and used to compile a profile of
each teacher that highlighted the teacher’s conceptions about mathemat-
ics teaching and stances toward using curriculum materials. The coding
scheme for the rest of the data (see Appendix B for the full list of codes)
focused on (1) the teachers’ reading of the task design, the enactment of
the task, and student engagement with the task; (2) how the teachers
adapted the tasks, with categories for sources of adaptations, the nature of
the adaptation, the rationale cited for the adaptations, how the adaptations
functioned to engage students (both reported and observed); and (3) the
teacher learning that resulted from the process of successive enactments.
These codes were used to organize the results from each teacher, with
additional codes used to identify teacher learning, as explained below.

Teacher Learning. Teacher learning was characterized according to per-
spectives that emphasize situated, experiential, and participatory qualities
(Dreyfus & Dreyfus, 1986; Lave & Wenger, 1991; Schön, 1983; Wertsch,
1991). Evidence for teacher learning was divided into four categories. The
first category implied that no evidence existed for teacher learning with
respect to the use of the curriculum materials. The necessary condition for
the development of CCK was that the teacher had to connect her discussion of a prior enactment (descriptions of student strategies, chronology of enactment, descriptions of logistical entailments, descriptions of challenges and obstacles) with the rationale for subsequent adaptations. In other words, the teacher needed to show that she was building from past experiences and reflections on those experiences.

The second category implied that the teacher learned to use the curriculum materials to better suit her original purposes. In this category the teacher adapted the curriculum materials so that students engaged more successfully with the mathematical concept or procedure that the teacher established in the initial enactment, as reported by the teacher.

In the third category, the teacher showed evidence of learning, but the learning extended to purposes related to students’ opportunity to learn mathematics with understanding, an explicit aim for Standards-based curriculum materials. Building from a considerable body of research in mathematics and science education, Carpenter et al. (2004) defined learning with understanding as a set of processes or activities rather than in terms of more inert qualities. They define the four primary activities as constructing relationships, extending and applying mathematics knowledge, justifying and explaining generalizations and procedures, and making mathematical knowledge one’s own.

The fourth category related to cases when teachers developed new conceptions of mathematics, teaching, and learning. An example for the fourth category was that the teacher reported new understanding of the mathematics in a task or of general views of how students learn mathematics. In sum, I characterized learning in the following ways:

1. For learning to occur, a teacher connected her evaluation of a prior enactment with her rationale for subsequent adaptations.
2. A teacher changed her use of curriculum materials to better achieve her original goals.
3. A teacher changed her use of curriculum materials, but in way that increased students’ opportunity to learn with understanding.
4. A teacher expressed new conceptions of mathematics, teaching, learning, or curriculum as a result of successive adaptations.

Data organization. After the data were analyzed, I selected two or three tasks for each teacher for which I had the most comprehensive data in terms of the extent to which the teacher discussed the enactments, the adaptations, and the rationale for the adaptations. I then created tables for each of those tasks that marked the year-by-year adaptations. In each table, I described the adaptations, the teacher’s rationale for the adaptations, how the adaptation functioned to engage students (both reported and observed), a description of the enactment, and what the teacher reported she would do differently as a result of her evaluation of the enactment.
RESULTS

Each teacher made noticeable adaptations over the course of three or four enactments that demonstrated learning according to the categories described above. That is, each of them learned how to use the materials to better suit their purposes, with varying outcomes in terms of learning to use the materials to teach for understanding. I present below a comprehensive analysis of one or two tasks for each teacher with occasional references to other tasks. The analysis describes the evolution of adaptations of the tasks over several enactments and how these evolutions represent the teachers’ curriculum orientation and the developing knowledge of how that task functioned to engage students.

Ms. Cartwright’s Adaptations

Cartwright was videotaped enacting the new version of the Growing, Growing, Growing unit for the first time and was subsequently interviewed before and during the second enactment, and before the third enactment. Cartwright had taught the prior version of Growing, Growing, Growing three times, during which she had followed worksheets developed by the eighth-grade team with whom she planned. The new version of the unit resulted in the team jettisoning the old plans and developing new pacing guidelines and worksheets. The description of the adaptations below were those made by the entire eighth-grade team of three teachers but which reflected Cartwright’s conceptions of the unit.

Description of Task. The goal of the Growing, Growing, Growing unit is to help students develop an understanding of exponential growth and different ways to represent that growth, including the general form of an exponential equation, usually expressed as \( y = a \cdot b^x \). The focus of the data collection and analysis was on the first investigation in the unit in which students engage in four problems that investigate the nature of exponential growth and compare it to linear growth. The first problem asks students to consider the number of ballots formed by cutting a piece of paper successively in half. The ballot problem yields a relatively simple relationship that can be represented as \( y = 2^x \), where \( x \) is the number of times the paper has been cut and \( y \) is the number of ballots. The students are then introduced to two problems in which a peasant is awarded a number of rubas (the lowest denomination of a fictitious currency) to be placed on squares on a checkerboard. In the first ruba plan, one ruba is placed on the first square and then the number of rubas doubles from square to square. Subsequently, students are asked to consider plans in which the number of rubas triples or quadruples.

Unlike the ballot problem, the ruba problems yield two likely equations. In the ruba problem where the pattern doubled, there is one ruba on the
first square, two on the second square, four on the third square, eight on
the fourth square, and so on, which contrasts with the ballot problem in
which there are two ballots after the first cut, four ballots after the second
cut, eight ballots after the third cut, and so on. The pattern for the ruba
problem can be discerned by looking at the number of times one multiplies
by 2 and relating that to the number of the checkerboard square where the
rubas are placed. For the fourth square, for example, there are eight rubas,
which can be found by multiplying $2 \times 2 \times 2$. For the fifth square, there are
16 rubas, or $2 \times 2 \times 2 \times 2$, and so on. Thus, the number of times that 2
appears as a factor is always one less than the number of squares, which can
be represented as $y = 2^{x-1}$. Alternatively, one could compare the pattern for
the number of rubas to that for the number of ballots and see that for the
third square, the number of rubas is half of the number of ballots for the
third cut, a relationship that holds for the fourth square and so on. Thus
the equation also can be written as $y = \frac{1}{2} \times 2^x$, although in the context of the
ruba problem, the $\frac{1}{2}$ has no meaning because the ruba is the smallest
denomination.

As is typical with most CMP units, the students explore multiple ways to
represent exponential relationships in the Growing, Growing, Growing
unit, including the use of tables, graphs, and equations. In the first inves-
tigation in relation to graphs, the students are asked to create a graph and
are asked questions about a graph that appears in the student text. The
apparent goal for the graph-related tasks is for students to notice the shape
of the graph of an exponential function to compare it to the shape of the
graph of a linear function. In Investigation One, the students are not asked
about the $y$-intercept, a point which has implications for the discussion of
Cartwright’s adaptations below. Furthermore, the teacher resource mate-
rials anticipate that students will generate the $b^{x-1}$ form of the exponential
rule. The $a^*b^x$ form is mentioned once in the first investigation, but stu-
dents are not expected to generate it.

**Description of Successive Adaptations.** Cartwright’s adaptations were aimed
at developing the standard form of the exponential equation. Cartwright
noted the similarity between the standard form of the exponential equa-
tion $a^*b^x$ and the standard form of a linear equation $a + b^x$, where $a$
stands for some starting value (graphically, where the curve hits the $y$-axis)
and $b$ for how the function grows over time. Cartwright’s adaptations and
interview data suggested that the generation of the standard exponential
equation was a strong emphasis for her in the unit. She stated that “every
year it’s always the equation, you know, that I want them to understand
more fully . . . every year I want to make this equation an ‘ah-ha’ for them
and it never is” (interview 02/16/07).

The analysis of Cartwright’s adaptations described below focuses on her
uncertainty about how to develop the standard form of the exponential
equation. In particular, she struggled to identify ways to utilize two
resources—the graphical representation and the $b^{x-1}$ form—to help students generate the $a \times b^x$ form. Her adaptations inexorably narrowed student activity to develop the standard form in the first investigation but ultimately proved futile.

In the first enactment, Cartwright’s efforts to emphasize the standard form were manifested in two adaptations. The first adaptation involved creating charts for each of the ballot and ruba plans that included a column for showing an expanded calculation. This would allow students to count the number of times a factor appeared and then relate that number to the number of cuts or the square on the checkerboard. For example, in the ruba problem involving doubling, on the fifth square students would notice that the expanded calculation was $2 \times 2 \times 2 \times 2$, and ultimately state that the number of 2s was always one less than the number for the square (hence the $2^{x-1}$ form). In relation to the problem that involved tripling the number of rubas, Cartwright stated:

We are constantly multiplying by three. And then taking this whole string and if you multiply that whole string by three again it will give you the next value. So I just want to make that a little bit more on where these numbers are coming from AND where the exponent is coming from. (Interview 01/09/07)

The equation for the doubling problem can also be found by comparing the ruba table with the ballot table. In doing so, the students can see that the values in the ruba table are always half of the corresponding values in the ballot table, and would thus generate the equation $y = \frac{1}{2} \times 2^x$.

For the second adaptation in the first enactment, Cartwright and her team also created a chart that included columns for the ballot problem and the three ruba problems, so that students could compare the four plans, with the goal of identifying the starting value (see Appendix A for a copy of the chart). Cartwright stated that in the enactments of the prior version of Growing “we never had anything beyond that to go back and compare . . . all four plans” (interview 02/16/07). In terms of finding the starting values, she said that

[The chart] is just to show that I think, if I recall, 2 to the x [for the ballot problem] I said “Well, what do you multiply with 2 to the x” and we came up with the “one” [which is the implicit starting value for $y = 2^x$] . . . so that was my decision, to show all of [the plans]. Just so they can see the similarities and differences. . . . The goal was again to really reinforce what that 1/3, 1/2 meant. It’s your initial starting value which is also the $y$-intercept on the graph. (Interview 01/09/07).

It should be noted that although $\frac{1}{3}$ and $\frac{1}{2}$ are the $y$-intercepts for the ruba equations, the students were not shown a graph that easily demonstrated this nor are the values $\frac{1}{3}$ and $\frac{1}{2}$ mentioned in the context of the
problem, as they would represent the “zero” square, which does not make
sense in the context of the problem.

Cartwright’s uncertainty about how to develop the standard form was
manifested in two primary ways. First, she was uncertain about how to use
the graphical representation, and second, she was uncertain about what to
do with the \( y = bx - 1 \) form. First, she wanted to use the graphical representa-
tion to help establish the starting value. She stated:

I think last year I remember just being at that zero cut . . . they understood it was the
\( y \)-intercept, but I don’t think they fully understood it. So I felt like maybe bringing
in the graph they’ll be able to see that it actually crosses the \( y \)-axis versus not having
it there, ’cause I . . . But I don’t know . . . ’cause I was hoping that that \( y \)-intercept
definitely would help with their equation, you know, that we use that just like we use
it in our linear. (Interview 02/16/07)

She related the development of the standard form for exponential
functions to the standard form for linear functions. In the case of linear,
however, discerning the \( y \)-intercept visually from the graph is usually fairly
easy because linear rates of increase do not dramatically affect scale.
However, exponential functions generally increase much more rapidly and
the increase visually obscures the \( y \)-intercept if other parts of the graph are
correctly displayed. Cartwright’s emphasis on the connections between
multiple representations was consistent with her experiences in other CMP
units; however, in the first investigation of the Growing unit, there is no
emphasis in the student text on the \( y \)-intercept as an important graphical
feature.

The second uncertainty centered on how to handle the \( y = bx - 1 \) form. The
ruba problems lend themselves to the \( bx - 1 \) form because in the expanded
calculation there is always one less factor than the number of squares. In
the first investigation, the student text presumes that students will generate
the \( bx - 1 \) form and explicitly makes the connection between it and the
general form, although it does not require that students generate the
general form. The purpose of the \( bx - 1 \) form is apparently to get students to
generate a symbolic representation that makes intuitive sense and then to
connect that form to another, more conventional form that will be empha-
sized later in the unit.

Cartwright, however, was unsure how to make use of the \( bx - 1 \) form and
consequently wanted to avoid it rather than help students see how they
were related. Her rationale was that the \( bx - 1 \) form was too confusing for
students. She stated:

I’m leading into the general equation, you know, pulling out, instead of the x-1,
pulling out the start, you know, the \( y \)-intercept. . . . I think that was my thinking. You
know, to kind of get that out there, of where, instead of having that x-1 out, ’cause
that was very confusing. (Interview 01/09/07)
The student text shifts exclusively to the standard form in Investigation Two, in part because the problems each have a starting value that is distinct from the growth factor and cannot be expressed in the $b^{x-1}$ form.

In the second enactment, Cartwright adapted the table so that each column began with a 0 instead of a 1, which was intended to focus students on finding the $y$-intercept (which occurs when the independent quantity, usually represented by $x$, equals 0). In the case of the ruba plans, this presented some problems. For the ballot problem, zero cuts could be thought of as having one ballot, which corresponds to a starting value of 1. For the ruba problems, however, the starting value would be $\frac{1}{2}$ or $\frac{1}{3}$, for which there is no number of rubas if rubas are the lowest denomination of currency in the mythical kingdom. Cartwright noted the dilemma:

I think that’s another reason why I wanted the zero in the chart. For here, if you cut, when you don’t make the cut, that’s zero, you have one ballot. But in the ruba plans, it’s not, because you start out putting one ruba on the first, versus having one on zero . . . it brings in the $y$-intercept even though it doesn’t make sense to the [rubas] problem. (Interview 03/16/07)

The creation of the chart, its subsequent revision, and the discussion of these adaptations signaled Cartwright’s commitment to generating the standard form in investigation one and the intense efforts she expended in doing so. The adaptations in Investigation One functioned to narrow student activity to the development of a particular representation. See Table 2 for a summary of the adaptations related to the chart of the four plans.

In discussing plans for the third enactment, Cartwright reported that she would abandon the table with the four plans because she felt it had not effectively focused the students and because she felt the students needed Investigations Two and Three to fully be able to generate the standard form. Cartwright’s plans for the third enactment are discussed in more detail below.

**Teacher Learning Evident in Adaptations.** Cartwright began to realize that her adaptations were largely ineffectual and did not take advantage of the way that the standard form of the exponential equation was developed in the unit. She stated that in relation to generating the standard form, students generally did poorly in Investigation One, but most understood it by the end of Investigation Three. Furthermore, the table itself proved cumbersome and the students had difficulty following the important details. The students were not able to focus on the important comparisons that might lead them to understand and generate the standard form. As a result, she stated in the final interview that she would probably drop the table and reduce the emphasis on developing the standard form:

So I definitely like taking my time with Investigation One. But I think I need to step back and relax a little bit about that equation. They get it, they get it, they don’t. You
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<tbody>
<tr>
<td>1</td>
<td>Created a worksheet that had patterns from four problems in investigation 1. The worksheet contained a column for each plan, with space at the bottom to write. Contemplates connecting to graphical representation but did not do so in chart.</td>
<td>To help students compare the four plans and to use the similarities and differences to generate the general form of the exponential equation.</td>
<td>Focus on patterns and development of symbolic representation.</td>
<td>There is ambiguity around the development of equation that does not fit ( a^b^x ).</td>
<td>The worksheet did not help students to develop the general form, although it is not clear if it helps them to see relationships between the different patterns.</td>
</tr>
<tr>
<td>2</td>
<td>Add a row of zeros to help students develop equation in ( a^b^x ) form. The original task comparing the growth factors is buried in this adaptation, which tries to accomplish too much.</td>
<td>Wanted to help students understand starting value and growth factor in investigation one, and to develop general form (akin to linear equations). Confounds the goals of seeing similarities between patterns and focusing on a particular form.</td>
<td>Did not really help students see the connection to the general form, and it apparently detracts from the goal of helping students see the similarities between patterns.</td>
<td>Table was confusing and students had a hard time dealing with all of the information, especially when it was publicly displayed.</td>
<td>This table does not serve its purpose and maybe it’s better to focus on general form in investigation 2. Forcing development too early is counterproductive.</td>
</tr>
</tbody>
</table>
know it’s fine. They’ll see it again in Investigation Two. They’ll see it again in Investigation Three. And then they’ll see it again next year. No because I think all through this unit I’ve always struggled with investigation 1. And I think I was trying to get too much out of it. I came to that realization after years and years. I am just like, you know, “Why am I forcing something that’s showing up again?” (Interview 10/04/07)

Cartwright began to realize that her frustration in pushing students to generate the standard form had broader implications in terms of understanding how students learned mathematical concepts. Cartwright compared learning the standard form of exponential equations to learning the standard linear form:

I guess like because...it seems like it always comes back. You know. They’re going to compare it to linear. And what amazes me about this unit is that all these students that have struggled with $y = a + bx$, the standard linear form or you know $y = mx + b$, another version of the standard linear form] whatever it is. They’ve struggled through it and all of the sudden they get to here where they have one of them is a linear and they’re able to pull that equation out like instantaneous. And then they struggle with this one. So then I’m thinking well maybe I’m trying to demand perfection before they’re ready and then maybe next year when they see it again. It’ll start to click for them easier. So that’s why I want to kind of back off forcing it. (Interview 10/04/07)

Ms. Karp’s Adaptations

Karp was videotaped enacting the Say It With Symbols unit for the first time and was subsequently interviewed before and after the next two enactments. Karp had an improvisational style of planning, which was reflected in her adaptations. She stated:

I usually read the investigation, read the teacher notes and suggestions, and then basically just try things in the classroom and see what happens, see how the kids pick it up. (Interview 01/10/07)

Karp primarily read the student text, rather than the teacher resources, to focus her instructional planning: “When I look at an investigation, you look at the ACE questions, and you say, ‘What’s this unit about?’ and you basically look at the ACE questions and say, ‘What are they asking?’” (interview 02/13/07). She planned individually, although she and her colleagues shared ideas and informally discussed the unit as they were enacting it.

Karp admitted that her improvisational style of planning caused her problems in the first enactment of the unit because she underestimated the impact of the activities in Investigation One on students’ ability to understand subsequent investigations. She skipped Investigation One in her first
enactment and then included it subsequently and reported that it made a difference in student understanding of Investigations Two and Three. The analysis below focuses on two tasks from Investigation Two that illustrate Karp’s process for adapting tasks in Investigation Two and what she learned from adapting those tasks.

**Description of Tasks.** The first two investigations of the Say It With Symbols (SIWS) unit are designed to help students interpret and write symbolic expressions by investigating order of operations, equivalent expressions, and the distributive property. The Tiling Pools problem is intended to help students consider equivalent expressions. The problem entails finding the number of tiles around the outside of a pool of a given size. For example, the number of 1-foot tiles around a square pool whose sides lengths are each 5 feet would be 24. There are multiple ways to arrive at this number and each way can be distinctly represented numerically or symbolically. The Diving In problem introduces rectangles (pools divided into two rectangular sections) as a visual representation for multiplying a monomial by a binomial (e.g., $3(10 + 5)$ or $4(x + 5)$). The rectangles can be used to show that the area of the whole triangle can be found by adding the areas of the two rectangles or by adding the two parts of the length and multiplying that sum by the width (e.g., $30 + 15 = 3 \times 15$) from first example above). See Figure 1 for an example involving a rectangle diagram.

**Description of Successive Adaptations.** For the Tiling Pools problem, Karp had participated in a professional development project that investigated a videotaped enactment of the Border Problem (Boaler & Humphreys, 2005) in which the teacher had solicited multiple strategies for finding the border and then had the students symbolically represent their numeric expressions. Karp’s first enactment followed this model, at least initially. She solicited four numeric expressions from students and in the next lesson tried to get them to convert the expressions into symbolic forms. This proved to be unsuccessful and Karp ultimately needed to be quite directive to generate the symbolic form, including pointing to numbers or symbols that she wanted students to call out.

In the second enactment of the Tiling Pools problem, Karp focused on developing students’ sense of variable expressions, which she suggested was the reason why students struggled in the first enactment. She chose to use algebra tiles, in part because a colleague decided to use them, but found that these manipulatives did not allow students “to exactly see it model-wise” (interview 10/10/07). The algebra tiles were not utilized in the third enactment. In the third enactment, Karp focused on only two of the students’ numerical strategies. She again tightly structured the development of the symbolic expressions for these strategies and had the students test out the expressions on larger squares (e.g., side length of 29), but did
not explicitly establish their equivalence. She stated that “they’ll accept it’s equal. . . . I don’t think it’s a hundred percent” (interview 10/10/07). See Table 3 for a description of the iterative adaptations of the Tiling Pools problem.

In the first enactment of the Diving In problem, Karp began by using the rectangle model to go through a series of examples with numbers and eventually variables as dimensions of the rectangles. She rapidly progressed to the expression 3(x + 5) without reference to the rectangles, at which point the students became confused, which Karp recognized. She stated that part of this adaptation related to her improvisational style:

I thought they were picking up on it and they were seeing it and I thought, “Oh. They are ready.” And that’s where I say when I go through a unit the first time I get a feel for how much I can push them. I have a tendency to push my students more than feed them. In other words, I let them choke on the water in the pool before I try “Oh maybe we should go here to the shallow water.” My teaching style is more that I don’t have a tendency to enable them at the beginning. I have a tendency to push first to see how far and if I see I pushed too far I’ll step back. And this is a case of I said, “Let’s see how far we can go with it.” (Interview 01/10/07)

She also referred to her prior experiences teaching from conventional curricula:
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<tbody>
<tr>
<td>1</td>
<td>Karp had students produce multiple numeric representations for the pool tiles and then tried to link the numeric representations to their symbolic counterparts.</td>
<td>Unclear, although it followed an example Karp had seen in a professional development project that year in which the group looked at the Border Problem from the Boaler book.</td>
<td>Students provided multiple strategies, which they translated into numeric expressions.</td>
<td>Students struggled to connect numeric expressions to symbolic expressions and to examine connections between symbolic expressions.</td>
<td>Rapid focus on symbolic forms was ineffective. Students need more support to develop symbolic expressions.</td>
</tr>
<tr>
<td>2</td>
<td>Followed the structure of the book more closely. Used algebra tiles to help students understand meaning of variable expressions.</td>
<td>Students had difficulty in the prior enactment understanding variable expressions.</td>
<td>Students used algebra tiles to model the borders.</td>
<td>Students had difficulty generating variable expressions and making connections between tiles and symbolic form.</td>
<td>Algebra tiles did not help students to better understand the meaning of variable expressions.</td>
</tr>
<tr>
<td>3</td>
<td>Karp focusing on two or three strategies to reduce the confusion from having to attend to multiple strategies. She introduced a verbal step between the numeric and symbolic representations, which she learned from another CMP unit.</td>
<td>The algebra tiles didn’t allow students &quot;to exactly see it model-wise.&quot; Introduction of too many strategies in year 1 confused the students.</td>
<td>Focused student attention on a few strategies, developed symbolic expressions for these strategies, and applied them to larger squares.</td>
<td>None reported, although students did not explicitly consider equivalence of symbolic expressions.</td>
<td>Focusing on a few student strategies helps to establish development of symbolic representations.</td>
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</table>
Well, this is the first time I’ve done distributive property with pictures. So my thinking was, like I said, earlier, when I’ve done my honors and accelerated classes or even summer school, I’ve always just done it numerically. The kids get it and they know the pattern. This time, because I’ve gone through the unit for the first time, I didn’t realize how powerful the pictures were for the students. So I will keep those. (Interview 01/10/07)

In subsequent enactments and adaptations, Karp recognized the value of using the rectangles as visual supports and extended the use of the rectangles to develop the inverse skills of expanding and factoring, which are applications of the distributive property and are emphasized on the state test. She ultimately relied on the rectangles to accomplish simultaneous goals:

I don’t want them to just go to the distributive property, I want them to be able to factor. Using the model they’re going to understand factoring better . . . I could teach distributive property faster, can’t teach factoring faster. (Interview 10/10/07)

See Table 4 for a description of the iterative adaptations of the Diving In problem.

Teacher Learning Evident in Adaptations. In both cases, Karp’s adaptations were focused on developing algebraic skills related to the state test. In the Tiling Pools problem, Karp’s adaptations were aimed at helping students translate between numeric and variable expressions and she ultimately deemphasized the equivalence theme that was highlighted in the student text. The use of algebra tiles highlights the focus on translation between numeric and symbolic forms and Karp’s improvisational style, as their use seemed to be motivated by a colleague’s request to borrow a set from her room. In the Diving In problem, Karp’s adaptations were aimed at developing student competency with the skills of factoring and expanding that were part of the state test. However, Karp’s growing realization that the rectangle diagrams were powerful learning tools facilitated adaptations that created connections between the algebraic skills and the distributive property that linked them. Her use of the SIWS unit helped her to recognize and utilize a resource that assisted students in constructing relationships and in justifying their procedures, two activities related to learning with understanding. Although Karp did not utilize the developmental characteristics of the SIWS unit, such as the trajectory implicated in the consecutive tasks, she developed a better understanding of some of the resources in the curriculum to enhance students’ opportunity to learn.

Pless’s Adaptations

Pless was videotaped enacting the new version of the Bits and Pieces II unit for the third time and was subsequently interviewed after the fourth
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<tbody>
<tr>
<td>1</td>
<td>Pushed symbolic representation for distributive property. Moved quickly from rectangle model to several numeric examples to symbolic example.</td>
<td>In the past, with other curricular materials, Karp had developed distributive property through using numerical examples, without the rectangle model.</td>
<td>Focused rapidly on the development of abstract symbolic expressions illustrating the distributive property.</td>
<td>Students struggled with rapid progression to abstract representation of distributive property.</td>
<td>Students could benefit from continued use of rectangle models. Could also use rectangles to develop factoring and expanding.</td>
</tr>
<tr>
<td>2</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>3</td>
<td>Karp gave the student multiple examples using rectangle models to both expand and factor. She varied the information she provided to help them factor and expand and to highlight misconceptions.</td>
<td>The students struggled with going from using the models to quickly focusing on developing symbolic representation from several numeric examples. Karp felt that students were able to use the models for expanding, factoring, and developing the distributive property.</td>
<td>NA</td>
<td>None reported.</td>
<td>Karp learned to use the models to more efficiently teach factoring and expanding, with implicit understanding of distributive property.</td>
</tr>
</tbody>
</table>
enactment. Pless had taught the prior version of Bits and Pieces II three times. Pless reported that she would typically enact a task as closely as possible to the way it was represented in the student text for the first one or two enactments. During those enactments, Pless would attend closely to the ways that students focused on the concepts identified as important in the teacher resources, on how students were able to develop a variety of strategies, and on how students were able to attend to other students’ explanations. Her close attention to student engagement was evidenced on multiple occasions in the interviews by lengthy descriptions of student strategies and of how students reacted to those strategies. Through the detailed observations, Pless identified aspects of a task that had not worked to her satisfaction and would then adapt the tasks to change those specific aspects. Pless stated two primary goals for her adaptations, which were to increase student attention to the central concepts in the task and to develop student dispositions through the emergence and presentation of a variety of strategies.

**Description of Task.** The Tupelo task is designed to help students develop the concepts of equivalent fractions and common denominators. The task involves a diagram of the township of Tupelo, which is divided into two squares each 1-mile wide. These squares are then divided into farms composed of shapes that can be decomposed into rectangles of varying sizes and shapes. The farms can be represented in terms of fourths, eighths, sixteenths, or thirty-seconds, with a few farms requiring the use of thirty-seconds. The students are then asked a series of questions about the size of the farms and different ways they can be combined.

**Description of Successive Adaptations.** The analysis of Pless’s adaptations of the Tupelo task begins with her third enactment, which is the enactment initially observed for this study. Pless reported that in the first two enactments (which happened in the same academic year because Pless taught regular and accelerated sections), she “did it directly out of the book” (interview 10/19/06). In the third enactment, she made two notable adaptations that influenced the “explore” phase of the investigation. First, she divided the questions into two sets, in part because the students were too worried about completing the questions rather than focusing on key ideas or strategies.

[The problem] was so hard for the kids, it felt like it would take forever. They were focusing on the fact that they had like 10 questions to answer and they still hadn’t gotten to the first one. The first question in the book is to find what fraction of a section each person owns. And that in and of itself is a huge part of the [task]. (Interview 10/19/06)

The first two questions on which Pless had the students work involved having students find the fractional size of each farm and then the number
of acres for each section (the student text stated that 640 acres equals 1 square mile). The second adaptation for the Tupelo task involved the ways Pless wanted to focus student activity in relation to these two questions. First, Pless divided the students into pairs of their choosing rather than larger groups. Pless stated:

I let the kids work by themselves or with a partner that they chose because what I found in the past was that as I introduced the problem, kids are automatically thinking about it and they automatically see what they want to do. And some kids want to break it up so that they have all equal parts. Some kids want to go based on all different denominators as their fractions, so I find that when you say they have to work with the group they’re sitting in, you’re taking away what a kid’s thinking is and I get more strategies by letting them do it this way. (Interview 10/19/06)

Second, Pless banned the use of calculators because she wanted students to use visual strategies in which they related the size of farms to each other rather than multiplying their fractional sizes by 640. She found that students could more quickly arrive at the number of acres for each farm and the process reinforced student awareness of relationships between farms, which proved helpful in subsequent tasks and discussions. Third, Pless provided students with scissors and copies of the Tupelo map in case students wanted to cut out the farms or draw on the maps. These adaptations helped to facilitate the emergence of strategies that involved different denominators, which Pless used in the summary phase of the investigation.

Pless made a key adaptation in the summary phase of the investigation, an adaptation designed to help students focus attention on the variety of strategies that emerged from the explore phase and how those strategies related to the concepts of equivalent fractions and common denominators. Pless had been dissatisfied with how students attended to their peers’ explanations in past enactments, especially when the students had used multiple denominators in determining the fractional sizes of the various farms. During one of her classes, Pless noted that some students were adding up pieces visually rather than numerically and this sparked an idea for organizing the summary discussion:

The first couple times I taught it I wasn’t thinking of this problem visually because I don’t think of it that way, so I wasn’t thinking that kids were making sense of this whole map by looking at it visually, and they were adding things up and thinking visually. I didn’t make sense of that until I started watching kids cut up the map and think of it that way. (Interview 11/27/07)

She recounted how this realization helped her to organize the summary:

And, so this I found was the easiest way to summarize it, when I actually had [the farms] cut out and I could place [the cutout of the farms] on top of different things, and I could show them visually. It made so much more sense to me and it helped the
kids see things, the similarities between different sections that were there. I’ve never
done it this way and I have to say, I felt the best about my summary of this than I ever
have before. (Interview 10/19/06)

In the fourth enactment, she further adapted the summary by using her
electronic whiteboard to create and manipulate the farm cutouts. Pless
reported that this made the visual demonstration of the strategies less
cumbersome in terms of her ability to show the relationships and students’
ability to see them, and thus even more effective. See Table 5 for a descrip-
tion of the iterative adaptations of the Tupelo problem.

**Teacher Learning Evident in Adaptations.** Pless’s adaptations increased stu-
dents’ opportunity to learn with understanding relative to her initial enact-
ments. Pless carefully attended to student strategies and to the ways in
which the task features structured student engagement. As a result, she was
able to increase student focus on the central concepts and the number of
strategies that emerged from student exploration. Furthermore, her adap-
tations relative to the summary portion of the lesson facilitated attention to
student strategies in a way that emphasized the main concept under con-
sideration. These adaptations were consistent with the design of CMP and
in fact accomplished the stated purposes of the curriculum better than the
task designs in the student textbooks.

**COMPARING THE LEARNING OF THE THREE TEACHERS**

Each of the teachers developed a greater understanding of the resources
in the respective CMP units as a result of repeated enactments, although
there was some important variation between the teachers. In particular, the
teachers varied in the extent to which their learning resulted in providing
students greater opportunity to learn, the ways in which their adaptations
signaled teacher learning, and the development of curricular knowledge
related to other forms of knowledge. These themes are explored below.

Each of the teachers learned to use the CMP materials to accomplish
their original purposes, but the teachers also showed evidence of
expanded purposes that would enhance students’ opportunity to learn.
Cartwright’s original goal was to help students develop the standard form
of the exponential equation. Through her adaptations, she came to a
better understanding of how the Growing unit facilitated students’ ability
to generate the standard form over several investigations instead of just
the beginning of the unit. This new understanding may allow her to more
effectively build from students’ intuitive reasoning to generate the stan-
dard form. Karp’s original goal was to help students develop fluency with
symbolic manipulations. Her adaptations helped her to realize that the
visual resources in the curriculum would help her students connect the
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<tr>
<td>1–2</td>
<td>Pless reported following the book very closely the first two times.</td>
<td>Pless stated that she tended to follow the book very closely the first couple of times to get a sense of how the tasks connected.</td>
<td>NA</td>
<td>Students not adequately engaged with concepts, each others’ explanations, or generating distinct approaches.</td>
<td>Pless, through observation of students’ strategies, noted unintended distractions, lack of engagement with ideas or explanations, or confusion.</td>
</tr>
<tr>
<td>3</td>
<td>Pless grouped students in pairs, provided scissors and extra maps, and banned calculators.</td>
<td>To promote the emergence of multiple strategies for finding fractional sizes of farms tied to different denominators.</td>
<td>Enhanced emergence of strategies related to different denominators.</td>
<td>None reported. Stable design.</td>
<td>In the past, Pless could tell that students each had their own strategies and she wanted as many of those to emerge as possible.</td>
</tr>
<tr>
<td>3</td>
<td>Pless divided problem into parts A and E (Day 1) and other parts (Day 2).</td>
<td>To focus students on finding fractions for each farm using different denominators and on using those fractions to generate the number of acres (relational understanding).</td>
<td>Students focused on their own strategies for finding fractions, finding the number of acres, and generating number sentences that demonstrated visual relationship between farms, which became the basis of the ensuing discussion.</td>
<td>None reported. Stable design.</td>
<td>Pless learned that students were better able to produce strategies and generate a variety of number sentences, which she was able to build from to discuss equivalent fractions and common denominators.</td>
</tr>
<tr>
<td>3</td>
<td>Summary organization, where Pless divided maps into 16ths and 32nds and cut out individual farms.</td>
<td>To help students to see how strategies related to different denominators. Prior piecemeal approaches were hard to follow.</td>
<td>Enhanced student attention to different strategies and to use of different denominators.</td>
<td>Handling the cutout farms was cumbersome at times.</td>
<td>Pless learned that the parsed maps and cutout farms helped her to focus discussion on the strategies and implications of those strategies in relation to different denominators and equivalent fractions.</td>
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<td>4</td>
<td>Used SmartBoard to slide cutouts of farms to demonstrate relationships between farms and to better visualize their fractional size.</td>
<td>To enhance student attention to the visual aspects of different strategies and to relate them to different denominators and equivalent fractions.</td>
<td>Further enhanced student attention to different strategies and to use of different denominators.</td>
<td>None reported. Stable design.</td>
<td>Pless is satisfied with current design.</td>
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</table>
different skills to each other and to the underlying concept of the distributive property. Pless’s original goals were to help students develop an understanding of equivalent fractions primarily through numeric strategies and to the serial presentation of student strategies. Her adaptations helped students to develop and then connect a variety of strategies to the notion of equivalent fractions.

The process of adapting the curriculum showed that the teachers learned in different ways. Cartwright and Karp adapted the curriculum without first carefully attending to how the instructional unit engaged students in its original form in the student text. Consequently, they both made adaptations that were not effective relative to their respective goals. For both of them, it took several rounds of adaptations before they came to an understanding of how to use the resources more effectively. Pless initially did not make substantive adaptations in her first two enactments, during which she carefully attended to how students engaged with the tasks and with each other’s explanations. As a result of her careful observation, Pless adapted the tasks; furthermore, these adaptations were stable and effective relative to activities associated with learning with understanding.

The teachers showed evidence that their increased knowledge of how to use the resources in the CMP units was connected to the development of PCK and, to a lesser extent, content knowledge. In terms of content, Cartwright developed a greater understanding of the connections between the patterns in the tables and the related symbolic forms, although she showed less flexibility with respect to the graphical representation. In terms of PCK, Cartwright showed an increased understanding of how tabular representations can help students make connections to other forms and more generally the processes by which students develop understanding of abstract notations. Karp’s learning was related to the development of PCK in that she came to a greater understanding of the ways in which students benefit from the use of visual resources. Pless similarly came to understand the impact of visual resources but in addition she learned how to better orchestrate student activity around the use of these resources with respect to multiple approaches, knowledge aligned with conceptions of PCK.

IMPACT OF CONTEXT

Much of the analysis and discussion has been focused on the curricular context and not as much on other features of context. Given that a central claim of the article is that the context in which a teacher uses curriculum materials affects the ways he or she uses them and consequently what is learned, this is an important consideration. However, the data show that features of context played a role in how teachers used the materials. For example, the Brookline teachers (Cartwright and Pless) did not feel pressured to explicitly address test content and were thus free to implement
units in their entirety at the pace recommended in the teachers’ resources. The Brookline teachers also had considerably more training in relation to the units and were thus informed of possible adaptations that had been tested elsewhere, something both teachers mentioned with respect to particular adaptations. Finally, the Brookline teachers planned regularly in their grade-level teams, providing opportunities for collaborative reflection that Karp did not have, possibly contributing to the more systematic nature of their adaptations.

DISCUSSION

The notion of CCK provides insight into the development of curricular knowledge and how it relates to other forms of knowledge that are relevant to the practice of teaching, such as content knowledge and PCK. The learning evidenced by the teachers in relation to the CMP units demonstrated their intricate knowledge of the curriculum and the way it engaged their students. Furthermore, this learning informed their instructional practices and was intertwined with their discussion of content and how best to teach it.

The results show the challenges related to developing an understanding of how to use curricular resources for given purposes, particularly with purposes aligned with learning for understanding. Although the teachers were familiar with other CMP units and even prior versions of the same unit, they found it challenging to enact new materials without encountering difficulties directly related to their unfamiliarity with the task features and sequences in the new unit. They each spoke of the challenges related to first-time implementations and how only by teaching a unit could they truly come to understand it.

There is a need to connect the study of CCK to larger issues related to teacher learning and curriculum implementation. The Confrey et al. (2008) quote echoes the larger need to account for the knowledge necessary to use Standards-based curricula and to relate the development and existence of well-elaborated knowledge components to evaluations of curricula. I would argue that intimate knowledge of curriculum materials would be one of the components that should be included in such evaluations.

A primary limitation of the study is I did not collect data on other measures, such as teacher knowledge, student achievement, or more conventional measures of curriculum implementation. Making these connections would buttress claims as to the relevance of CCK and of curricular knowledge more generally. Given the existence of cases in which Standards-based curricula have been implemented for a number of years, it will hopefully be possible to scale up data collection on teachers’ CCK and connect it to broader data sources and research programs.
ACKNOWLEDGMENTS

I would like to thank my colleagues Nancy Ares, Raffaella Borasi, Corey Drake, and Janine Remillard for providing feedback on earlier versions of this manuscript. This research was supported in part by the National Science Foundation under grant No. DRL-0746573. The opinions expressed herein are those of the author and do not necessarily reflect the views of the National Science Foundation.

NOTES

1. The names of the teachers are pseudonyms. The pseudonyms are those of noted female mathematicians.

2. The names of the districts are pseudonyms.

3. I am characterizing improvisation as one of two things. The first is adapting a task in the process of enactment rather than in the planning stages. The second is adapting a task without much forethought about how the adaptations will work. This characterization is consistent with the definition for “improvise,” which is to “compose and perform or deliver without previous preparation” (retrieved April 23, 2008, from dictionary.com).

REFERENCES


APPENDIX A

Copy of Plan Chart

NAME: _______________ PERIOD: ______________

GROWING, GROWING, GROWING INVESTIGATION 1 DATA TABLE

<table>
<thead>
<tr>
<th>CUT # OR SQUARE #</th>
<th>MAKING BALLOTS INV 1.1</th>
<th>RUBAS PLAN #1 (KING’S PLAN #1)</th>
<th>RUBAS PLAN #2 (KING’S PLAN #2)</th>
<th>RUBAS PLAN #3 (QUEEN’S PLAN)</th>
<th>RUBAS PLAN #4 (ADVISER’S PLAN)</th>
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APPENDIX B

Data Codes

Reading of task/engagement/enactment
   Reading task
      Intent of task
         Development of a concept
         Connections to other tasks in unit
         Development of a procedure, skill, or fact
         Development of a representation
         Affords multiple approaches
         Reflect on learning experiences
         Connect to real life
         Introduce a concept
         Connections between representations
         Ascertain students’ prior experiences
         Create artifact for future reference
      Anticipates student strategies
      Anticipates challenges
   Reading student engagement
      Details of strategies/difficulties
      Evaluation of performance
      Comparison between student ability and difficulty of task
   Reading enactment
      Evaluates enactment
      Describes chronology of enactment

Planning practices

Adaptations

Source for adaptation
   Teacher resources
   Interactions with colleague
   Workshop
   Prior enactment

Nature of adaptation
   Omit (deleted parts of a unit or investigation)
   Supplement (added additional materials not part of student text)
   Revise (changed questions, directions, resources)
   Reorganize (maintained materials but changed order)

Rationale cited for adaptation
   Specific student strategies
   Overall student engagement
   Specific difficulties faced by students
   Preparation for the state test
Function of adaptation
  Enhance emergence of strategies
  Focus on attending to other students’ strategies
  Focus on connections between concepts
  Focus on development of a concept
  Focus on development of a representation
  Focus on development of skill or procedure
States adherence to book
Discusses subsequent adaptation

Teacher learning
  No learning
  Maintained orientation but learned to more effectively use materials in relation to initial purposes
  Changed orientation (beliefs/practices related to mathematics, teaching, learning, use of curriculum materials)
  Related to learning/teaching
  Related to use of curriculum
    Gained new understanding of characteristics of CMP
    Gained new understanding of how to use CMP to help students learn mathematics with understanding